

Foundations of Absolute Arithmetic

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Abstract

In a paper which was published in 1957, Tits made a seminal and provocative remark which alluded to the fact that through a certain analogy between the groups $\mathbf{GL}_n(q)$ — q a prime power — and the symmetric group \mathbf{S}_n , one should interpret the latter as a Chevalley group “over the field of characteristic one”, \mathbb{F}_1 .

In this introductory lecture, I will survey some of Tits’s ideas from a combinatorial point of view, as well as mention the linear algebra in characteristic one which was developed by Kapranov and Smirnov.

Then I will describe Anton Deitmar’s geometry of commutative monoids, which is a fundamental attempt to introduce an algebraic geometry over \mathbb{F}_1 which lives deeply “below $\mathrm{Spec}(\mathbb{Z})$ ”.

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