Linear (blocking) sets

Geertrui Van de Voorde

Linear sets in $\text{PG}(n, q)$ were introduced by Lunardon, and are characterised as the projection of a canonical subgeometry [2]. They are frequently used, but not (yet) well studied. Natural questions about their intersection remain largely unanswered. The first part of this talk deals with linear sets: after an extensive introduction dealing with three different points of view on linear sets, we discuss the intersection of a subline with a linear set [1].

A small minimal $k$-blocking set $B$ in $\text{PG}(n, q)$ is a set of less than $3(q^k + 1)/2$ points, intersecting every $(n - k)$-space, such that no proper subspace of $B$ has this property. Linear sets gave rise to the first examples of small minimal blocking sets that were not of Rédei-type [3], disproving the wide-spread belief that all small minimal blocking sets are of Rédei-type. This construction led to the linearity conjecture for blocking sets, stating that every small minimal $k$-blocking set is linear (see [4]). In the second part of the talk, we use the result on the intersection of a subline and a linear set to reduce the linearity problem to the plane (at least for large $p$); more precisely, we show that if the linearity conjecture holds for blocking sets in the plane, then it also holds for 1-blocking sets in $\text{PG}(n, p^h)$, where $p > h + 3$. Similar ideas can be used to show that the truth of the linearity conjecture for 1-blocking sets in $\text{PG}(n, p^h)$ implies the truth of the linearity conjecture for $k$-blocking sets in $\text{PG}(n, p^h)$ for large $p$.

References


