OM representations of prime divisors of curves over finite fields

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Let k be a finite field and let $f(x,y) \in k[x,y]$ be a polynomial, which is irreducible and separable over $k(x)$. Let C/k be the projective smooth curve defined as the normalization of the projective closure of the affine curve $f(x,y)=0$. Let $p\colon C \to \mathbb{P}^1$, be the separable morphism determined by $(x,y) \mapsto x$.

Given any point $u \in \mathbb{P}^1$, there is an algorithm, due to Jesús Montes, that computes what we call *OM representations* of the prime divisors of $p^{-1}(u)$ [2, 3]. This algorithm is based on pioneering work of Øystein Ore and Saunders MacLane at the beginning of the last century, and it has a low complexity and an excellent practical performance. These OM representations provide extremely fast algorithms for the resolution of several tasks on the curve C, like the computation of the genus, local conductors and local indices, integral closures of subrings of the function field, reduction of divisors, bases of Riemann-Roch spaces, etc.

In this talk we shall present these OM representations, and we shall discuss a concrete application: an algorithm of Jens-Dietrich Bauch for the computation of the genus [1].

References

[1] J.-D. Bauch, *Genus computation of global function fields*, in preparation.

[2] J. Guàrdia, J. Montes, E. Nart, *Higher Newton polygons in the computation of discriminants and prime ideal decomposition in number fields*, J. Théor. Nombres Bordeaux **23** (2011), no. 3, 667–696.

[3] J. Guàrdia, J. Montes, E. Nart, *Newton polygons of higher order in algebraic number theory*, Trans. Amer. Math. Soc. **364** (2012), no. 1, 361–416.

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