# From curves to surfaces: a walk through Algebraic Geometry codes

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Reed–Solomon codes are well-studied linear codes that have optimal parameters but suffer from one limitation: their length is bounded by the cardinality of the field. Algebraic Geometry (AG) codes allow to overcome the main drawback of Reed–Solomon codes while still benefiting from good parameters. AG codes also provided the first example of codes beating the Gilbert–Varshamov bound, and they were extensively studied since this breakthrough. Even if the original construction of AG codes relies on the use of algebraic curves, it can be extended to algebraic varieties of any dimension.

In this talk, I will present and compare AG codes from curves and surfaces. After introducing the construction of AG codes, I will survey the study of their parameters, their implementation [1] and decoding [4, 6], their *local* properties [2, 5], and their use in quantum error-correction [3].

## References

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