

## $\mathbb{F}_1$ -Analytic Functions and Borger's Descent

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### Abstract

The existence of algebraic geometry over  $\mathbb{F}_1$ , a nonexistent “field with one element”, was tentatively suggested by Jacques Tits in 1957. In the last ten years or so the interest of mathematicians in this idea was steadily growing, and by now it has developed into an active (if small) research field. Not one but about a dozen versions of  $\mathbb{F}_1$ -geometry appeared. In this talk I will survey two recent ideas that emerged and were tested in these studies:

1. Analytic geometry over  $\mathbb{F}_1$ , suggested by the speaker and based on the notion of multivariable Habiro rings, that (in the one variable version) was initially introduced in totally different contexts as generating series for certain knot and other topological invariants.
2. Borger's geometry, based on the idea that the “descent data” on a commutative ring coming from an affine  $\mathbb{F}_1$ -scheme are given by the lambda-structure upon this ring. In particular, I will discuss Borger's descent for Habiro rings and its relation to Witt rings.

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