A bound for the maximum weight of a linear code

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(Joint work with Aart Blokhuis)

Let C be a linear code over $\mathbb{F}_q$ of length n, dimension k, minimum weight d and maximum weight m.

The main result presented in this talk wll be the following.

If $m \geq n-d+1$ then, for all $\epsilon \in \mathbb{N}$ and $\gamma \geq n-d$, the coefficient of $X^{\gamma q - m + \epsilon}$ in

$$(1+X)^{-m}(1+X^p)^{\gamma q/p}$$

is divisible by q^{k-1}.

In the case that $q = p$ is a prime, this leads to the bound

$$m \leq (n-d)p - e(p-1),$$

where $e \in \{0, 1, \ldots, k-2\}$ is maximal with the property that

$$\binom{n-d}{e} \neq 0 \pmod{p^{k-1-e}}.$$

Thus, if C contains a codeword a length n then $n \geq d/(p-1) + d + e$.

This is probably best compared with the Griesmer bound which states that for a linear code of length n, dimension k and minimum distance d over $\mathbb{F}_q$,

$$n \geq \sum_{i=0}^{k-1} \lceil d/q^i \rceil.$$

We shall make such a comparison and also translate the bounds obtained to bounds for (n,t)-arcs and t-fold blocking sets of hyperplanes in $\mathrm{AG}(k-1, q)$, the Desarguesian affine space.

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