Old and new results on the MDS-conjecture

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(Joint work with Simeon Ball)

An arc of a projective space $\mathrm{PG}(k-1,q)$ is a set of points $\mathcal{K}$ such that any k points of $\mathcal{K}$ span the whole space. The set

$$S = \{(1, t, t^2, \ldots, t^{k-1}) \mid t \in \mathrm{GF}(q)\} \cup \{(0, \ldots, 0)\}$$

is a set of $q+1$ points in $\mathrm{PG}(k-1,q)$ satisfying the required property. It is well known that linear MDS codes and arcs of projective spaces are equivalent objects. The following conjecture goes back to a series of questions of Segre in [1].

Conjecture 1. *An arc of* $\mathrm{PG}(k-1,q)$*,* $k \leq q$*, has size at most* $q+1$*, unless* q *is even and* $k = 3$ *or* $k = q-1$*, in which case it has size at most* $q+2$*.*

In the talk, we will overview old results and some examples of arcs different from the above one, and discuss the most recent result showing the MDS-conjecture for $k \leq 2p-2$, with $q = p^h$, $h \geq 1$, which is joint work with Simeon Ball.

References

[1] Beniamino Segre. Curve razionali normali e k-archi negli spazi finiti. *Ann. Mat. Pura Appl. (4)*, 39:357–379, 1955.