Generalized Ovals in PG$(3n-1,q)$, with q odd

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Abstract

In 1954 Segre proved that every oval of PG$(2,q)$, with q odd, is a nonsingular conic. The proof relies on the "Lemma of Tangents". A generalized oval of PG$(3n-1,q)$ is a set of q^n+1 $(n-1)$-dimensional subspaces of PG$(3n-1,q)$, every three of them generate PG$(3n-1,q)$; a generalized oval with $n=1$ is an oval. The only known generalized ovals are essentially ovals of PG$(2,q^n)$ interpreted over GF(q). If the oval of PG$(2,q^n)$ is a conic, then we call the corresponding generalized oval classical. Now assume q odd. Several properties of classical generalized ovals will be stated. Further we obtain a strong characterization of classical generalized ovals in PG$(3n-1,q)$ and an interesting theorem on generalized ovals in PG$(5,q)$, developing a theory in the spirit of Segre's approach. So for example a "Lemma of Tangents" for generalized ovals is obtained. We hope such an approach will lead to a classification of all generalized ovals in PG$(3n-1,q)$, with q odd.