Relative m–ovoids of elliptic quadrics

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(joint work with A. Cossidente)

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Definitions

- T. Penttila, J. Williford, New families of Q–polynomial association schemes, *J. Combin. Theory Ser. A* 118 (2011), 502-509.

Hermitian surface $\mathcal{H}(3, q^2)$,

$\Sigma \simeq \mathrm{PG}(3, q)$, $\Sigma \cap \mathcal{H}(3, q^2) = \mathcal{W}(3, q)$

ℓ generator of $\mathcal{H}(3, q^2)$, $|\mathcal{W}(3, q) \cap \ell| \in \{0, q + 1\}$

$P \in \mathcal{H}(3, q^2) \setminus \mathcal{W}(3, q)$, q generators of $\mathcal{H}(3, q^2)$ disjoint from $\mathcal{W}(3, q)$ through P

Relative hemisystem

A *relative hemisystem* of $\mathcal{H}(3, q^2)$, q even, is a set $\mathcal{S}$ of generators of $\mathcal{H}(3, q^2)$ disjoint from $\mathcal{W}(3, q)$ such that each point of $\mathcal{H}(3, q^2) \setminus \mathcal{W}(3, q)$ lies on $q/2$ generators of $\mathcal{S}$.

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Known results

Theorem (Bamberg, Lee, 2017)

Let $\mathcal{R}$ be a nontrivial relative m–cover of Γ (with respect to Γ'), then q is even and $\mathcal{R}$ is a relative hemisystem (that is $m = q/2$).

Corollary

A nontrivial relative m–cover of $\mathcal{H}(3, q^2)$ with respect to a symplectic subgeometry $\mathcal{W}(3, q)$ is a relative hemisystem.

- M. Lee, *The m–covers and m–ovoids of generalised quadrangles and related structure*, Master Thesis, UWA, 2016.

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Definitions

Klein correspondence

$$\mathcal{H}(3, q^2) \longrightarrow \mathcal{Q}^-(5, q),$$

$$\mathcal{W}(3, q) \longrightarrow \mathcal{Q}(4, q),$$

relative m–cover of $\mathcal{H}(3, q^2)$ (with respect to $\mathcal{W}(3, q)$) $\longrightarrow$ $\mathcal{R} \subset \mathcal{Q}^-(5, q) \setminus \mathcal{Q}(4, q)$

every generator of $\mathcal{Q}^-(5, q)$, not contained in $\mathcal{Q}(4, q)$ meets $\mathcal{R}$ in m points.

Relative m–ovoid

A relative m–ovoid of $\mathcal{Q}^-(2n+1, q)$ (with respect to a $\mathcal{Q} := \mathcal{Q}(2n, q) \subset \mathcal{Q}^-(2n+1, q)$) is a subset $\mathcal{R}$ of points of $\mathcal{Q}^-(2n+1, q) \setminus \mathcal{Q}$ such that every generator of $\mathcal{Q}^-(2n+1, q)$ not contained in $\mathcal{Q}$ meets $\mathcal{R}$ in precisely m points.

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Properties

a relative m–ovoid is *nontrivial* if $0 < m < q^{n-1}$,

a *relative hemisystem* is a relative m–ovoid with the same size as its complement in $\mathcal{Q}^-(2n+1,q) \setminus \mathcal{Q}$.

$\mathcal{R}$ relative m–ovoid of $\mathcal{Q}^-(2n+1,q)$ (with respect to $\mathcal{Q}$),

$|\mathcal{R}| = mq(q^n - 1)$.

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Properties

P point of $\mathcal{Q}^-(2n+1, q)$,

iv) $|P^\perp \cap \mathcal{R}| = m(q^n+1) - q^n$, if $P \in \mathcal{R}$,

v) $|P^\perp \cap \mathcal{R}| = m(q^n+1)$, if $P \notin \mathcal{Q} \cup \mathcal{R}$,

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Theorem

Let $\mathcal{R}$ be a nontrivial relative m–ovoid of $\mathcal{Q}^-(2n+1, q)$, then q is even and $\mathcal{R}$ is a relative hemisystem.

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Corollary

$\mathcal{R}$ relative hemisystem of $\mathcal{Q}^-(2n+1, q)$ (with respect to $\mathcal{Q}$),
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Relative hemisystem of $\mathcal{Q}^-(4n+1, q)$

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- T. Penttila, J. Williford, New families of *Q*–polynomial association schemes, *J. Combin. Theory Ser. A* 118 (2011), 502-509.
- A. Cossidente, Relative hemisystems on the Hermitian surface, *J. Algebraic Combin.* 38 (2013), 275-284.
- A. Cossidente, A new family of relative hemisystems on the Hermitian surface, *Des. Codes Cryptogr.* 75 (2015), 213-221.
- A. Cossidente, F. Pavese, Intriguing sets of $\mathcal{W}(5, q)$, q even, *J. Combin. Theory Ser. A* 127 (2014), 303-313.

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$\phi : \mathrm{PG}(2n-1, q^2) \longrightarrow \mathrm{PG}(4n-1, q)$

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$\mathcal{W}(2n-1, q^2)$ symplectic polar space of $\mathrm{PG}(2n-1, q^2)$

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$\pi \subset \mathcal{W}(2n-1, q^2) \longrightarrow \phi(\pi) \subset \mathcal{W}(4n-1, q)$

$\mathcal{D} = \{\phi(P) \mid P \in \mathcal{W}(2n-1, q^2)\}$

line–spread of $\mathcal{W}(4n-1, q)$

$\mathcal{W}(4n-1, q)$ isomorphic to $\mathcal{Q} := \mathcal{Q}(4n, q)$

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Relative hemisystem of $\mathcal{Q}^-(4n+1, q)$, $n \geq 2$

K–orbits on points

K has two orbits $\mathcal{O}_1, \mathcal{O}_2$ on points of $\mathcal{Q}^-(4n+1, q) \setminus \mathcal{Q}$,

$$|\mathcal{O}_1| = |\mathcal{O}_2|$$

$$\mathcal{O}_1^\tau = \mathcal{O}_2.$$

$\mathcal{L}$ set of lines of $\mathcal{Q}^-(4n+1, q)$ not contained in $\mathcal{Q}$.

K–orbits on lines

K has three orbits on lines of $\mathcal{L}$.

Theorem

$\mathcal{O}_i$ is a relative hemisystem of $\mathcal{Q}^-(4n+1, q)$, $1 \leq i \leq 2$, admitting $\mathrm{PSp}(2n, q^2)$ as an automorphism group.

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With the aid of MAGMA we checked that $\mathcal{Q}^-(7,2)$ has no relative hemisystems!

Open Problem

Does there exist a relative hemisystem of $\mathcal{Q}^-(4n-1,q)$, $(n,q) \neq (2,2)$, $n \geq 2$?

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Applications

$\mathcal{R}$ relative hemisystem of $\mathcal{Q}^-(2n+1, q)$

$\left(\frac{q^n-1}{q-1}\right)$–ovoid $\mathcal{X}$ of $\mathcal{W}(2n+1, q)$,

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vertices of $\mathcal{G}$ are the points of $\mathcal{R}$

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THANK YOU