On representations of the symplectic groups on exterior powers of vector spaces

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Let V be a vector space of dimension $2n \geq 4$ over a field $\mathbb{F}$ and let $k \in \{1, \ldots, 2n-1\}$. Let f be a nondegenerate alternating bilinear form on V and let $Sp(V,f) \cong Sp_{2n}(\mathbb{F})$ denote the symplectric group associated with (V,f). The symplectic group $Sp(2n,\mathbb{F})$ has a natural representation $\mathcal{R}_1$ on the k-th exterior power $\bigwedge^k V$ of V and in case that $k \leq n$ also a natural representation $\mathcal{R}_2$ on the $\binom{2n}{k} - \binom{2n}{k-2}$-dimensional subspace of $\bigwedge^k V$ generated by all vectors $\bar{v}_1 \wedge \bar{v}_2 \wedge \cdots \bar{v}_k$ for which $< \bar{v}_1, \bar{v}_2, \ldots, \bar{v}_k >$ is totally isotropic. In the talk, we will discuss some properties of these representations, invoking only elementary linear algebra. E.g., we will give a sketch of the proof of the irreducibility criterion for the representation $\mathcal{R}_2$, a result originally due to Premet, Suprunenko and Adamovich. The motivation for the above study arose from some specific problems regarding projective embeddings of symplectic dual polar spaces and projective Grassmannians.