# Direction problems in affine spaces, related problems, and applications

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Let $\mathrm{AG}(n,q)$ denote the $n$-dimensional affine space over the finite field $\mathbb{F}_q$. Consider an arbitrary set of points $U \subset \mathrm{AG}(n,q)$. A point at infinity $P$ is called a *direction determined by* $U$ if and only if there exist two points in $U$ determining an affine line containing $P$ as point at infinity. Clearly all points at infinity are directions determined by $U$ if $|U| > q^{n-1}$. Denote by $U_D$ the set of directions determined by $U$. The following research questions are of our interest.

- What are the possible sizes of $U_D$ given that $|U| = q^{n-1}$? What is the possible structure of $U_D$?
- What are the possible sets $U$, $|U| = q^{n-1}$, given that $U_D$ (or its complement at infinity) or only $|U_D|$ is known?
- Given that a set $N$ of directions is not determined by a set $U$, $|U| = q^{n-1} - \epsilon$, can $U$ be extended to a set $U'$, $|U'| = q^{n-1}$, such that $U'$ does not determine the given set $N$?

Many results are known for $n = 2$. We focus on particular results, their application in the theory of blocking sets of finite Desarguesian projective planes, and the use of so-called *lacunary polynomials* to obtain results. Then we describe more recent results for $n \geq 3$, of which some of them are joint work with A. Gács, P. Sziklai and M. Takáts. Finally, we present connections between direction problems in $\mathrm{AG}(3,q)$ and the *Cylinder conjecture*.

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