Linear (blocking) sets

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Linear sets in PG(n, q) were introduced by Lunardon, and are characterised as the projection of a canonical subgeometry [2]. They are frequently used, but not (yet) well studied. Natural questions about their intersection remain largely unanswered. The first part of this talk deals with linear sets: after an extensive introduction dealing with three different points of view on linear sets, we discuss the intersection of a subline with a linear set [1].

A *small minimal k-blocking set B* in PG(n, q) is a set of less than $3(q^k + 1)/2$ points, intersecting every $(n-k)$-space, such that no proper subspace of B has this property. Linear sets gave rise to the first examples of small minimal blocking sets that were not of Rédei-type [3], disproving the wide-spread belief that all small minimal blocking sets are of Rédei-type. This construction led to the *linearity conjecture* for blocking sets, stating that every small minimal k-blocking set is linear (see [4]). In the second part of the talk, we use the result on the intersection of a subline and a linear set to reduce the linearity problem to the plane (at least for large p); more precisely, we show that if the linearity conjecture holds for blocking sets in the plane, then it also holds for 1-blocking sets in PG(n, p^h), where $p > h + 3$. Similar ideas can be used to show that the truth of the linearity conjecture for 1-blocking sets in PG(n, p^h) implies the truth of the linearity conjecture for k-blocking sets in PG(n, p^h) for large p.

References

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[4] P. Sziklai. On small blocking sets and their linearity. *J. Combin. Theory, Ser. A*, **115 (7)** (2008), 1167–1182.