Strongly regular graphs and substructures of finite classical polar spaces

Jan De Beule
Ghent University
jdebeule@cage.ugent.be

Let Γ be a strongly regular graph with parameters (n, k, λ, μ). Let A be its adjacency matrix. if $0 < k < n - 1$, then it is well known that, under certain conditions,

- the matrix A has three eigenvalues k, e^+ and e^-;
- the eigenvalue k has multiplicity 1 and its eigenspace is generated by the all-one vector $\mathbf{j}$;
- let V^+, V^- respectively, be the eigenspace corresponding to the eigenvalue e^+, e^- respectively, then $\mathbb{C}^n = \langle \mathbf{j} \rangle \perp V^+ \perp V^-$.

A finite classical polar space is the geometry of totally isotropic sub vector spaces with relation to a sesquilinear or quadratic form on a vector space. Such a geometry is embedded in a projective space and contains points, lines, planes, etc. The point graph of a finite classical polar space is the graph with vertex set the set of points, and two vertices are adjacent if and only if the two points are collinear in the polar space. This graph Γ will be strongly regular and its parameters are well known. A *weighted intriguing set* is a vector orthogonal to either V^- or V^+. In the polar space, such a vector represents a generalization of the geometrical concepts *ovoid* and *tight* sets.

In the talk we report on some recent results on substructures of finite classical polar spaces that were obtained by studying them in a graph theoretical context. These results include: recent results (jointly with J. Bamberg and F. Ihringer) on the non-existence of ovoids of certain finite classical polar spaces, recent results (jointly with J. Demeyer, K. Metsch, and M. Rodgers) on the construction of certain tight sets of the quadric of Klein.