

Noncommutative Geometry and \mathbb{F}_1

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Abstract

There is already a touch of noncommutativity present in most approaches to commutative algebraic geometry over \mathbb{F}_1 via natural actions of the Bost-Connes algebra. This is a canonical object associated to the group of all roots of unity and the action of $\text{Gal}(\mathbb{Q}^{ab}/\mathbb{Q})$ on them, two main ingredients in most approaches to \mathbb{F}_1 .

We will explore generalizations of Soulé's and other approaches to \mathbb{F}_1 -geometry to the truly noncommutative realm. We will give an example in which the role of the roots of unity and their Galois action is replaced by that of Grothendieck's "dessins d'enfants" and the action of the absolute Galois group $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$.

We will use techniques from noncommutative algebraic geometry, in particular work by Kontsevich and Soibelman, to define and calculate substitutes of Habiro's new topology on roots of unity and its associated functions in this wilder noncommutative setting.

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