Hilbert's Tenth Problem and Elliptic Curves

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Abstract

Hilbert's Tenth Problem was the following question: find an algorithm which, given a polynomial in any number of variables with integer coefficients, tells whether or not it has a solution over the integers. It was shown in 1970 by Y. Matiyasevich, building on earlier work by M. Davis, H. Putnam and J. Robinson, that this question has a negative answer: such an algorithm does not exist. In other words: diophantine equations are undecidable.

Many authors have generalized this undecidability from the integers to other rings and fields. Perhaps surprisingly, elliptic curves play an important role in many of the proofs. In 1978, Denef was the first to use an elliptic curve in his proof of the undecidability of $\mathbb{R}(t)$. In the first part of the talk, we will give the complete proof of this. In the second part of the talk, we will give an idea of the proof of the undecidability of $\mathbb{C}((t))(x)$, a result by the second speaker. This proof also uses elliptic curves, but in a different way.