Hodge theory and the definition of the Hodge-Deligne polynomial
(Jan Schepers)

On a compact Kähler manifold X (e.g. a non-singular projective variety over $\mathbb{C}$), the cohomology with complex coefficients $H^k(X,\mathbb{C})$ can be written in a natural way as a direct sum $\bigoplus_{p+q=k} H^{p,q}(X)$, with $H^{p,q}(X) = \overline{H^{p,q}(X)}$. To deal with possibly singular quasi-projective varieties, Deligne developed the theory of mixed Hodge structures, which gives a more difficult decomposition of the cohomology. The data of the mixed Hodge structure can then be used to define the Hodge-Deligne polynomial of an algebraic variety, which is an important generalized Euler characteristic.