$\mathbb{F}_1$-Geometry and its Applications

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Abstract

The original motivation to consider a geometry over the *field* $\mathbb{F}_1$ *with one element* is based on a remark by Jacques Tits in a paper from 1957: it seemed that analogies between geometry over finite fields and combinatorial geometry could find an explanation in such a theory. In the early nineties, Manin, Smirnov, et al., connected this viewpoint to arithmetic problems like the abc-conjecture and the Riemann hypothesis. From then on, the field with one element attracted more and more attention.

In the last decade, there appeared more than a dozen different definitions of a geometry over $\mathbb{F}_1$. Though a solution of the mentioned arithmetic problems is out of sight yet, it became clear that $\mathbb{F}_1$-geometry connects to many other fields of mathematics and that it is the natural formulation to understand problems with a combinatorial flavour in an algebro-geometric language. This touches stable homotopy of spheres; tropical geometry and log-schemes; non-archimedean analytic spaces and Arakelov theory; reductive groups and buildings; canonical bases and cluster algebras; moduli of quiver representations and quiver Grassmannians.

In this talk, I will explain some of the ideas of $\mathbb{F}_1$-geometry and review its achievements so far.

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