Eigenvalue techniques applied to polar spaces

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In order to obtain purely geometric results, we will consider concepts from algebraic combinatorics like association schemes, Bose-Mesner algebras, inner distributions, eigenvalues,.....
Bose and Shimamoto first introduced the notion of a D-class *association scheme* as a pair $(\Omega, \{R_0, \dots, R_D\})$, with Ω a finite set and $\{R_0, R_1, \dots, R_D\}$ a set of symmetric relations on Ω such that the following axioms hold:

(i) R_0 is the identity relation,
(ii) $\{R_0, R_1, \dots, R_D\}$ is a partition of Ω^2,
(iii) there are *intersection numbers* p_{ij}^k such that for $(x, y) \in R_k$, the number of elements z in Ω for which $(x, z) \in R_i$ and $(z, y) \in R_j$ always equals p_{ij}^k.

If one orders the elements of Ω as $\omega_1, \dots, \omega_{|\Omega|}$, one can associate with each relation R_i a square $|\Omega| \times |\Omega|$-matrix A_i over the real numbers, with:

$$\begin{cases} (A_i)_{rs} = 1 \text{ if } (\omega_r, \omega_s) \in R_i, \\ (A_i)_{rs} = 0 \text{ if } (\omega_r, \omega_s) \notin R_i. \end{cases}$$

The *Bose-Mesner algebra* of the association scheme is the space of matrices spanned by these $D + 1$ matrices. The eigenvalues that we need, are the eigenvalues of these matrices. We can use these to obtain properties of certain subsets S of the set Ω.
Prior knowledge of these concepts should not be required, as they will be introduced during the talk, together with examples. After that, we will discuss examples of applications to incidence geometry, by considering the association scheme on generators in a polar space. In particular, we will obtain both bounds for and properties of partial spreads (subsets of generators, all mutually disjoint), which can be seen in this context as cliques of one relation of this association scheme.
The talk should be accessible to anyone who is familiar with the classical polar spaces
(i.e. $Q^-(2n + 1, q), Q(2n, q), Q^+(2n + 1, q), W(2n + 1, q)$ and $H(n, q^2)$), including students.