The celebrated Multiplicative Ergodic Theorem of Oseledets shows that under a finite first moment assumption, the product of random real iid matrices behaves asymptotically like the sequence of powers of some fixed positive definite symmetric matrix. In 1989 Vadim Kaimanovich showed that this property can be expressed in purely geometric terms using the symmetric space associated to $\mathrm{GL}_n(\mathbb{R})$. This observation lead to the notion of a 'regular sequence' in a symmetric space, and Kaimanovich gave a complete characterisation of these sequences in terms of spherical and horospheric coordinates in the symmetric space. As a consequence of this characterisation Kaimanovich obtained a Multiplicative Ergodic Theorem for noncompact semisimple real Lie groups with finite centre, generalising the original theorem of Oseledets.

In this talk we will discuss a $p$-adic analogue of this story. In this setting the symmetric space is replaced by the affine building of the $p$-adic group. We define regular sequences in affine buildings, and give a characterisation of these sequences in terms of analogues of the spherical and horospherical coordinates from the real theory. We then discuss applications to a Multiplicative Ergodic Theorem for Lie groups defined over $p$-adic fields, and random walks on affine buildings. This is joint with W. Woess.