

Witt designs, Mathieu groups, involutions and a tower of graphs

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27 October 2006
SEMINAR INCIDENCE GEOMETRY

When studying transitive decompositions of the Johnson graphs we encountered a very intriguing graph on 165 vertices and valency 8 whose automorphism group is M_{11} . It can be constructed on the set of 3-subsets of an 11-set or on a set of 4-subsets of a 12-set. When trying to understand the two definitions of this graph we were led to look at the involutions of M_{11} . This in turn revealed that the graph was the third member of a tower of graphs defined on a conjugacy class of involutions of the groups A_5 , $PSL(2, 11)$, M_{11} and M_{12} with the first graph being the line graph of the Petersen graph and the fourth graph being the Johnson graph $J(12, 4)$.