Congress Irsee

Improvement of the sunflower bound

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September 2017

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2 Definition Subspace code

- Codewords are subspaces
- Constant or mixed dimension subspace code
- Subspace distance:

$$d(U, V) = \dim(U + V) - \dim(U \cap V)$$

- Speeds up the transmission of information through a wireless network

3 Sunflower

t-intersecting constant dimension subspace codes

Codewords are k-dimensional subspaces, where distinct codewords intersect in a t-dimensional subspace.

Sunflower

All codewords pass through the same t-dimensional subspace.

4 Sunflower Bound

Sunflower Bound

Large t-intersecting constant dimension subspace codes are sunflowers if

$$|C| > \left(\frac{q^k - q^t}{q-1}\right)^2 + \left(\frac{q^k - q^t}{q-1}\right) + 1$$

- This bound is probably too high.

5 Decrease the sunflower bound

- Codewords are three-dimensional subspaces, where distinct subspaces intersect in a point. ($k = 4, t = 1$).
- The sunflower bound is in this case

$$\left(\frac{q^4 - q^1}{q-1}\right)^2 + \left(\frac{q^4 - q^1}{q-1}\right) + 1 = q^6 + 2q^5 + 3q^4 + 3q^3 + 2q^2 + q + 1$$

Purpose

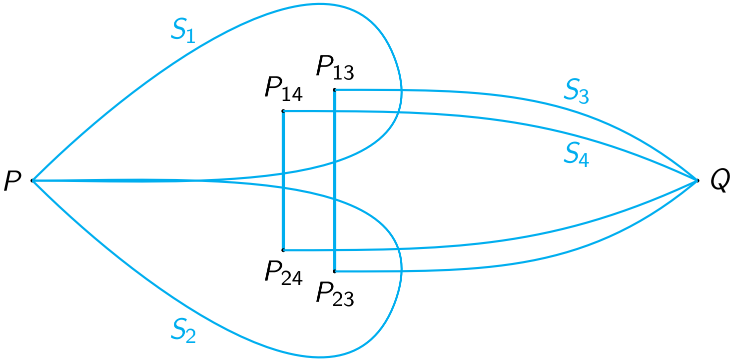
Decreasing the sunflower bound, for $k = 4, t = 1$, to q^6.

Summary of the proof

Suppose we have a collection of q^6 solids that pairwise intersect in a point, and doesn't form a sunflower.
We look for a contradiction.

- All solids are located in a $7, 8$ or 9-dimensional space
- Entropy and Shearer's Lemma
- Looking for a contradiction by countings

All solids are located in a $7, 8$ or 9-dimensional space.



Entropy and Shearer's lemma

Entropy

The entropy $H(X)$ of a random variable $X = \{x_1, x_2, \ldots, x_n\}$ measures the quantity of uncertainty in X.

Shearer's lemma

Suppose n different points in $\mathbb{F}^3$, that have n_1 projections on the XY-plane, n_2 projections on the XZ-plane, and n_3 projections on the YZ-plane. Then $n^2 \leq n_1 n_2 n_3$.

Shearer's lemma in our Situation

Suppose L is a collection of lines, and $P_1, P_2, \ldots, P_{k+1}$ are $k+1$ linearly independent points in $PG(k, q)$ so that every point P_i is located on at most l lines of L. Suppose n is the number of points in $PG(k, q)$ so that $PP_i, \forall i \in [1, \ldots, k+1]$ is a line of L. Then we find that $n \leq l^{k/(k-1)} + l$.

Looking for a contradiction by countings

- Double counting of the collection of points with specific characteristics.
- An inequality in function of q.
- A contradiction when q becomes large enough.

Theorem

Every 1-intersecting constant dimension subspace code of 4-spaces having size at least q^6 is a sunflower.

11 Further research

- Generalisation: Decreasing the sunflower bound when the codewords are k-dimensional subspaces that pairwise intersect in a point.
- Generalisation: Decreasing the sunflower bound when the codewords are k-dimensional subspaces that pairwise intersect in t-dimensional subspace.

Thank you very much for your attention.