On representations of the symplectic groups on exterior powers of vector spaces

Bart De Bruyn

Let $V$ be a vector space of dimension $2n \geq 4$ over a field $F$ and let $k \in \{1, \ldots, 2n-1\}$. Let $f$ be a nondegenerate alternating bilinear form on $V$ and let $Sp(V, f) \cong Sp_{2n}(F)$ denote the symplectic group associated with $(V, f)$. The symplectic group $Sp(2n, F)$ has a natural representation $\mathcal{R}_1$ on the $k$-th exterior power $\bigwedge^k V$ of $V$ and in case that $k \leq n$ also a natural representation $\mathcal{R}_2$ on the $(\binom{2n}{k} - \binom{2n}{k-2})$-dimensional subspace of $\bigwedge^k V$ generated by all vectors $\vec{v}_1 \wedge \vec{v}_2 \wedge \cdots \vec{v}_k$ for which $<\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k>$ is totally isotropic. In the talk, we will discuss some properties of these representations, invoking only elementary linear algebra. E.g., we will give a sketch of the proof of the irreducibility criterion for the representation $\mathcal{R}_2$, a result originally due to Premet, Suprunenko and Adamovich. The motivation for the above study arose from some specific problems regarding projective embeddings of symplectic dual polar spaces and projective Grassmannians.