New maximum scattered linear sets of the projective line

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joint work with
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Università della Campania “Luigi Vanvitelli”

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Authors

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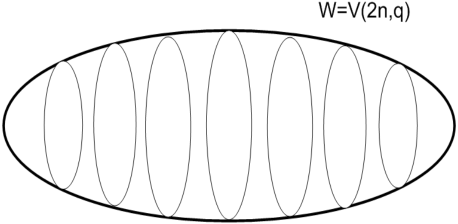

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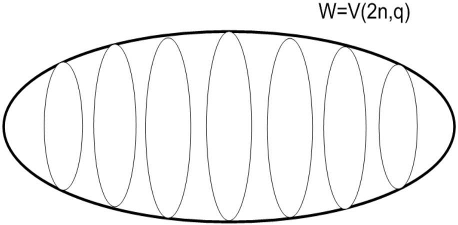
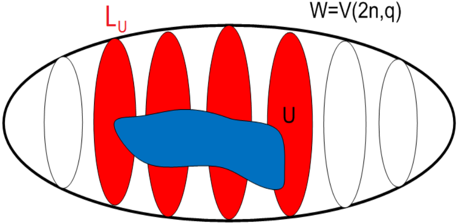


$S = \{\langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} : \mathbf{u} \in W\}$ is a Desarguesian spread of W

Definition of linear set

U $\mathbb{F}_q$-subspace of W



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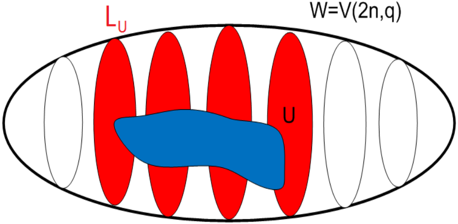

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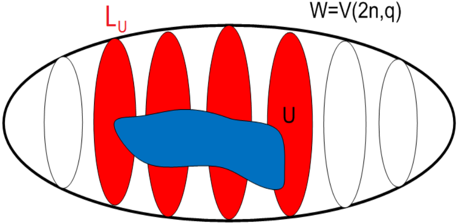

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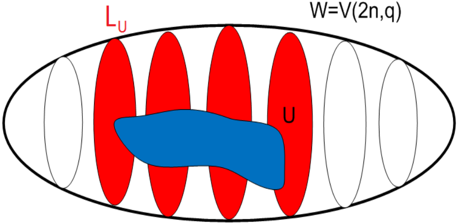

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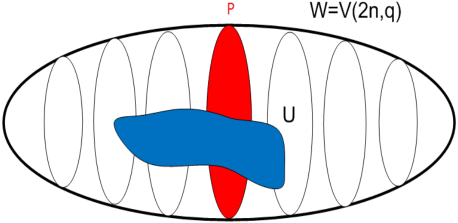
$$\dim_{\mathbb{F}_q} U \leqslant n$$

The weight of a point

$$P = \langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} \in \mathrm{PG}(1, q^n)$$

The **weight of P in L_U** is

$$w_{L_U}(P) = \dim_{\mathbb{F}_q}(U \cap \langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}})$$



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A linear set L_U of rank k and size $\dfrac{q^k-1}{q-1}$ $\rightarrow$ **scattered linear set**

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If $k=n \rightarrow$ **maximum scattered linear set of** $\mathrm{PG}(1,q^n)$

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L_U and L_V are **equivalent** if there exists a collineation ϕ_f of the line such that $L_U^{\phi_f} = L_{U^f} = L_V$.

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If $V = U^f$ for some $f \in \mathrm{\Gamma L}(2, q^n) \Rightarrow L_U$ and L_V are equivalent.

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Example

The linear sets of $\mathrm{PG}(1, q^n)$ of rank $n+1, n+2, \ldots, 2n$.

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Giuseppe Marino's talk!

$\Gamma\mathrm{L}$-class and MRD-codes

If $k = n$

$$L_f = \{\langle (x, f(x)) \rangle_{\mathbb{F}_{q^n}} : x \in \mathbb{F}_{q^n}\}$$

$f(x) = \sum_{i=0}^{n-1} a_i x^{q^i}$ q-polynomial over $\mathbb{F}_{q^n}$

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If L_f is maximum scattered then

$$\{x \in \mathbb{F}_{q^n} \mapsto ax + bf(x) \in \mathbb{F}_{q^n} : a, b \in \mathbb{F}_{q^n}\}$$

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ΓL-class of L_f = number of inequivalent MRD-codes obtained from L_f

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$$\Gamma\mathrm{L}\text{-class} \leqslant \mathcal{Z}(\Gamma\mathrm{L})\text{-class}$$

$\mathcal{Z}(\Gamma\mathrm{L})$-class

$$\mathrm{PG}(W, \mathbb{F}_{q^n}) = \mathrm{PG}(1, q^n) \to \mathrm{PG}(W, \mathbb{F}_q) = \mathrm{PG}(2n-1, q)$$

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variety of $\mathrm{PG}(2n-1,q)$ **associated to** L_U

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S. **Ball**, A. **Blokhuis** and M. **Lavrauw**: *Linear (q+1)-fold blocking sets in* $\mathrm{PG}(2, q^4)$, Finite Fields Appl., 6 (2000), 294-301.

M. **Lavrauw**, J. **Sheekey** and C. **Zanella**: *On embeddings of minimum dimension of* $\mathrm{PG}(n, q) \times \mathrm{PG}(n, q)$, Des. Codes Cryptogr. 74 n.2 (2015), 427-440.

$\mathcal{H} = PG(V, \mathbb{F}_q) = \mathrm{PG}(n-1, q)$ is a **transversal space of** $\mathcal{V}(L_U)$

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If L_U is maximum scattered $\Rightarrow$
Number of transversal spaces through $Q \in \mathcal{V}(L_U) = \mathcal{Z}(\Gamma\mathrm{L})$-class of L_U

Known maximum scattered linear sets: L_1

$$U_1 := \{(x, x^{q^s}) \colon x \in \mathbb{F}_{q^n}\}$$

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Classes of L_1

- $\mathcal{Z}(\Gamma\mathrm{L})$ – class $= \varphi(n)$ (**Lavrauw, Sheekey and Zanella** - 2015);
- $\Gamma\mathrm{L}$ – class $= \varphi(n)/2$ (**Csajbók and Zanella** - 2016).

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$$U_2 = \{(x, \delta x^{q^s} + x^{q^{n-s}}) \colon x \in \mathbb{F}_{q^n}\}$$

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Theorem (Lunardon-Polverino 2001)

For $q>3$, $n \geqslant 4$ and $s=1$, L_2 is not equivalent to L_1.

Known maximum scattered linear sets: L_3

$$U_3 := \{(x, \delta x^{q^s} + x^{q^{s+n/2}}) \colon x \in \mathbb{F}_{q^n}\}$$

$n \in \{6, 8\}$, $q > 2$, $\gcd(s, n/2) = 1$, $\mathrm{N}_{q^n/q^{n/2}}(\delta) \notin \{0, 1\}$ and other conditions on δ and q

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Bence Csajbók's talk!

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Theorem (Csajbók, Marino and FZ)

The linear set $L_2 = \{\langle (x, \delta x^{q^s} + x^{q^{n-s}}) \rangle_{\mathbb{F}_{q^n}} : x \in \mathbb{F}^*_{q^n}\}$ *is not of pseudoregulus type for each* $n \geqslant 4$, s, δ *and* $q > 3$.

Our results

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Theorem (Csajbók, Marino and FZ)

For $n = 6, 8$ and for any choice of the parameters, the linear sets L_1, L_2 and L_3 are pairwise non-equivalent.

Our results

$$L_4 = \{\langle (x, x^q + x^{q^3} + bx^{q^5}) \rangle_{\mathbb{F}_{q^6}} : x \in \mathbb{F}^*_{q^6}\}$$

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- it is **simple** if $q \equiv 0 \pmod 5$;
- MRD-codes obtained from L_4 are **new**.

Sketch of the proof

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is scattered $\Leftrightarrow \omega_{L_4}(P) = 1$ for each $P \in L_4$

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Note that $P \neq \langle (0,1) \rangle_{\mathbb{F}_{q^6}} \to P = \langle (1, -m) \rangle_{\mathbb{F}_{q^6}}$ with $m \in \mathbb{F}_{q^6}$.

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$\Leftrightarrow r(x) = mx + x^q + x^{q^3} + bx^{q^5}$ has rank $\geqslant 5$

Sketch of the proof

Theorem

The rank of a q-polynomial over $\mathbb{F}_{q^n}$ is equal to the rank of its Dickson matrix.

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Dickson matrix of $r(x) = mx + x^q + x^{q^3} + bx^{q^5}$

Sketch of the proof

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Thank you for your attention!