Rota's Conjecture and partial fields

Stefan van Zwam

SEMINAR INCIDENCE GEOMETRY
UGENT
June 12, 2009

A matroid, sometimes called "combinatorial pregeometry", consists of a finite set, and a partition of its subsets into "independent" and "dependent", subject to some axioms. Matroids arise in many contexts, and an important example is a set of vectors in some vector space, where the "independent" subsets are those that are linearly independent.

We distinguish two ways to reduce a matroid: "deletion" and "contraction". In the example of a set of vectors, deleting a vector is obvious, and contracting v consists of the following two operations. First, we project all vectors onto the subspace orthogonal to v. Then we delete v. A matroid obtained from M through deletions and contractions is called a "minor" of M. This relation defines a partial order on matroids.

A matroid M is representable over a field F if there exists a set of vectors over F having exactly the dependencies prescribed by M. The class of F-representable matroids is closed under taking minors, and we wish to characterize this class by the set of minor-minimal matroids that are not in the class. It is known that, for F in $\{GF(2), GF(3), GF(4)\}$, this set is finite. Rota's Conjecture is that this set is finite for every finite field.

Now consider a class of matroids representable over a set of fields $F_1, ..., F_k$. Such a class can be characterized as the set of matroids representable over a more general algebraic structure, a "partial field". Quite aside from their intrinsic interest, partial fields may be helpful in a proof of Rota's Conjecture. In this talk I show, among other things, how they helped to get us very close to a proof of the conjecture for $GF(5)$.