LDPC codes derived from finite geometries

Peter Vandendriessche

Department of Mathematics, Ghent University
Krijgslaan 281 - S22, 9000 Ghent

Definition 1 *Let $\mathbb{F}$ be a finite field and let H be an $m \times n$ matrix of rank $n - k$ over $\mathbb{F}$. The* linear $[n, k]$-code *defined by H is the k-dimensional subspace of $\mathbb{F}^n$ consisting of all vectors which have a zero inproduct (in $\mathbb{F}$) with all rows of H. The matrix H is called the* parity check matrix *of this code. This code is called* LDPC *if its defining matrix has much more zero than nonzero elements.*

Originally introduced by Gallager [1], LDPC ('low density parity check') codes are used frequently these days due to their excellent empirical performance under belief-propagation/sum-product decoding. In some cases, their performance is even near to the Shannon limit [2], but only for very large block lengths. For such large codes, however, even the low-complexity iterative decoding algorithms for LDPC codes are too complex for practical usage.

Definition 2 *A* finite geometry $(\mathcal{P}, \mathcal{B})$ *consists of a finite set $\mathcal{P}$, the elements of which we call* points*, and a finite collection $\mathcal{B}$ of subsets of $\mathcal{P}$, the elements of which we call* blocks*. The* incidence matrix *of a finite geometry $(\mathcal{P}, \mathcal{B})$ is the $|\mathcal{P}| \times |\mathcal{B}|$ matrix H, in which we label the rows by the points of $\mathcal{P}$ and the columns by the blocks of $\mathcal{B}$, and which has the entries 1 or 0 depending on whether the corresponding point is contained in the corresponding block or not. Since 0 and 1 are elements of every field, this technique can be used to define codes over any field $\mathbb{F}$.*

Recently, there has been a growing interest in LDPC codes constructed from finite geometries, since some of these achieve reasonable performance on much shorter block lengths. From a purely theoretical point of view, studying these codes has also proven to be fruitful for understanding the underlying geometry. Hence, for both practical and theoretical reasons, there has been a growing interest in the properties and internal structure of these codes. In this talk, we give an overview of the recent results on these finite geometry LDPC codes.

References

[1] R.G. Gallager, Low density parity check codes, *IRE Trans. Inform. Theory* **8** (1962), 21–28.

[2] D.J.C. MacKay and R.M. Neal, Near Shannon limit performance of low density parity check codes, *Electron. Lett.* **32** (1996), 1645–1646.