$\mathcal{S} = (\mathcal{P}, \mathcal{B}, \mathrm{I})$ is a finite generalized quadrangle if:

(i) Each point is incident with $1 + t$ $(t \geqslant 1)$ lines and two distinct points are incident with at most one line

(ii) Each line is incident with $1 + s$ points $(s \geqslant 1)$ and two distinct lines are incident with at most one point

(iii) If $x$ is a point and $L$ is a line not incident with $x$, then there is a unique pair $(y, M) \in \mathcal{P} \times \mathcal{B}$ for which $x \,\mathrm{I}\, M \,\mathrm{I}\, y \,\mathrm{I}\, L$

points:

(i) the points of $\mathrm{PG}(3,q) \setminus \mathrm{PG}(2,q)$

(ii) the hyperplanes $X$ of $\mathrm{PG}(3,q)$ for which $|X \cap \mathcal{O}| = 1$

(iii) one new symbol $(\infty)$

lines:

(a) the lines of $\mathrm{PG}(3,q)$ not contained in $\mathrm{PG}(2,q)$ meeting $\mathcal{O}$ in a unique point

(b) the points of $\mathcal{O}$

incidence: the point $(\infty)$ is not incident with a line of type (a) and incident with every line of type (b). Incidence between the other points and lines is the inherited incidence of $\mathrm{PG}(3,q)$

- A *spread* of a GQ is a set of lines which partition the point set

- A *partial spread* of a GQ is a set of lines such that every point of the GQ lies on at most one line.

- A *maximal partial spread* is a partial spread which is not contained in a bigger partial spread.

# Known Results (G. Tallini)

- if $q$ is odd, $\mathrm{Q}(4,q)$ has no spreads, and if $S$ is a partial spread then $|S| \leqslant q^2 - q + 1$
- if $q$ is even, $q \geqslant 4$ and $S$ is a maximal partial spread, $|S| < q^2 - \frac{q}{2}$

A $\{f, m; N, q\}$ minihyper is a pair $(F, w)$, where $F$ is subset of the point set of $\mathrm{PG}(N, q)$ and $w$ is a weight function $w : \mathrm{PG}(N, q) \to \mathbb{N} : x \mapsto w(x)$, satisfying

(i) $w(x) > 0 \iff x \in F$

(ii) $\sum_{x \in F} = f$

(iii) min $\{\sum_{x \in H} w(x) \| H \in \mathcal{H}\} = m$, where $\mathcal{H}$ is the set of hyperplanes of $\mathrm{PG}(N, q)$.

**Theorem 1.** *(Blokhuis, Storme, Szőnyi) Let* $B$ *be a blocking set in* $\mathrm{PG}(2,q)$*,* $q = p^h$*, prime of size* $q+1+c$*. Let* $c_2 = c_3 = 2^{\frac{-1}{3}}$ *and* $c_p = 1$ *for* $p > 3$

*(i) If* $q = p^{2d+1}$ *and* $c < c_p q^{\frac{2}{3}}$ *then* $B$ *contains a line*

*(ii) If* $4 < q$ *and* $q$ *is a square an* $c < c_p q^{\frac{2}{3}}$*, then* $B$ *contains a line or a Baer subplane*

**Theorem 2.** *(Govaerts, Storme) Let* $(F,w)$ *be a* $\{\delta(q+1), \delta; N, q\}$*-minihyper,* $q > 2$*, satisfying* $0 \leqslant \delta < \epsilon$*, where* $q+\epsilon$ *is the size of the smallest non trivial blocking set in* $\mathrm{PG}(2,q)$*. Then* $w$ *is the weight function induced on the points of* $\mathrm{PG}(N,q)$ *by a sum of* $\delta$ *lines.*

**Theorem 3.** *Let $q$ be even and $S$ be a partial spread of $T_2(\mathcal{O})$ of size $q^2 + 1 - \delta$ which covers $(\infty)$. $S$ can be extended if $\delta < \epsilon$, with $q + \epsilon$ the size of the smallest non trivial blocking set in $\mathrm{PG}(2, q)$*

**Theorem 4.** *Let $q$ be even and square, and $S$ be a partial spread of $T_2(\mathcal{O})$ of size $q^2 + 1 - \delta$ which covers $(\infty)$. $S$ can be extended if $\delta < c_p q^{\frac{2}{3}}$*

**Theorem 5.** *Let $q$ be even and $S$ be a maximal partial spread of $T_2(\mathcal{O})$ of size $q^2 + 1 - \delta$ which does not cover $(\infty)$. $S$ cannot be maximal if $\delta < q$.*