On the (linear) MDS Conjecture

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(Joint work with Simeon Ball and Ameera Chowdury)

A linear $[n, k, d]$-code C over the finite field $\mathbb{F}_q$, $q = p^h$, is the set of vectors of a k-dimensional subspace of the n-dimensional vector space $V(n, q)$ over $\mathbb{F}_q$. The Hamming distance between two codewords is the number of positions in which they differ. The minimum distance d is the minimum of the distances between all pairs of codewords of C. The Singleton bound for linear codes is the following relation between the parameters k, n and d of a linear code:

$$k \leq n - d + 1\,.$$

If C reaches the Singleton bound, then $n - k = d - 1$. By the fundamental theorem of linear codes, every $d - 1$ columns of the parity check matrix of C are linearly independent. Hence, its columns represent a *set of vectors in* $V(n - k, q)$ *with the property that every subset of size* $n - k$ *is a basis*, and vice versa. The following statement is known as the MDS-conjecture.

> An arc S of a vector space $V(r, q)$, $r < q$, has size at most $q + 1$, except when q is even and $r = 3$ or $r = q - 1$, in which case it has size at most $q + 2$.

The most recent result on this conjecture is the following.

Theorem. (**[2]**) *The MDS-conjecture is true for* $r \leq 2p - 2$.

This theorem was obtained by further elaborating on essential ideas found in [1], in which the MDS-conjecture was shown for $r \leq p$. In this talk we report on recent joint work with Simeon Ball and Ameera Chowdury on the MDS conjecture. The talk will include the essential parts of [1] and [2], together with the ideas of Ameera Chowdury that lead to a short proof of the theorem so far which is based on the use of inclusion matrices. We will particularly focus on the connection between the rank of the matrix, the weight of the vectors in its columnspace and the extendability of an arc, and how evidence based on computational results indicates that this connection might lead to an improvement of the theorem. Finally, if time permits, we will also discuss the connection of this problem with algebraic hypersurfaces.

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References

[1] S. Ball, *On sets of vectors of a finite vector space in which every subset of basis size is a basis*, J. Eur. Math. Soc. (JEMS), 14 (2012), pp. 733–748.

[2] S. Ball and J. De Beule, *On sets of vectors of a finite vector space in which every subset of basis size is a basis II*, Des. Codes Cryptogr., 65 (2012), pp. 5–14.