An introduction to Hilbert's Tenth Problem and Recursively Enumerable Subsets of $\mathbb{F}_q[Z]$

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Abstract

Hilbert's Tenth Problem is the following: find an algorithm which can decide whether or not a given a Diophantine equation (a polynomial equation over the integers in any number of variables) has a solution. The famous result by Davis, Putnam, Robinson and Matiyasevič (1970) states that such an algorithm *does not exist*. Actually, they proved something stronger, namely the equivalence of recursively enumerable and Diophantine sets.

Obviously, this question can be asked for other rings besides the integers. I plan to introduce some basic concepts regarding Hilbert 10, with focus on general rings.

In the second half of my talk, I will give an overview of my recent result that every recursively enumerable subset of $\mathbb{F}_q[Z]$ is Diophantine over $\mathbb{F}_q[W, Z]$. In other words, if we take a recursively enumerable subset of $\mathbb{F}_q[W, Z]$, where all elements happen to depend only on the variable Z, then that set is Diophantine.

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