Characterising point sets from intersection numbers

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Joint work with Kris Coolsaet, Alessandro Siciliano and Péter Sziklai

The motivation of this work is the study of maximal partial ovoids and minimal blocking sets of the parabolic quadric $Q(4, q)$. Some results are known, but for $q = p^h$, $h > 1$, p odd, the existence of a minimal blocking set of size $q^2 + 2$ is open, while for $q > 11$ and prime, the existence of a maximal partial ovoid of size $q^2 - 1$ is open.

First we provide a geometrical characterisation of the known examples of maximal partial ovoids of $Q(4, q)$, $q \in \{3, 5, 7, 11\}$, and an alternative characterisation in terms of sharply transitive *subsets* of the group $SL(2, q)$, [1].

Then we explain the translation of both problems into a direction problem in $AG(3, q)$. This setting gave us in [2] a non-existence proof for q non-prime, and can also be used as a setting to study minimal blocking sets of $Q(4, q)$. We explain how these problems translate into a stability, respectively a super stability problem.

Finally, we explain the connection with the cylinder conjecture, stating that any set of q^2 points in $AG(3, q)$, q prime, having $0 \mod q$ points in common with every plane of $AG(3, q)$, is a cylinder, i.e. the points on q lines in one parallel class. Our approach is to "compute" as much coefficients of the Rédei polynomial of the point set as possible, which is done using the intersection numbers (modulo q). At the time of writing this abstract, this is ongoing work, and it seems an approach that is applicable to the three problems addressed here.

References

[1] K. Coolsaet, J. De Beule and A. Siciliano, *The known maximal partial ovoids of* $Q(4, q)$ *of size* $q^2 - 1$, *J. Comb. Des.*, to appear, DOI 10.1002/jcd.21307

[2] J. De Beule and A. Gács, *Complete arcs on the parabolic quadric* $Q(4, q)$, *Finite Fields Appl.*, **14**(1):14–21, 2008.