

Irsee 2017

In/out the **jungle**
of differences

Marcos Bweatti

Università di Perugia **ITALY**

DIFFERENCE FAMILIES

G additive group of order v ; $H \leq G$

$\mathcal{F} = \{B_1, B_2, \dots, B_m\}$ is a (G, H, k, λ) -DF if

- $B_i \in \binom{G}{k} \quad \forall i$

- $\bigcup_{i=1}^m \Delta B_i = \lambda \text{ times } G \setminus H$

THM $H = \{0\} \Rightarrow \text{Orb } B_1 \cup \dots \cup \text{Orb } B_m$ is
the block-collection of a $2-(v, k, \lambda)$ design

THM $|H| = k \Rightarrow \text{Orb } B_1 \cup \dots \cup \text{Orb } B_m \cup \lambda(\text{Orb } H)$ is
the block-collection of a $2-(v, k, \lambda)$ design



Example

$\mathcal{F} = \{ \{1, 3, 9\}, \{2, 5, 6\} \}$ is a disjoint $(13, 3, 1)$ -DF

	1	3	9
1	•	11	5
3	2	•	7
9	8	6	•

	2	5	6
2	•	10	9
5	3	•	12
6	4	1	•

Remark (MB 19??) G abelian $\Rightarrow \Delta B = \Delta(-B) \Rightarrow$

If $\mathcal{F} = \{B_1, \dots, B_m\}$ is an abelian $(G, H, k, 1)$ -DF,

then $\mathcal{F}' = \{(-1)^{i_1} B_1, \dots, (-1)^{i_m} B_m\}$ is also a

$(G, H, k, 1)$ -DF $\forall (i_1, \dots, i_m) \in \{0, 1\}^m$.

The 2^m designs generated by these DFs are pairwise different

Let us say that \mathcal{F} and \mathcal{F}' are **SIMILAR**

PROBLEM

How many "non-isomorphic" difference families are contained in the "similarity class" of \mathcal{F} ?

Some computer results giving the number S of non-isomorphic DFs in the similarity class of a given $(v, k, 1)$ -DF:

v	k	S
19	3	2
25	3	4
49	3	10
55	3	6
37	4	2

v	k	S
49	4	8
61	4	4
61	5	4
91	6	2
151	6	7

At least 7
non-isomorphic
 $S(2, 6, 151)$

[MB - Beugmoli, New designs by changing... the signs, 2013]

I reevaluated the work with my student in view of
[V. Kecadimac and R. Vlahovic 2016
[New quasi-symmetric designs by **Kramer-Mesner method**]

Here 3 NEW **cyclic** $S(2,3,217)$ are found via **heavy computer work**

I noticed that the same designs could be obtained as follows:
take the only known $(217, 3, 1)$ -DF [Bagchi & Bagchi 1989] which is

$\{B, 8B, 15B, 29B, 190B\}$ with $B = \{0, 1, 37, 67, 88, 92, 149\}$;
then the 3 designs by Kecadimac/Vlahovic are those generated by

$$\{B, 8B, 15B, -29B, 190B\}$$

$$\{B, 8B, 15B, -29B, -190B\}$$

$$\{B, 8B, -15B, -29B, 190B\}$$

Applying

THM (Bays-Lambassy 1931 / Muzychuk 1999)

The number of isomorphisms between two **cyclic** designs with v points is at most $\varphi(v)$

We get:

If there exists a $(v, k, 1)$ -DF with m blocks,
then there exist $\left\lfloor \frac{2^m}{\varphi(v)} \right\rfloor$ pairwise non-isomorphic

cyclic 2 -($v, k, 1$) designs

On (q, k, λ) Disjoint Difference Families

Necessarily,

$$1 \leq \lambda \leq k-1$$

Results for $\lambda = 1$:

Dimitz-Rodney 1997: COMPLETE SOLUTION in the cyclic case for $k=3$

Some constructions for RADICAL $(\mathbb{F}_q, k, 1)$ -DDFs with k ODD:

Bose 1939; Wilson 1972; Goig 1990; MB 1995

$(q, k, k-1)$ -DDFs

Many papers \nRightarrow many results

The cosets of k -th roots of 1 in F_q^* is a $(F_q, k, k-1)$ -DDF
[Wilson, JNT 1972]

$$q_1 \equiv q_2 \equiv \dots \equiv q_m \equiv 1 \pmod{k} \Rightarrow \\ \Rightarrow \exists \text{ a } (F_{q_1} \oplus F_{q_2} \oplus \dots \oplus F_{q_m}, k, k-1)\text{-DDF}$$

Jungnickel DM 1978

Fireino DM 1991

Boykett SJDM 2001

Cay-Zeng-Helleseth-Tang-Yang IEEE 2013 (with ZDBF language)

Li-Wei-Ge DCC 2017

$p \equiv 1 \pmod{k}$ for any prime p in $V \implies$

\exists a $(\mathbb{Z}_V, k, k-1)$ -DDF

for V a prime power: Phelps ADM 1987
Furino DM 1991

for general V : Ding-Wang-Xiong IEEE 2014
with an involved proof using
the language of ZDBFs.

A quick proof could be obtained using

Phelps/Furino DDFs + DIFFERENCE MATRICES

ALL previous results on $(G, k, k-1)$ -DDFs
are immediate corollaries of the following
reformulation of a result that I found in
[J.R. Clay, Nearings: Geneses and Applications. Oxford 1992]

THM

F Frobenius group with kernel G and complement A
of order $k \Rightarrow$ the A -orbits on G form a $(G, k, k-1)$ -DDF

LAST REMARK

THM [MB, JCD 1998]

\exists a $(\mathbb{Z}_p, k, \lambda)$ -DF for any prime p in $v \Rightarrow$

\exists a (G, k, λ) -DF in ANY G of order v

Replacing DF with DDF in the hypothesis,
we can do the same in the thesis!

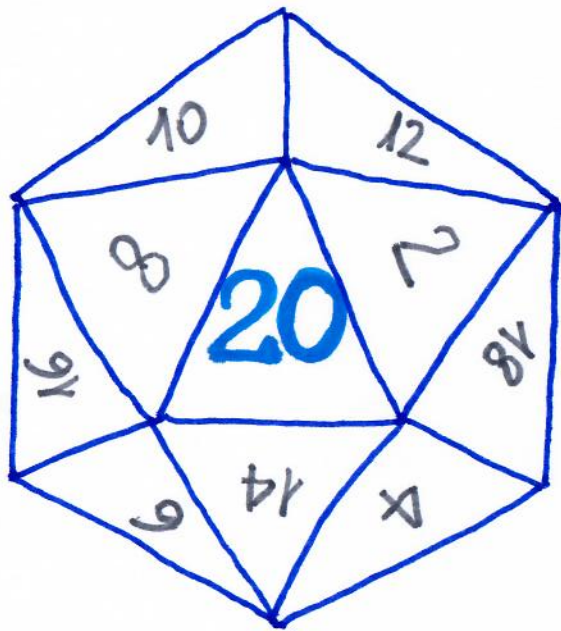
COROLLARY

$p \equiv 1 \pmod{k}$ for any prime p in $v \Rightarrow$

\exists a $(G, k, k-1)$ -DDF in ANY G of order v

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