A matroid, sometimes called "combinatorial pregeometry", consists of a finite set, and a partition of its subsets into "independent" and "dependent", subject to some axioms. Matroids arise in many contexts, and an important example is a set of vectors in some vector space, where the "independent" subsets are those that are linearly independent.

We distinguish two ways to reduce a matroid: "deletion" and "contraction". In the example of a set of vectors, deleting a vector is obvious, and contracting $v$ consists of the following two operations. First, we project all vectors onto the subspace orthogonal to $v$. Then we delete $v$. A matroid obtained from $M$ through deletions and contractions is called a "minor" of $M$. This relation defines a partial order on matroids.

A matroid $M$ is representable over a field $F$ if there exists a set of vectors over $F$ having exactly the dependencies prescribed by $M$. The class of $F$-representable matroids is closed under taking minors, and we wish to characterize this class by the set of minor-minimal matroids that are not in the class. It is known that, for $F$ in $\{GF(2), GF(3), GF(4)\}$, this set is finite. Rota’s Conjecture is that this set is finite for every finite field.

Now consider a class of matroids representable over a set of fields $F_1,...,F_k$. Such a class can be characterized as the set of matroids representable over a more general algebraic structure, a "partial field". Quite aside from their intrinsic interest, partial fields may be helpful in a proof of Rota’s Conjecture. In this talk I show, among other things, how they helped to get us very close to a proof of the conjecture for $GF(5)$. 