The smallest sets of points meeting all generators of $\mathrm{H}(2n, q^2)$, $n \geqslant 2$

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It is known that the Hermitian varieties $\mathrm{H}(2n, q^2)$, $n \geqslant 2$, have no ovoids. The question arises how the smallest sets of points meeting every generator look like.

We start with the case $\mathrm{H}(4, q^2)$. Supposing that $\mathcal{K}$ is a minimal blocking set of size $q^5 + \delta$, $\delta \leqslant q^2$, we obtain a contradiction in several steps if $\delta < q^2$. The main step is to consider intersections with hyperplanes. The considered $\mathrm{H}(3, q^2)$ also contain lines as generators and hence the subsets of $\mathcal{K}$ in the corresponding hyerplanes must block these generators and contain a minimal number of points. This leads to $\delta = q^2$ and hence $|\mathcal{K}| \geqslant q^5 + q^2$. If we have equality, a short extra argument shows that $\mathcal{K}$ is the set of points of a cone with base $\mathrm{H}(2, q^2)$ and vertex a point of $\mathrm{H}(4, q^2)$, minus the point itself. This is quite classical and analogue results are known for other polar spaces.

Having the result for $\mathrm{H}(4, q^2)$, it is now possible to characterise the smallest minimal blocking sets of $\mathrm{H}(2n, q^2)$, $n > 2$. Looking in quotient geomtries and using the result for $\mathrm{H}(4, q^2)$, together with standard combinatorial arguments, proves that $\mathcal{K}$ is a *truncated cone* with base a Hermitian curve $\mathrm{H}(2, q^2)$ and vertex an $(n-2)$-dimensional subspace contained in $\mathrm{H}(2n, q^2)$

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