Functional codes arising from quadrics and Hermitian varieties

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Definition 1 *([4]) Let $\mathcal{X}$ be a fixed algebraic variety in* $\mathrm{PG}(n, q)$*, with point set* $\{P_1, \ldots, P_N\}$*, where we normalize the coordinates of the points with respect to the leftmost non-zero coordinate. Let $\mathcal{F}_h$, resp. $\mathcal{F}_{Herm}$, be the set of the homogeneous polynomials in the variables $X_0, \ldots, X_n$, of degree h, resp. of the form $(X_0 \ \ldots \ X_n)A(X_0^{\sqrt{q}} \ \ldots \ X_n^{\sqrt{q}})^t$ with A a Hermitian matrix, over the finite field $\mathbb{F}_q$, q square. The functional codes $C_h(\mathcal{X})$ and $C_{Herm}(\mathcal{X})$ are given by*

$$C_h(\mathcal{X}) = \{(f(P_1), \ldots, f(P_N)) \mid f \in \mathcal{F}_h\} \cup \{0\}, \qquad h \in \mathbb{N} \cup \{Herm\}.$$

In general, it is easy to determine the dimension of these functional codes. Most research about these codes hence focuses on the minimum distance. Mostly, the algebraic variety $\mathcal{X}$ is chosen to be a non-singular quadric $\mathcal{Q}$ or a non-singular Hermitian variety $\mathcal{H}$.

The functional codes $C_2(\mathcal{Q})$ and $C_{Herm}(\mathcal{H})$ are the functional codes that were studied first (e.g in [2]). Recently, also the codes $C_2(\mathcal{H})$ and $C_{Herm}(\mathcal{Q})$ were studied (see [1, 3]). In this talk, we present the main results about these four functional codes.

References

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