Group representations in incidence geometry

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A representation of a group G is a homomorphism ρ from G to $GL(V)$ with V a complex vector space, where $GL(V)$ denotes the group of invertible linear transformations of V. Representations can be decomposed as a direct sum of indecomposable subrepresentations, which is unique under some assumptions.
If a group G acts transitively on a set Ω, we can consider the vector space $\mathbb{C}\Omega$, a basis of which is indexed by elements of Ω. For every $g \in G$, we can now let $\rho(g)$ denote the element of $GL(\mathbb{C}\Omega)$ permuting the basis elements accordingly. This is a *permutation representation* of G.
It is our goal to consider automorphisms groups G of incidence structures, which act on the objects of the differents types (points, lines, planes,...). Each type yields a permutation representation of the same automorphism group G. Examples of the incidence structures under consideration are generalized polygons, projective spaces, polar spaces.... What these structures have in common, is that they are *buildings*, and hence a (finite) Coxeter group can be associated with them. It turns out that understanding the decompositions of the permutation representations of this (relatively) small group already provides a good deal of information. This will allow us to discuss many important types of substructures in geometry: Cameron-Liebler line classes, tight sets, (partial) m-systems,.... Some old results will be proved in an alternative way in this setting, as well as some new results.
In the first part of the talk, a general discussion of representations of groups, decompositions, homomorphisms... will be given, including several simple examples. No prior knowledge of representation theory, Coxeter groups or buildings should be required.