On the structure of 3-nets embedded in a projective plane

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We investigate finite 3-nets embedded in a projective plane over a (finite or infinite) field of any characteristic p. Such an embedding is *regular* when each of the three classes of the 3-net comprises concurrent lines, and *irregular* otherwise. It is *completely irregular* when no class of the 3-net consists of concurrent lines. We are interested in embeddings of 3-nets which are irregular but the lines of one class are concurrent. For an irregular embedding of a 3-net of order $n \geq 5$ we prove that, if all lines from two classes are tangent to the same irreducible conic, then all lines from the third class are concurrent. We also prove the converse provided that the order n of the 3-net is smaller than p. In the complex plane, apart from a sporadic example of order $n = 5$ due to Stipins (2004), each known irregularly embedded 3-net has the property that all its lines are tangent to a plane cubic curve. Actually, the procedure of constructing irregular 3-nets with this property works over any field. In positive characteristic, we present some more examples for $n \geq 5$ and give a complete classification for $n = 4$.

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