Translation hyperovals and $\mathbb{F}_2$-linear sets of pseudoregulus type

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1. Research question
2. The set of directions $\mathcal{D}$ is a linear set
3. The set $\mathcal{D}$ is of pseudoregulus type
4. Hyperoval in the André/Bruck-Bose plane $\mathrm{PG}(2, q^k)$
5. Generalization Barwick and Jackson

3 Article Barwick and Jackson

Theorem

Consider PG$(4, q)$, *q even, $q > 2$. Let C be a set of q^2 affine points, called C-points and consider a set of planes called C-planes which satisfies the following:*

- *Each C-plane meets C in a q-arc.*
- *Any two distinct C-points lie in a unique C-plane.*
- *The affine points, not in C, lie on exactly one C-plane.*
- *Every plane which meets C in at least three points either meets C in exactly four points or is a C-plane.*

Then there exists a regular spread S in Σ_∞ s. t. in the ABB plane $P(S) \equiv$ PG$(2, q^2)$, *the C-points, together with two extra points on l_∞, form a translation hyperoval of* PG$(2, q^2)$.

4 Problem

Techniques used in article Barwick and Jackson

They use

- the existence of a design, isomorphic to an affine plane, of which they later need to use the parallel classes.
- the Klein correspondence to represent lines in $\mathrm{PG}(3, q)$ in $\mathrm{PG}(5, q)$.

Both techniques cannot be extended to q^k, $k > 2$.

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5 Main Theorem

Theorem

Let $\mathcal{Q}$ be a set of q^k affine points in $\mathrm{PG}(2k, q)$, $q = 2^h$, $h \geq 2$ determining a set $\mathcal{D}$ of $q^k - 1$ directions in the hyperplane at infinity $H_\infty = \mathrm{PG}(2k-1, q)$. Suppose that every line at infinity has 0, 1, 3 or $q-1$ points in common with the point set $\mathcal{D}$. Then

(1) *$\mathcal{D}$ is an $\mathbb{F}_2$-linear set of pseudoregulus type.*

(2) *There exists a Desarguesian spread $\mathcal{S}$ in H_∞ such that in the André/Bruck-Bose plane $\mathcal{P}(\mathcal{S}) \cong \mathrm{PG}(2, q^k)$, the points of $\mathcal{Q}$ together with 2 extra points on ℓ_∞ form a translation hyperoval in $\mathrm{PG}(2, q^k)$.*

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7 $\mathcal{D}$ is a linear set

Lemma

Let $P_0, P_1, P_2 \in \mathcal{Q}$ so that $P_1'P_2'$ is a 3-secant to $\mathcal{D}$, then the plane in $\mathrm{PG}(2kh, 2)$ spanned by $\tilde{P}_0$, $\tilde{P}_1$ and $\tilde{P}_2$ is contained in $\tilde{\mathcal{Q}}$.

$\mathcal{D}$ is a linear set

Proof by Lemma 3 and induction argument.

9 $\mathcal{D}$ is linear set of pseudoregulus type

Definition

Let S be a scattered $\mathbb{F}_q$-linear set of $\mathrm{PG}(2k-1, 2^h)$ of rank kh, $h, k \geq 2$. We say that S is of *pseudoregulus type* if

1. there exist $m = \frac{2^{hk}-1}{2^h-1}$ pairwise disjoint lines of $\mathrm{PG}(2k-1, 2^h)$, say $s_1, s_2, \ldots, s_m$, such that

$$|S \cap s_i| = 2^h - 1 \quad \forall i = 1, \ldots, m,$$

2. there exist exactly two $(k-1)$-dimensional subspaces T_1 and T_2 of $\mathrm{PG}(2k-1, 2^h)$ disjoint from S such that $T_j \cap s_i \neq \emptyset$ for each $i = 1, \ldots, m$ and $j = 1, 2$.

11 Construction of $(k-1)$-spread in H_∞

Lemma

There exists a Desarguesian $(k-1)$-spread $\mathcal{S}$ in $\mathrm{PG}(2k-1, q)$, so that

- $T_1, T_2 \in \mathcal{S}$,
- *every other element of $\mathcal{S}$ has one point in common with $\mathcal{D}$.*

12 Hyperoval in $\mathrm{PG}(2, q^k)$

Theorem

The set $\mathcal{Q}$, together with T_1 and T_2, defines a translation hyperoval in $\mathcal{P}(\mathcal{S}) \cong \mathrm{PG}(2, q^k)$.

13 Other direction

The set of affine points of a translation hyperoval in $\mathrm{PG}(2, q^k)$, $q = 2^h$, $k \geq 2$.

$\downarrow$ ABB construction

Set $\mathcal{Q}$ of q^k affine points in $\mathrm{PG}(2k, q)$ whose set of determined directions $\mathcal{D}$ is an $\mathbb{F}_2$-linear set of pseudoregulus type.

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15 Generalization Barwick and Jackson

Theorem

Consider $\mathrm{PG}(2k, q)$*, q even,* $q > 2$*. Let* $\mathcal{C}$ *be a set of* q^k *affine points, called* $\mathcal{C}$*-points and consider a set of planes called* $\mathcal{C}$*-planes which satisfies the following:*

(A1) *Each* $\mathcal{C}$*-plane meets* $\mathcal{C}$ *in a q-arc.*

(A2) *Any two distinct* $\mathcal{C}$*-points lie in a unique* $\mathcal{C}$*-plane.*

(A3) *The affine points that are not in* $\mathcal{C}$ *lie on exactly one* $\mathcal{C}$*-plane.*

(A4) *Every plane which meets* $\mathcal{C}$ *in at least* 3 *points either meets* $\mathcal{C}$ *in* 4 *points or is a* $\mathcal{C}$*-plane.*

Then there exists a Desarguesian spread $\mathcal{S}$ *in* Σ *such that in the Bruck-Bose plane* $\mathcal{P}(\mathcal{S}) \cong \mathrm{PG}(2, q^k)$*, the* $\mathcal{C}$*-points, together with* 2 *extra points on* ℓ_∞ *form a translation hyperoval in* $\mathrm{PG}(2, q^k)$*.*

Thank you very much for your attention.