Minimal blocking sets of size $q^2 + 2$ of $Q(4, q)$, q an odd prime, do not exist

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Consider the finite generalized quadrangle $Q(4, q)$, q odd. An *ovoid* is a set $\mathcal{O}$ of points of $Q(4, q)$ such that every line of the quadric contains exactly one point of $\mathcal{O}$. A *blocking set* is a set $\mathcal{B}$ of points of $Q(4, q)$ such that every line of the quadric contains at least one point of $\mathcal{B}$. A blocking set $\mathcal{B}$ is called *minimal* if for every point $p \in \mathcal{B}$, the set $\mathcal{B} \setminus \{p\}$ is not a blocking set.

The GQ $Q(4, q)$ has always ovoids. Recently, it was proved that all ovoids of $Q(4, q)$, q an odd prime, are elliptic quadrics [2]. The main step is proving that all elliptic quadrics intersect the ovoid in 1 mod p points. This is a result for all q odd, $q = p^h$, p prime, but when p is a prime, it is shown in [2] that this result implies that the ovoid is an elliptic quadric. The 1 mod p result for ovoids of $Q(4, q)$ was obtained also in an earlier paper [1].

We consider a minimal blocking set $\mathcal{B}$ of size $q^2 + 2$ of $Q(4, q)$, q odd. Using the same algebraic description of $W(3, q)$ (the dual of $Q(4, q)$) as in [1], and the structure of the multiple-blocked lines with respect to $\mathcal{B}$, derived from a theorem from [3], we obtain the intersectionnumbers of $\mathcal{B}$ with elliptic quadrics contained in $Q(4, q)$). Using again the structure of the multiple-blocked lines with respect to $\mathcal{B}$, intersectionnumbers with hyperbolic quadrics and cones contained in $Q(4, q)$ can also be obtained.

If we suppose now that q is a prime, all these intersectionnumbers are strong enough to exclude the existence of $\mathcal{B}$, using geometrical and combinatorial arguments.

References

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