Non-existence of maximal partial ovoids of $\mathrm{Q}(4, q)$, $q = p^h$, $h > 1$, p odd prime, of size $q^2 - 1$

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(Joint work with András Gács)

We consider the classical generalized quadrangle $\mathrm{Q}(4, q)$. An *ovoid* is a set $\mathcal{O}$ of points of $\mathrm{Q}(4, q)$ such that everly line of $\mathrm{Q}(4, q)$ meets $\mathcal{O}$ in exactly one point. An ovoid of $\mathrm{Q}(4, q)$ contains exactly $q^2 + 1$ points.

A *partial ovoid* of $\mathrm{Q}(4, q)$ is a set $\mathcal{O}$ of points such that everly line of $\mathrm{Q}(4, q)$ meets $\mathcal{O}$ in at most one point. A partial ovoid is *maximal* if it cannot be extended to a larger partial ovoid.

In general, it is known that maximal partial ovoids of $\mathrm{Q}(4, q)$, of size $q^2 + 1 - \delta$, $\delta \leq \sqrt{q}$, can only exist when δ is even [3]. When q is even, maximal partial ovoids of size $q^2 + 1 - \delta$, $0 < \delta < q$, do not exist [1]. Hence the existence of maximal partial ovoids of size $q^2 - 1$ for odd q is open. Furthermore, examples of that size are known for $q = 3, 5, 7$ and 11, while the non-existence is proved for $q = 9$ by computer [2].

We show that maximal partial ovoids of $\mathrm{Q}(4, q)$, $q = p^h$, p odd prime, $h > 1$ do not exist. We translate the problem in the representation $T_2(\mathcal{C})$ and discuss how it becomes a direction problem in $\mathrm{AG}(3, q)$. We discuss how to attack the direction problem using the Rédei polynomial, but we emphasize the geometrical aspects, rather then explaining the technical details.

References

[1] M. R. Brown, J. De Beule, and L. Storme. Maximal partial spreads of $T_2(\mathcal{O})$ and $T_3(\mathcal{O})$. *European J. Combin.*, 24(1):73–84, 2003.

[2] M. Cimrakova and V. Fack. Searching for maximal partial ovoids and spread in generalized quadrangles. *Bull. Belgian Math. Soc. – Simon Stevin.* to appear.

[3] P. Govaerts, L. Storme, and H. Van Maldeghem. On a particular class of minihypers and its applications. III. Applications. *European J. Combin.*, 23(6):659–672, 2002.