Lecture series:

Chromatic products in algebra and geometry

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Ultraproducts of rings, and variant constructions, can be used to prove some deep theorems in (pure) algebra and algebraic geometry. One such application, which I have already lectured about in Leuven a few years back, is tight closure theory in characteristic zero. In this lecture series, I will describe further applications, obtained by similar principles.

All lectures take place in building B of K.U. Leuven Campus Arenberg III, Celestijnenlaan 200, Leuven.

Tentative schedule:

Lecture 1. Ultraproducts and uniform bounds.

(a) Introduction of ultraproducts: logical, algebraic, and geometric constructions;

(b) Lefschetz hulls: flat embeddings of rings inside ultraproducts,

(c) Application 1: uniform Artin Approximation

(d) Application 2: uniform bounds in algebra for linear properties

Monday June 21st, 14:00-16:30 (including coffee break), in room B02.18.

Lecture 2. Cataproducts and classification of singularities

(a) Cataproducts as generalizations of standard parts: Noetherianity recovered

(b) Application 1: singularities up to similarity form a Polish space via the jet metric

(c) Application 2: characterization of ring theoretic properties

Wednesday June 23rd, 14:00-16:30 (including coffee break), in room B00.16.

Lecture 3. Protoproducts and homological conjectures

(a) Protoproducts as bounded subrings of ultraproducts

(b) Ax-Kochen principle in higher dimensions

(c) Application 1: asymptotic homological conjectures in mixed characteristic

(d) Application 2: etale bounds for non-linear properties

Date and room TBA.

Lecture 4. Ultra-Frobenius and rational singularities

(a) Ultra-cohomology and its action of the ultra-Frobenius

(b) Application 1: quotient singularities are rational

(c) Application 2: vanishing theorems on quotients of Fano varieties

Date and room TBA.