Applications of exponential sums to systems of homogeneous forms defined over *p*-adic fields. (Work of David Leep)

Let $\mathcal{F} = \{F_1, \ldots, F_r\}$ be a system of r homogeneous forms of degree d in s variables. Denote with $\nu_l(\mathcal{F})$ the number of solutions of the system \mathcal{F} in $(\mathbb{Z}_p/p^l\mathbb{Z}_p)^s$. Using exponential sums a formula expressing $\nu_l(\mathcal{F})$ in terms of the quantities $\nu_l(a_1F_1 + \cdots + a_rF_r)$, where (a_1, \ldots, a_n) runs through all primitive vectors in $\mathbb{Z}_p/p^l\mathbb{Z}_p)^s$, is given.

We also explain the relation of the problem to different questions in the theory of quadratic forms and forms of higher degree over local fields and over fields finitely generated over local fields.