

Generalized Hyperfocused Arcs in Finite Desarguesian Planes

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Let $PG(2, q)$ be the projective plane over F_q , the finite field with q elements. A *generalized hyperfocused arc* \mathcal{H} in $PG(2, q)$ is a k -arc with the property that the $k(k-1)/2$ secants can be blocked by a set \mathcal{B} of $k-1$ points not belonging to the arc. Points of the arc \mathcal{H} will be called *white points* and points of the blocking set \mathcal{B} *black*. In case $k > 1$, since every secant to the arc contains a unique black point, the $k-1$ black points induce a factorization, i.e. a partition into matchings, of the white k -arc and k is forced to be even. For $k = 2$, we only have a trivial example: \mathcal{B} consists of a unique point out of \mathcal{H} on the line through the two points of \mathcal{H} .

An non trivial example of generalized hyperfocused arc is any 4-arc of white points with its three black diagonal points and **our main result is that this is the only non trivial example, provided q is an odd prime.**

For q even, there are many examples with all black points on a line; in this case \mathcal{H} is simply called a *hyperfocused arc*. As a consequence of the main result of [3], hyperfocused arcs only exist if q is even. When q is even, a nice result is that generalized hyperfocused arcs contained in a conic are hyperfocused [1]; moreover it is known that there exist examples of generalized hyperfocused arcs which are not hyperfocused [6]. However, although much more is known about hyperfocused arcs, there are still many open problems concerning them [1, 5, 6].

The study of these arcs is motivated by a relevant application to cryptography in connection with constructions of efficient secret sharing schemes [7, 8]. Interestingly, our problem is also related to the S.Ball (strong) cylinder conjecture [2].

References

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