Consider the symmetric group S_n with the Hamming metric. A **permutation code** on n symbols is a subset $C \subseteq S_n$. If C has minimum distance $\geq n-1$, then $|C| \leq n^2 - n$. Equality can be reached if and only if a projective plane of order n exists. Call C **embeddable** if it is contained in a permutation code of minimum distance $n-1$ and cardinality n^2-n. Let $\delta = \delta(C) = n^2 - n - |C|$ be the **deficiency** of the permutation code $C \subseteq S_n$ of minimum distance $\geq n-1$.

We prove that C is embeddable if either $\delta \leq 2$ or if $(\delta^2 - 1)(\delta + 1)^2 < 27(n+2)/16$. The main part of the proof is an adaptation of the method used to obtain the famous Bruck completion theorem for mutually orthogonal latin squares.