

Linear systems on generic $K3$ surfaces

Joint work with Antonio Laface

Cindy De Volder

Let S be a generic $K3$ surface (i.e. $\text{Pic } S$ is generated by one element H), let P_1, \dots, P_r be general points on S and fix r natural numbers m_1, \dots, m_r .

By \mathcal{L} we denote the linear system of curves on S in $|dH|$ ($d > 0$) which have multiplicities at least m_i at P_i for all $i = 1, \dots, r$.

Define the virtual dimension v of \mathcal{L} as $\dim |dH| - \sum r_i m_i (m_i + 1)/2$ and its expected dimension by $e = \max\{v, -1\}$.

Obviously, the dimension l of \mathcal{L} is always bigger or equal to e , and if $l > e$ we say that the system is *special*.

In this lecture we introduce two conjectures concerning the non-speciality of linear systems through fat points (in general position) on generic $K3$ surfaces, we prove the equivalence of these two conjectures and we give some results in favor of them.