Linear codes arising from geometrical structures

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A *linear* $[n, k]$*-code* C is a k-dimensional subspace of $V(n, q)$, where $V(n, q)$ denotes the n-dimensional vector space over the finite field $\mathbb{F}_q$. A *generator matrix* G is a $k \times n$-matrix generating C. The columns of G can be regarded as projective points in $\mathrm{PG}(k-1, q)$. In this way, we obtain a connection between linear codes and projective geometry which allows us to study certain coding theoretical problems in an equivalent geometrical way. Perhaps the most famous example of this equivalence is that of MDS codes and *arcs* in a projective space.

In this talk, we explain how in the code generated by the incidence matrix of points and lines in $\mathrm{PG}(2, q)$, codewords of small weight are related to another well-known geometrical concept: blocking sets. This code was studied in the book of Assmus and Key [1], by McGuire and Ward [5] and Chouinard [2]. We will present improvements to these results, recently found by Gács, Szőnyi, and Weiner, and generalisations to higher dimensions, published in [4]. The situation is different for the dual of this code: in general, not much is known, and for q odd, determining the minimum weight is still an open problem.

The dual of the code of polar spaces was studied in [6]. For q even, codewords in this code correspond to certain *sets of even type*. This connection enabled us to study codewords of small and large weight. Moreover, we found a gap in the weight enumerator of the code of $\mathcal{Q}(4, q)$, q even, using a new result on covers of $\mathcal{Q}(4, q)$.

We show how the *functional code* arises from attaching a linear code to finite geometrical objects. The codewords of the smallest weights of the functional codes related to quadrics have been determined by Edoukou, Hallez, Rodier and Storme [3]; they correspond to quadrics having a large intersection with a fixed quadric.

References

[1] E.F. Assmus, Jr. and J.D. Key. Designs and their codes. *Cambridge University Press*, 1992.

[2] K. Chouinard. Weight distributions of codes from planes (PhD Thesis, University of Virginia) (August 1998).

[3] F.A.B. Edoukou, A. Hallez, F. Rodier, and L. Storme. On the small weight codewords of the functional codes $C_2(Q)$, Q a non-singular quadric. *J. Pure Appl. Algebra*, submitted.

[4] M. Lavrauw, L. Storme, P. Sziklai, and G. Van de Voorde. An empty interval in the spectrum of small weight codewords in the code from points and k-spaces of $\mathrm{PG}(n, q)$. *J. Combin. Theory, Ser. A* **116** (2009), 996–1001.

[5] G. McGuire and H. Ward. The weight enumerator of the code of the projective plane of order 5. *Geom. Dedicata* **73** (1998), 63–77.

[6] V. Pepe, L. Storme, and G. Van de Voorde. On codewords in the dual code of classical generalised quadrangles and classical polar spaces. *Discrete Math.*, to appear.