Stringy invariants of singular algebraic varieties

PhD thesis
Jan Schepers

For nonsingular projective complex algebraic varieties, the *Hodge numbers* are well known classical invariants. Batyrev has tried to generalize these invariants for a class of singular projective varieties as follows (the allowed singularities are called Gorenstein canonical). He defined a rational function in two variables, the *stringy E-function*, by using data from a resolution of singularities. He showed that this function does not depend on the chosen resolution, and thus it is an invariant of the singular variety. When this function is a polynomial, he defined the *stringy Hodge numbers*, essentially as the coefficients of this polynomial. For a nonsingular projective variety, the stringy Hodge numbers coincide with the classical Hodge numbers. Moreover, they have a lot of analogous properties. But there is one problem: classical Hodge numbers are certainly nonnegative, since they are dimensions of subspaces of the de Rham cohomology, but for stringy Hodge numbers the nonnegativity is not at all clear. It was conjectured by Batyrev, and it is the subject of the thesis.

We have been able to prove Batyrev's conjecture for varieties with certain mild isolated singularities. A nice corollary is the proof for the conjecture for threefolds in full generality (for surfaces the conjecture is trivially true). Moreover, the proofs suggested that a more general question for the power series development of not necessarily polynomial stringy E-functions is worth further investigation. However, we found an example that gives a negative answer to this question. In our opinion this provides some evidence that Batyrev's conjecture might not be true. In the thesis, we also compute explicit formulae for the contribution of so called A-D-E singularities to the stringy E-function in arbitrary dimension.