13th International ISAAC Congress
August 2–6, 2021 - Ghent, Belgium

Conference programme

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with technical support by Geert Vernaeve

Overview

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00–10:00	10	2,4,8,10 11,13	4,6,10,11,14 + JOINT (7,8,13)	3,6,7,8 12,14,16	1,3,6,7 8,14,16
10:00–10:30	Break	Break	Break	Break	Break
10:30–12:00	10,15	2,4,8,10 11,13,15	4,6,10,11,14,15 + JOINT (7,8,13)	3,6,7,8 12,14,16	1,3,6,7 8,14,16
12:15–13:00	**Opening**	Break	Break	Break	Break
13:00–14:00	de Hoop	Break	Break	Break	13:30 **Closing**
14:00–15:00	De Philippis	Kaltenbacher	Kuijlaars	Jaffard	Seip
15:00–16:30	2,3,5,7,8,14	1,3,5,7 9,12,14	1,3,7,8,10,12,14	2,4,5,8,10 13,14,15	4,9,10,15
16:30–17:00	Break	Break	Break	Break	Break
17:00–19:00	2,3,5,7,8,14	1,5,7 9,12,14	1,7,8,10,12,14	2,4,5,8,10 13,14,15	4,10

All times are in time zone UTC+2.

Plenary lectures and events

Monday

12:15		Opening ceremony
13:00	Maarten de Hoop	Inverse problems in anisotropic elasticity and waves
14:00	Guido De Philippis	(Boundary) Regularity for Mass Minimising currents

Tuesday

14:00	Barbara Kaltenbacher	Nonlinear Acoustic Waves: Modeling - Analysis - Numerics - Optimization

Wednesday

14:00	Arno Kuijlaars	Universality for eigenvalues of random matrices

Thursday

14:00	Stéphane Jaffard	Multivariate Multifractal Analysis

Friday

13:30		Closing ceremony
14:00	Kristian Seip	Contractive inequalities for Hardy spaces

Session 1 Applications of Dynamical Systems Theory in Biology

Organizer: Torsten Lindström

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00 8:30 9:00 9:30					Serovajsky Zhunussova Cheban Gaiko
10:00					Break
10:30 11:00 11:30					C. C. Wanjala Silva Lindström
12:00	12:15 **Opening**				Break
13:00	**de Hoop**				13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00 15:30 16:00		Malaguti Chakraborty Martiradonna	Szymańska-Dębowska Söderbacka Guillamon		
16:30		Break	Break		
17:00 17:30 18:00 18:30		Bulai Peixe Hritonenko Sørensen	Wyller Alcála Garrido Ivanov Wolkowicz		

Session 2 Challenges in STEM Education

Organizers: Ján Gunčaga and Vladimir Mityushev

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30		Wijaya			
9:00		Putra			
9:30		Toiganbayeva			
10:00		Break			
10:30		Zhunussova			
11:00		Jancarik			
11:30		Ferdianova			
12:00	12:15 **Opening**	Break			
13:00	**de Hoop**	Break			13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Körtesi			Sajka	
15:30	Václavíková			Zemanová	
16:00	Velichová			Santos	
16:30	Break			Break	
17:00	Korenova			Hritonenko	
17:30	Lavicza			Hvorecky	
18:00	Papp			Nevřelová	

Session 3 Complex Analysis and Partial Differential Equations

Organizers: Sergei Rogosin, Ahmet Okay Celebi and Carmen Judith Vanegas

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00				Gulua	Muratbekov
8:30				Chinchaladze	Ng Tuen Wai
9:00				Samer Ahmed	Riche
9:30				Bouallala	Giorgobiani
10:00				Break	Break
10:30				Duduchava	Jaiani
11:00				Tephnadze	Celebi
11:30				Break	Break
12:00	12:15 **Opening**			Break	Break
13:00	**de Hoop**			Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Rogosin	Xu	Vanegas		
15:30	Adukova	Jimenez	Esquivel		
16:00	Bosiakov	Gilbert	Aksoy		
16:30	Break				
17:00	Manjavidze				
17:30	Spitkovsky				
18:00	Katz				
18:30	Mikhailov				

Session 4 Complex Variables and Potential Theory

Organizers: Tahir Aliyev Azeroglu, Massimo Lanza de Cristoforis, Anatoly Golberg and Sergiy Plaksa

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30		Golberg	Ishikawa		
9:00		Karp	Kalmykov		
9:30		Klishchuk	Kaptanoglu		
10:00		Break	Break		
10:30		Prilepkina	Örnek		
11:00		Sete	Fernandez		
11:30		Afanasjeva	Break		
12:00	12:15 **Opening**	Break	Break		
13:00	**de Hoop**	Break	Break		13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00				Akyel	Denega
15:30				Reznichenko	Gryshchuk
16:00				Lanza de Cristoforis	Osipchuk
16:30				Break	Break
17:00				Luzzini	Pérez
17:30				Musolino	Plaksa
18:00				Kohr	Vyhivska

Session 5 Constructive Methods in the Theory of Composite and Porous Media

Organizers: Vladimir Mityushev, Natalia Rylko and Piotr Drygaś

	Monday	Tuesday	Wednesday	Thursday	Friday
12:00	12:15 **Opening**				
13:00	**de Hoop**				13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Kolpakov	Andrianov		Zaccaron	
15:30	Natroshvili	Kurtyka		Wojnar	
16:00	Kakulashvili	Stawiarz		Bosiakov	
16:30	Break	Break		Break	
17:00	Gric	Necka		Ashimov	
17:30	Gulua	Nasser		Krzaczek	
18:00	Giorgadze	Drygas			
18:30	Paszkuta	Mityushev			

Session 6 Function spaces and their applications to nonlinear evolution equations

Organizers: Baoxiang Wang and Mitsuru Sugimoto

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00			Hao	Tomita	Bez
8:30			Guo	Iwabuchi	Liu
9:00			Chen	Huo	Y. Wang
9:30			Kinoshita	Kishimoto	Taniguchi
10:00			Break	Break	Break
10:30			Xu	Zhao	Tsutsui
11:00			T. Kato	X. Wang	Kobayashi
11:30			Break	Han	K. Kato
12:00	12:15 **Opening**		Break	Break	Break
13:00	**de Hoop**		Break	Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**

Session 7 Generalized Functions and Applications

Organizers: Michael Kunzinger and Marko Nedeljkov

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00			(joint with 8,13)		Gordić
8:30			Coriasco		Melnikova
9:00			Cardona	Choi	Prangoski
9:30			Garello	Hwang	Seleši
10:00			Break	Break	Break
			(joint with 8,13)		
10:30			Morando	Kmit	Singh
11:00			Garetto	Kunstek	Stoeva
11:30			Sugimoto	Nedeljkov	Teofanov
12:00	12:15 **Opening**		Break	Break	Break
13:00	**de Hoop**		Break	Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Pilipović	Abdelkader	Atanacković		
15:30	Atanasova	Chatzakou	Broucke		
16:00	Bargetz	Giordano	Debrouwere		
16:30	Break	Break	Break		
17:00	Ben Cheikh	Marti	Break		
17:30	Kunzinger	Oberguggenberger	Minchava-Kamińska		
18:00	Velinov	Vernaeve	Kleiner		
18:30	Vindas	Verriest	Neyt		

Session 8 Harmonic Analysis and Partial Differential Equations

Organizers: Vladimir Georgiev, Michael Ruzhansky and Jens Wirth

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00		Kubo	(joint with 7,13)		Zhou
8:30		Kitamura	Coriasco		Lu
9:00		Garrisi	Cardona	Maes	Kerimbekov
9:30		Lai	Garello	Trushin	Dosmagulova
10:00		Break	Break	Break	Break
			(joint with 7,13)		
10:30		Bez	Morando	Restrepo	Suragan
11:00		Chikami	Garetto	Pombo	Tushir
11:30		Taniguchi	Sugimoto	Omarova	Break
12:00	12:15 **Opening**	Break	Break	Break	Break
13:00	**de Hoop**	Break	Break	Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Turunen		D'Ancona	Yessirkegenov	
15:30	Bastianoni		Federico	Loiudice	
16:00	Brinker		Scandone	Guliyev	
16:30	Break		Break	Break	
17:00	El Ouadih		Sabitbek	Daher	
17:30	E. M. Berkak		Abdullaev	Safouane	
18:00	Haouala		Van Bockstal	I. Berkak	
18:30	Azzedine		Vougalter		

Session 9 Integral Transforms and Reproducing Kernels

Organizer: Zouhaïr Mouayn

	Monday	Tuesday	Wednesday	Thursday	Friday
12:00	12:15 **Opening**				
13:00	**de Hoop**				13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00 15:30 16:00		Negzaoui Hashimoto Nelson			Sonoda Jakšić Cho
16:30		Break			Noda
17:00 17:30		Schlosser Saitoh			

Session 10 Operator Theory and Harmonic Analysis

Organizers: Alexey Karapetyants and Vladislav Kravchenko

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00	Sawano	Thangavelu	Nakai		
8:30	Skubachevskii	Dyakonov	Karagulyan		
9:00	Taskinen	Capone	Shargorodsky		
9:30	Stratis	Almeida	Virtanen		
10:00	Break	Break	Break		
10:30	Blasco	Kokilashvili	Lamberti		
11:00	Meskhi	Kusraeva	Leon Saavedra		
11:30	Break	Shishkina	Karapetyants		
12:00	12:15 **Opening**	Break	Break		
13:00	**de Hoop**	Break	Break		13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00			Perez Moreno	Daher	Sehba
15:30			Karlovich	Tyr	Restrepo
16:00			Zampogni	Britvina	Üster
16:30			Bondarenko	Break	Break
17:00			Gonessa	Hitrik	Torba
17:30			Vicente-Benitez	Mazzucato	Loredo
18:00				Harutyunyan	Barrera-Figueroa
18:30				Aslan	

Session 11 Operator Theory and Time-dependent PDEs

Organizer: Marcus Waurick

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30 9:00 9:30		Behrndt Schwenninger Seifert	Solonukha Kennedy Trunk		
10:00		Break	Break		
10:30 11:00 11:30		Philipp Waurick Picard	Cooper Trostorff Pauly		
12:00	12:15 **Opening**	Break	Break		
13:00	**de Hoop**	Break	Break		13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**

Session 12 Partial differential equations on curved spacetimes

Organizers: Anahit Galstyan, Makoto Nakamura and Karen Yagdjian

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00				Takamura	
8:30				Takeda	
9:00				Tsuchiya	
9:30				Nakamura	
10:00				Break	
10:30				Liu	
11:00				Yang	
11:30				Palmieri	
12:00	12:15 **Opening**			Break	
13:00	**de Hoop**			Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00		Break	Break		
15:30		Natario	Karp		
16:00		Wrochna	Ebert		
16:30		Break	Break		
17:00		Baskin	Grigorian		
17:30		Yagdjian	Balogh		
18:00			Galstyan		

Session 13 Pseudo Differential Operators

Organizer: Man Wah Wong

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00		Toft	(joint with 7,8)		
8:30		Ivec	Coriasco		
9:00		Vučković	Cardona		
9:30		Abdeljawad	Garello		
10:00		Break	Break		
			(joint with 7,8)		
10:30		Wahlberg	Morando		
11:00		Vojnovic	Garetto		
11:30		Break	Sugimoto		
12:00	12:15 **Opening**	Break	Break		
13:00	**de Hoop**	Break	Break		13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00				V. Kumar	
15:30				Catană	
16:00				M. Kumar	
16:30				Break	
17:00				Rodino	
17:30				Cohen	
18:00				Ben-Benjamin	
18:30				Lu	

Session 14 Quaternionic and Clifford Analysis

Organizers: Swanhild Bernstein, Uwe Kähler, Irene Sabadini and Franciscus Sommen

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00			Baghal Ghaffari	W. Wang	Hitzer
8:30			Dizon	Massopust	El Gargati
9:00			Hogan	L. Wang	Bryukhov
9:30			De Martino	de Fabritiis	Yuksel
10:00			Break	Break	
10:30			Ferreira	Ren	
11:00			Legatiuk	Gentili	
11:30			Grigoriev	Bisi	
12:00	12:15 **Opening**		Break	Break	
13:00	**de Hoop**		Break	Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Kimsey	Guzman Adan	Vieira	Kraußhar	
15:30	Cerejeiras	Libine	Bernstein	Hu	
16:00	Lavicka	Eelbode	Faustino	Diki	
16:30	Break	Break	Break	Break	
17:00	Colombo	Ryan	Luna-Elizarraras	Morais	
17:30	Eriksson	Pinton	Tellez Sanchez	Moreira	
18:00	Mertens	Abreu Blaya	Vajiac	di Teodoro	
18:30	De Zayas	Ariza Garcia	Nolder		

Session 15 Recent Progress in Evolution Equations

Organizers: Marcello D'Abbicco and Marcelo Rempel Ebert

	Monday	Tuesday	Wednesday	Thursday	Friday
10:30 11:00 11:30	Hirosawa Chen Break	Georgiev Pham Trieu Dao	Faminskii Kubo Lin		
12:00	12:15 **Opening**	Break	Break		
13:00	**de Hoop**	Break	Break		13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00 15:30 16:00				Cappiello Ascanelli da Silva	Gobbino Lucente Charao
16:30				Break	
17:00 17:30 18:00				Picon Girardi Arias	

Session 16 Wavelet theory and its Related Topics

Organizers: Keiko Fujita and Akira Morimoto

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30				Aihara	Morimoto
9:00				K. Fujita	Ashino
9:30				Mandai	M. Fujita
10:00				Break	Break
10:30				Dades	Machigashira
11:00				Li	Aremua
11:30				Suzuki	Break
12:00	12:15 **Opening**			Break	Break
13:00	**de Hoop**			Break	13:30 **Closing**
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**

Plenary talks

Inverse problems in anisotropic elasticity and waves

Maarten de Hoop

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We present advances in the analysis of inverse problems associated with the theory of wave propagation and scattering in models of anisotropic elasticity, which are expected to describe material properties in crust and mantle of terrestrial planets. Propagation of singularities lead to geometric inverse problems; we will discuss how techniques from microlocal analysis and Finsler geometry come into play. We include the case of data generated by unknown interior sources. We will summarize consequences of elastic anisotropy through the lens of algebraic geometry. Finally, we will show results pertaining to time-harmonic and dynamic inverse boundary value problems associated with elastic waves, exploiting unique continuation, relevant in exploration seismics.

(Boundary) Regularity for Mass Minimising currents

Guido De Philippis

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Plateau problem consists in finding a surface of minimal area among the ones spanning a given curve. It is among the oldest problem in the calculus of variations and its study lead to wonderful development in mathematics.

Federer and Fleming integral currents provide a suitably weak solution to the Plateau problem in arbitrary Riemannian manifolds, in any dimension and co-dimension. Once this weak solution has been found a natural question consists in understanding whether it is classical one, i.e. a smooth minimal surface. This is the topic of the regularity theory, which naturally splits into interior regularity and boundary regularity. After the monumental work of Almgren, revised by De Lellis and Spadaro, interior regularity is by now well understood. Boundary regularity is instead less clear and some new phenomena appear.

Aim of the talk is to give an overview of the problem and to present some boundary regularity results we have obtained in the last years.

This is based on joint works with C. De Lellis, J. Hirsch and A. Massaccesi.

Multivariate Multifractal Analysis

Stéphane Jaffard

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Joint work with Patrice Abry, Roberto Leonarduzzi, Stéphane Seuret, Herwig Wendt

Multifractal analysis quantifies the fluctuations of regularities in functions through the estimation of their ”multifractal spectrum” which encapsulates the fractional dimensions of the singularity sets of a function. Wavelet techniques supply robust tools in order to perform multifractal analysis, and they became a standard signal/image processing method for classification or model selection, successfully used in a large variety of applications. Yet, successes were confined to the analysis of one signal or image at a time (univariate analysis). In view of many modern real-world applications that rely on the joint (multivariate) analysis of collections of signals or images, the need for extensions to multivariate settings became a major challenge.

We will describe the theoretical foundations of multivariate multifractal analysis, which proposes to estimate the multivariate multifractal spectrum of several signals as a new way to reveal the correlations between their singularity sets. We will study the properties and limitations of the most natural extension of the univariate formalism to a multivariate formulation, and explain why, while performing well for some models, this natural extension is not valid in general. We will illustrate these theoretical results by numerical computations worked out for several classical models of stochastic processes.

Nonlinear Acoustic Waves: Modeling - Analysis - Numerics - Optimization

Barbara Kaltenbacher

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High intensity (focused) ultrasound HIFU is used in numerous medical and industrial applications ranging from litotripsy and thermotherapy via ultrasound cleaning and welding to sonochemistry. We will highlight certain mathematical and computational aspects related to the relevant nonlinear acoustic phenomena, namely

- modeling of high intensity ultrasound phenomena as second and higher order wave equations
- some parameter asymptotics
- absorbing boundary conditions for the treatment of open domain problems
- optimal shape design and boundary control problems in the context of HIFU

The talk is based on joint work with Christian Clason (University of Graz), Manfred Kaltenbacher (TU Vienna), Irena Lasiecka (University of Memphis), Vanja Nikolic (Radboud University), Petronela Radu (University of Nebraska at Lincoln), Gunther Peichl (University of Graz), Igor Shevchenko (Imperial College London), and Mechthild Thalhammer (University of Innsbruck).

Universality for eigenvalues of random matrices

Arno Kuijlaars

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Eigenvalues of random matrices show a remarkable regular pattern as the size tends to infinity. This has been studied in great detail for the class of unitarily invariant ensembles of Hermitian matrices, where powerful tools from orthogonal polynomials and potential theory can be used. I will give an overview of this development and some of its extensions that include multiple orthogonal polynomials.

Contractive inequalities for Hardy spaces

Kristian Seip

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It has been recognized by many authors that contractive inequalities involving norms of H^p spaces can be particularly useful when the objects in question (like the norms and/or an underlying operator) lift in a multiplicative way from one (or few) to several (or infinitely many) variables. This has been my main motivation for looking more systematically at various contractive inequalities in the context of Hardy spaces on the d-dimensional torus. I will discuss results from recent studies of Hardy–Littlewood inequalities, Riesz projections, idempotent Fourier multipliers, and Hilbert points (which in one variable is another word for inner functions). We will see interesting phenomena occurring both in the transition from low to high dimension and from low to infinite dimension. The talk builds on joint work with Sergei Konyagin, Hervé Queffélec, and Eero Saksman and with Ole Fredrik Brevig and Joaquim Ortega-Cerdà.

Session talks

Session 1: Applications of Dynamical Systems Theory in Biology

Organizer: Torsten Lindström

Analysis and Design of a Wireless Sensor Network Based on the Residual Energy of the Nodes and the Harvested Energy from Mint Plants.

Hassel Aurora Alcalá Garrido

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Joint work with Víctor Barrera-Figueroa, Mario E. Rivero-Ángeles, Yunia V. García-Tejeda, Hosanna Ramírez-Pérez

In this talk, we present the analysis and study of a wireless sensor network (WSN) based on a harvesting energy system that harvests bio- electricity from mint plants. This energy aids in powering custom designed wireless sensors operating in a continuous monitoring mode. The WSN is based on randomly turning nodes ON (active nodes) and OFF (inactive nodes) to avoid their energy depletion. While a node is inactive, it is allowed to harvest energy but not to report data. Conversely, an active node is allowed to report data but not to harvest energy. As such, a clear compromise is established between the amount of information reported by the nodes and the lifetime of the network. We focus on extending the network's lifetime by taking advantage of energy harvesting techniques. To finely tune the system's parameters and provide a suitable operation, we derive a mathematical model based on a discrete Markov chain that describes the main dynamics of the system. Applications for the proposed scheme can be found in the cases where a WSN should operate during long times even if data reporting is reduced after long working periods. For instance, when nodes are not accessible or are placed in dangerous or remote locations like polar regions, radioactive zones, wildfire monitoring in forests, or even space exploration missions where the objective is to obtain as much information as possible from the environment for as long as possible periods, and if energy is scarce, conserve it as much as possible even if only occasional reports are available.

Stopping waves: Geometric analysis of coupled bursters in an asymmetric excitation field

Iulia Martina Bulai

University of Basilicata, Italy
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Joint work with Morten Gram Pedersen

Bursting is a type of electrical activity seen in many neurons and endocrine cells where episodes of action potential firing are interspersed by silent phases. Here we investigate partial synchrony and wave propagation in a population of square-wave bursters. In particular, by using a prototypical polynomial bursting model and slow/fast bifurcation analysis, we study why electrically coupled model bursters typically synchronize very easily, as reflected in the tendency for simulated excitation waves to propagate far into the region of silent cells when an excitation gradient is imposed. Such simulation are inspired by, but do not reproduce, experimentally observed Ca^{2+} waves in pancreatic islets exposed to a glucose gradient. Our analyses indicate a possible modification of the model so that the excitation waves stop at the border between "active" and "silent" cells. We verify this property by simulations using such a modified model for a chain, and for a cubic cluster, of coupled cells. Furthermore, we show how our one- and two-parameter bifurcation analyses allow us to predict where the simulated waves stop, for both the original model and the modified version.

A Covid-19 SEIR Model for People Living With Underlying Medical Conditions in Kenya

Iyaya C.C.Wanjala

Multimedia University of Kenya, Kenya
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Joint work with George M Mocheche, Irene Wattanga

The corona virus disease (COVID-19) is a novel infection caused by SARS-CoV-2. Its effects have caused tremendous economic, social and political challenges to nearly all countries around the world, albeit to varying degrees. Kenya reported its first documented case on the 13th of March, 2020. Out of an estimated number of global fatalities of 3.3 million people, Kenya has as of mid-May, 2021 recorded about 3,000 deaths. For a developing country with a health system that is not well equipped, the trend of mortality rate is worrying and therefore requires urgent attention. This paper generally looks at a high risk group of Kenyans living with underlying medical conditions. It is estimated that one in every three adults above the age of 58 years has underlying medical conditions. Alongside the frontline health workers, the government has prioritized this group for vaccination and other intervention measures against contracting the disease. It is estimated that the bulk of reported fatalities in Kenya comprise of people living with underlying medical conditions. In this paper, we propose a modified SEIR mathematical transmission model with a focus on all persons living with underlying medical conditions with a specific attention to those who are resident in city of Nairobi. The paper will rely on reported data from the Kenya Government's Ministry of Health to fit the model parameters. The next generation approach is employed to ascertain the level of infection or the stability condition of the model based on the basic reproduction number, . Results obtained will show that inclusion of Kenyans of all ages living with underlying medical conditions in the already prioritized group for vaccination and other mitigating measures put in place by the government, will significantly reduce COVID-19 fatalities. Numerical simulations will be used to generate results that will support arguments put forth in this paper.
Keywords: COVID-19; SEIR model, underlying medical conditions, quarantine, basic reproduction number.

Quantifying nitrogen fixation by heterotrophic bacteria in sinking marine particles

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Joint work with K. Andersen, A. Visser, K. Inomura, M. Follows, L. Riemann

Nitrogen fixation by heterotrophic bacteria associated with sinking particles contributes to marine N cycling, but a mechanistic understanding of its regulation and significance is not available. We develop a mathematical model for unicellular heterotrophic bacteria growing on sinking marine particles and can fix nitrogen under suitable environmental conditions. We find that the interactive effects of polysaccharide and polypeptide concentrations, sinking speed of particles, and surrounding oxygen and nitrate concentrations determine the nitrogen fixation rate inside particles. Nitrogen fixation inside sinking particles is mainly fueled by sulfate respiration rather than nitrate respiration. Our model suggests that anaerobic processes, including heterotrophic nitrogen fixation, can take place in anoxic microenvironments inside sinking particles even in fully oxygenated marine waters. The modelled rates are similar to bulk rates measured in the aphotic ocean, and our study consequently suggests that particle-associated heterotrophic nitrogen fixation contributes significantly to oceanic nitrogen fixation.

Almost periodic solutions and compact global attractors of equations modelling growth processes and gonorrhea epidemics

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This talk is dedicated to the study the problem of Bohr/Levitan almost periodic solutions and structure of

compact global attractor (Levinson center) a class of delay differential equations

$$x'(t) = f(t-1, x(t-1)) - f(t, x(t)). \tag{1}$$

Equation (1) may be viewed as the non-autonomous form of a model growth processes and gonorrhea epidemics introduced by K.Cooke and J.Yorke (1973).
We establish the main results in the framework of general non-autonomous (cocycle) dynamical systems. Assume that:

(F1) $f \in C(\mathbb{R}^2, \mathbb{R})$ is C^1 admissible in the sense that $f(t,u)$ and $\frac{\partial f}{\partial u}(t,u)$ are almost periodic in $t \in \mathbb{R}$ uniformly with respect to (w.r.t.) u on every compact from $\mathbb{R}$;

(F2) $f(t,0) = 0$ and $\frac{\partial f}{\partial u}(t,u) \geq 0$ for any $(t,u) \in \mathbb{R}^2$.

Denote by $H(f)$ the closure in $C(\mathbb{R}^2, \mathbb{R})$ w.r.t. compact-open topology of $\{f_\tau |\ \tau \in \mathbb{R}\}$, where $f_\tau(t,u) := f(t+\tau, u)$ for any $(t,u) \in \mathbb{R}^2$.
Let $\mathcal{C} := C([-1,0], \mathbb{R})$ and $\varphi_t(\phi, g)$ be the solution of equation (1), with f replaced by $g \in H(f)$, passing through $\phi \in \mathcal{C}$ at initial moment $t = 0$.
A family $\{I_g |\ g \in H(f)\}$ of compact subsets $I_g \subset \mathcal{C}$ is said a compact global attractor (Levinson center) of equation (1) if $I_g \neq \emptyset$ for any $g \in H(f)$, $\mathbf{I} := \bigcup\{I_g |\ g \in H(f)\}$ is pre-compact, $\varphi_\tau(I_g, g) = I_{g_\tau}$ for any $g \in H(f)$ and $\tau \geq 0$ and $\mathbf{I}$ attracts every compact subset from $\mathcal{C}$ uniformly w.r.t. $g \in H(f)$, i.e.,

$$\lim_{t \to +\infty} \sup_{g \in H(f)} \beta(\varphi_t(M, g), \mathbf{I}) = 0$$

for any compact subset M from $\mathcal{C}$, where $\beta(A,B) := \sup_{a \in A} \rho(a, B)$, $\rho(a,B) := \inf_{b \in B} \rho(a,b)$ and $\rho(a,b) := \max_{s \in [-1,0]} |a(s) - b(s)|$ $(a, b \in \mathcal{C})$.

Theorem. Suppose that $f \in C(\mathbb{R}^2, \mathbb{R})$ is almost periodic in $t \in \mathbb{R}$ uniformly w.r.t. u on every compact subset from $\mathbb{R}$ and equation (1) is compact dissipative.
Then under conditions (F1)–(F2) the following statements hold:

1. $\alpha_g, \beta_g \in I_g$ for any $g \in H(f)$ and, consequently, $I_g \subseteq [\alpha(g), \beta(g)]$, where α_g (respectively, β_g) is the greatest lower bound of I_g (respectively, the least upper bound of I_g);
2. $\varphi(t, \alpha_f, f) = \alpha_{\sigma(t,f)}$ (respectively, $\varphi(t, \beta_g, g) = \beta_{\sigma(t,g)}$) for any $t \geq 0$ and $g \in H(f)$;
3. equation (1) has at least two almost periodic solutions $\varphi(t, \alpha_g, g)$ and $\varphi(t, \beta_g, g)$ belonging to Levinson center of (1).

Bifurcation analysis and applications of multi-parameter dynamical systems

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We carry out a global bifurcation analysis of multi-parameter polynomial dynamical systems and consider their applications. To control limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know properties and combine effects of all rotation parameters. It can be done by means of development of new bifurcational geometric methods based on the Wintner–Perko termination principle for planar polynomial dynamical systems. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this approach, we have solved, e. g., Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the general Liénard polynomial system with an arbitrary number of singular points, the Kukles cubic-linear system, the Euler–Lagrange–Liénard polynomial mechanical system, Leslie–Gower systems which model the population dynamics in real ecological or biomedical systems and a reduced planar quartic Topp system which models the dynamics of diabetes. Finally, applying a similar approach, we have considered various applications of three-dimensional polynomial dynamical systems and, in particular, completed the strange attractor bifurcation scenario in Lorenz type systems globally connecting the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of their limit cycles.

Neural synchronization and control using phase and amplitude response functions

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We will explain how tools coming from differential geometry and dynamical systems help understanding the phase and amplitude dynamics of neural oscillators. First, we will revise the concept of *isochrons* (in fact, a specific type of stable manifolds introduced in biology by A. Winfree), how it is linked to Lie symmetries and the computational scheme we have proposed to compute them, based on the so-called *parameterization method*. Second, we will focus on the advantages that this quantitative information about isochrons brings us in terms of dimension reduction and improvement of accuracy with respect to other reductions. Third, we will exploit these advantages in several applications: (a) construction of "transient" *phase and amplitude response functions* for neuron models and other biological systems, (b) refinement of the prediction of the entrainment of neurons by external stimuli and (c) uses of the phase-amplitude reduction both to exert control and design induced sustained oscillations in damped systems. We will conclude with a discussion of how the methods explained are extendable to any model presenting sustained or damped oscillations, often present in many biological and mechanical systems. The contents of the lecture are based on collaborations with several authors: Oriol Castejón, Gemma Huguet, Kevin Martínez-Añón and Román Moreno.

Integral epidemiologic model with realistic infectiousness distributions

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Joint work with Yuri Yatsenko, Victoria Hritonenko

The integral model with finite delay is suggested to describe epidemic dynamics. The model clearly distinguishes two important delayed distributions: the varying infectiousness that depends on the contact time and the time passed after the infection occurs (infectiousness distribution) and the fraction that infectious individuals survive and remain in the infectious stage after becoming infected (survivor distribution). The model describes a realistic distribution of infectious period and other patterns of infectivity and latency not covered by ODE-based models Such patterns are not necessarily exponentially or gamma distributed. It will be shown that the SIR, SEIR, multi-compartment epidemiological models, Erlang SEIR, and some other well- known epidemiologic models are special cases of the proposed model. The key feature of the suggested model is the assumption about a finite delay (a finite length of process prehistory). Indeed, all epidemic processes have a finite length that depends on the nature of disease, and the assumption of no or infinite memory makes the models less accurate. The finite memory of processes leads to new phenomena in modeled dynamics and makes the model more realistic and flexible.

Global asymptotic stability and periodic solutions in a differential delay model of megakaryopoiesis

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Dynamical properties of solutions of scalar differential delay equation

$$x'(t) = -\mu x(t) + f(x(t))\, g(x(t-\tau)) \tag{1}$$

are studied where f and g are positive continuous functions of the positive semiaxis $\mathbb{R}_+ = \{x \in \mathbb{R} | x > 0\}$ and $\mu > 0$. Equation (1) was recently proposed as a mathematical model of the megakaryopoiesis and platelet production in human body. Its other partial cases include Mackey-Glass physiological models, Nicholson's blowflies differential delay equations, and several models in population dynamics.
Sufficient conditions are derived when the unique positive equilibrium of equation (1) is globally asymptotically stable. One criterion is based on the global attractivity of the unique fixed point of an underlying

interval map and is delay independent. The other one uses a similar approach via an enveloping interval map and is explicitly dependent of the size of delay τ.
Conditions for the existence of periodic solutions in model (1) are established. It is shown that slowly oscillating periodic solutions always exist when the linearized about positive equilibrium equation (1) is unstable (i.e. its characteristic equation has a solution with the positive real part).

On the stochastic engine of transmittable diseases in exponentially growing populations

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The purpose of this paper is to analyze the interplay of deterministic and stochastic models for epidemic diseases. Deterministic models for epidemic diseases are prone to predict global stability. If the natural birth and death rates are assumed small in comparison to disease parameters like the contact rate and the recovery rate, then the globally stable endemic equilibrium corresponds to a tiny proportion of infected individuals. Asymptotic equilibrium levels corresponding to low numbers of individuals invalidate the deterministic results.
Diffusion effects force frequency functions of the stochastic model to possess similar stability properties as the deterministic model. Particular simulations of the stochastic model are, however, oscillatory and predict oscillatory patterns. Smaller or isolated populations show longer periods, more violent oscillations, and larger probabilities of extinction.
Evolution maximizes the infectiousness of the disease as measured by the ability to increase the proportion of infected individuals provided the stochastic oscillations are moderate enough to remain in the vicinity of the deterministic equilibrium. Physiological bounds are likely to be reached on a short time-scale for the contact rate. The general long-run evolutionary pattern for diseases is therefore, to develop less virulent variants ensuring that infectious individuals remain in the population.

Diffusion-aggregation models in biological invasions and in crowd dynamics

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Joint work with Diego Berti, Andrea Corli, Juan Campos

This talk accounts on various dynamical systems which frequently meet in the study of biological invasions and in crowd dynamics. Both a convective effect and a reaction term are usually included. These models are given by suitable partial differential equations of parabolic type. They may also display aggregative behaviours, depending on the density in the habitat.
The main focus of this discussion is about the presence of constant-profile solutions, usually denoted wavefront solutions, that connect two stationary states of the system. We discuss their existence and provide an estimate of their admissible speeds.
Different types of diffusivities are introduced, inside the system, and corresponding additional properties of their solutions are discussed. In particular, we show that wavefronts can further be right compactly supported, left compactly supported, or both and sharp when associated to degenerate diffusivities while they may display jump discontinuities corresponding to diffusion terms of saturated type.
Some concrete examples, in correspondence of real data, complete this discussion.

RothC models for Soil Organic Carbon dynamics

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Soil Organic Carbon (SOC) refers to the carbon component in the organic matter compounds. It is one of the key indicators of the status of land degradation neutrality (LDN), the latter defined by the United Nations Convention to Combat Desertification (UNCCD) as "a state whereby the amount and quality of land resources, necessary to support ecosystem functions and services and enhance food security, remains stable or increases within specified temporal and spatial scales and ecosystems". Predictive discrete and continuous models are used to simulate the SOC dynamics in order to estimate the carbon sequestration

under a given land use and evaluate the deviation from the target indicator. The prediction of long-term solutions gives a synthetic view of the system in specific agro-climatic conditions and makes it possible to realize when a studied soil reaches an equilibrium. We analyse the continuous and discrete versions of Rothamsted Carbon Model (RothC) [1, 4, 3], a model of soil organic carbon turnover in non-waterlogged soils and propose a novel non-standard scheme [2] for the approximation of the continuous model. We compare it with both the original discrete formulation of RothC and the Exponential Rosenbrock-Euler approach introduced in [4].
This work received fundings from the REFIN project N.0C46E06B (Regione Puglia, Italy) and from the H2020 project eLTER PLUS.

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Permanence in Polymatrix Replicators

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Generally a biological system is said to be permanent if under small perturbations none of the species goes to extinction. In 1979 P. Schuster, K. Sigmund, and R. Wolff introduced the concept of permanence as a stability notion for systems that models the self-organization of biological macromolecules. After, in 1987 W. Jansen, and J. Hofbauer and K. Sigmund give sufficient conditions for permanence in the replicator equations. In this work we extend these results for polymatrix replicators.

Discrete model of the epidemic propagation with a limited time spent in compartments

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Joint work with O. Turar, D. Zhakebaev

In connection with the Covid pandemic, the development of effective mathematical models of epidemiology is of particular relevance. Starting with the classic works of A.G. McKendrick and W.O. Kermack's most popular are ?ompartmental models in epidemiology, in which the population is divided to compartments.
In this paper, we propose a discrete model of the propagation of the epidemic, a distinctive feature of which is the direct consideration of the limited time in the category. In this model, the population is divided into the following compartments: susceptible, contact, asymptomatic, mildly ill, hospitalized, recovered and deceased. It is assumed that contact, as well as all categories of patients are in this compartment for a limited time, different for each compartment.
Each category is associated with a function of a discrete argument (time in days), which characterizes the number of this compartment. At the same time, the number of contacts and patients of all compartments is the sum of the numbers of representatives of this compartment for each day of being here. Thus, all representatives of the compartment of a given day become representatives of the same compartment of the next day every other day, and representatives of the last day of this compartment leave it the next day, replenishing other compartments.
A qualitative analysis of this model is carried out, as well as calculations on the example of the propagation of Covid in Kazakhstan.

Complex network model for COVID-19 with piecewise constant parameters

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Joint work with G. Cantin, C. Cruza, R. Fonseca-Pinto, R. Fonseca, E. S. Santos, D. F. M. Torres

In this talk we propose a complex network model for COVID-19 with piecewise constant parameters and analyze the impact of human behavior and mobility in the transmission dynamics of SARS-CoV-2. We prove the existence of pseudo-periodic solutions, which are related to epidemic waves, in the context of the COVID-19 pandemic. Through numerical simulations, we propose optimal topologies for the management of epidemic outbreaks, aiming to minimize the number of active infected individuals. This talk is based on the results proved in [1].

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Review on the behaviour of a many predator - one prey system

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In this presentation we make a review of the behaviour of a known system of many predators and one prey. We mainly consider the case with two predators. We consider the question below and give results and formulate open problems.

- Extinction conditions for one predator
- Different types of coexistence
- Simpler case, where a one dimensional model map works
- More complicated chaos, spiral chaos etc
- Attractor appearing from a contour
- Existence of many attractors
- Bifurcations of attractors

Modelling of growth and disinfection of bacterial biofilms in a centrifugal Lab-on-a-disc platform

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Joint work with T. Laksafoss, L. Serioli, K. Zor, A. Boisen, M. Efendiev and J.A.J Haagensen

We have set up a mathematical model of the bacterial growth pattern in a newly developed rotating Lab-on-a-disc microfluidics system. The model is based on a reaction diffusion advection equation for the bacterial growth, supply of probiotics and antibiotics. The cells are divided into two states, an active state and a dormant state. In the active state the bacteria can move, reproduce and conduct bio chemical signaling. In the dormant state the bacteria are passive within the biofilm in order to protect themselves against antibiotics.

The movement of the bacteria is assumed to be governed by diffusive processes and in part advection, where the diffusion transport depends on the biomass of bacteria. This dependence is assumed to be singular, such that the diffusion increase strongly as the biomass approach a threshold value. In addition we have cross diffusion. From numerical simulations, we observe solution patterns, showing high concentration of bacteria in small regions and very low concentration outside these regions. The resulting spot like and highly nonlinear pattern resembles the patterns found in experiments. The solution patterns are markedly different from the case of constant diffusion, pointing toward highly nonlinear and mass dependent diffusion processes for bacterial growth and biofilm formation.

Existence of canard solutions to singularly perturbed predator-prey models.

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Joint work with J. Banasiak, M.S. Seuneu Tchamga

We consider two predator-prey models in the case when the prey dynamics is very fast. The quasi-steady manifolds intersect and a backward bifurcation occurs along their intersection. We give a proof of the existence of canards and provide an exact value of time at which the stability switch occurs.

A Delay Model for Persistent Viral Infections in Replicating Cells

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Joint work with H. Gulbudak and P. Salceanu

Persistently infecting viruses remain within infected cells for a prolonged period of time without killing the cells and can reproduce via budding virus particles or passing on to daughter cells after division. The ability for populations of infected cells to be long-lived and replicate viral progeny through cell division may be critical for virus survival in examples such as HIV latent reservoirs, tumor oncolytic virotherapy, and non-virulent phages in microbial hosts. We consider a model for persistent viral infection within a replicating cell population with time delay in the eclipse stage prior to infected cell replicative form. We obtain reproduction numbers that provide criteria for the existence and stability of the equilibria of the system and provide bifurcation diagrams illustrating *transcritical (backward and forward), saddle-node, and Hopf* bifurcations, and provide evidence of *homoclinic bifurcations* and a *Bogdanov-Takens bifurcation*. We investigate the possibility of long term survival of the infection (represented by chronically infected cells and free virus) in the cell population by using the mathematical concept of *robust uniform persistence*. Using numerical continuation software with parameter values estimated from phage-microbe systems, we obtain two-parameter bifurcation diagrams that divide the parameter space into regions with different dynamical outcomes. We thus investigate how varying different parameters, including the time spent in the eclipse phase, can influence whether or not the virus survives.

Pattern formation in a neuronal network model

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In the last decades great advances have been made in mapping neural circuitry of the brain. This has been facilitated by novel experimental techniques for studies both at the single-cell and systems levels. It still remains, though, to combine all the pieces in the puzzle to a coherent picture of brain function. While single nerve cells are fairly well understood, the signal - processing properties of the nerve-cell networks in cortex are still obscure. The growth of experimental data has led to a revival of so - called rate equation models for cell networks in nervous tissue (neural networks). In these models, the probability for firing action potentials, the key information carriers in the brain, is the main dynamical variable. These models assume the form of coupled integral and integro - differential equations, and they describe non-linear interactions between different neuron populations.

In the present talk I will discuss the properties of the continuum limit of a nonlocal Hopfield type of neuronal network model, with spatial heterogeneity incorporated. I will discuss pattern formation though Turing type of instabilities within the framework of this model.

INVESTIGATION OF THE STABILITY OF MOTION WITH RESPECT TO SOME OF THE VARIABLES UNDER CONSTANT DISTURBANCES

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The stability of the motion of nonlinear equations has been the subject of research by many authors. The works of I.G. Malkin and S.I. Gorshin are devoted to consideration of the stability of systems under the assumption that constantly acting disturbing forces are small at each moment of time. N.N. Krasovsky proved that if the unperturbed motion is asymptotically stable uniformly with respect to t_0 and $x(t_0)$, then it is stable under constant perturbations that are small on average. Stability under constantly acting disturbances, small integral, is considered in the work of I. Vorch. these concepts and results were generalized to the problem of stability with respect to a part of the variable in the works of A. Oziraner. In the works of T. Biyarov the question of stability with respect to a part of the variables under permanently acting perturbations, small on the average and disappearing at infinity was investigated. This article discusses the issue of stability with respect to some of the variables under permanent disturbances in a particular system. Consider the equation of the disturbed motion

$$\frac{dx}{dt} = X(t,x), x(t,x) = 0 \tag{1}$$

where $x = (y_1, \ldots, y_n, z_1, \ldots, z_{\rho p})$, $m > 0, p > 0, n = m + p$. Suppose that: a) the right-hand sides of the system in the domain $\Omega = t > 0, ||y|| < H > 0, ||z|| < \infty$ are continuous and satisfy the conditions for the uniqueness of the solution; b) the solution of system (1) is continued. Along with system (1), consider the perturbed equation

$$\frac{dx}{dt} = X(t,\tilde{x}) + R(t,\tilde{x}), R(t,0) <> 0 \tag{2}$$

with respect to which conditions a) and b) are assumed to be satisfied and, in addition, the perturbations $R = (R_1, \ldots, R_n)$ in the domain Ω satisfy the conditions

$$\int t_0^t |(R_s(\tau, \tilde{x}_1 \ldots \tilde{x}_n) d\tau| < \rho p, t > t_0, s = 1.n,$$

where $\rho p > 0$ is a small enough number.

Session 2: Challenges in STEM Education

Organizers: Ján Gunčaga and Vladimir Mityushev

Teaching of STEM lectures during the COVID-19 time

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Joint work with J. Guncaga, M. Billich

The COVID-19 pandemic situation has adversely affected mobility and international cooperation of students and workers throughout Europe. The transfer of knowledge from foreign experts who can point out the issue in another point of view is an integral part of university studies. However, the reaction of international agency CEEPUS have been greatly flexible – it allowed online and blending mobility to several countries. Thus, the aim of this article is to introduce the possibility how to implement online teaching with the use of foreign workers using available tools and means. The presentation shows practical experience within direct online teaching of STEM subjects in university courses. Primarily, the advantage of using GeoGebra software in online teaching of mathematical subjects is pointed out. Thanks to the aspect of GeoGebra visualization, there was no discomfort in transition of full - time teaching to distance teaching, because understanding of the given topic is not affected by changing the form of teaching. As a part of direct online teaching, a presentation of historical mathematical problems were created; with use of GeoGebra software it facilitated the conversion of historical tasks into a more modern form.

Engaging strategies to promote STEM learning

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Challenges in STEM Education have been recognized by most countries throughout the world and outlined in numerous reports and publications. STEM disciplines are shown to be among the most challenging subjects in high schools, colleges, and universities. Students' failure in these disciplines hindering them in their desired studies and prevent them from pursuing their carrier in STEM-related fields. Various teaching strategies will be discussed and suggested to enhance students' interest and readiness for taking STEM courses. Emphasis will be made on interdisciplinary research projects, puzzles, and other captivating activities proven to work well in both face-to-face and virtual classroom settings. The presented strategies and their variations go beyond traditional tasks and surpass routine boring homework assignments and are designed to reduce subject anxiety, stimulate learning of fundamentals, and demonstrate the versatility of science and unity of STEM disciplines. Educational elements are integrated into all these activities that can be easily incorporated into any courses across disciplines. Assessment of students' progress, examples of students' submissions, and their evaluations of challenges and benefits of these extra-curricular activities will be presented.

STEM a STEAM Education in the Future Teacher Preparation

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Joint work with J. Guncaga, L. Korenova

According to the European Commissions' data, STEM-oriented education facilitates the countries' competitiveness in global economy, supports stability and prosperity as well as contributes to the sustainable development. It indicates its importance in the preparation of the future teachers. In our contribution we discuss a method of development of mathematical, technology and digital awareness of primary-school teachers using augmented reality. We present such activities and their learning aims. The opinion of the

students on the method was evaluated using a questionnaire. Its key outcomes are also part of our talk.

PCR pooling - an example of the connection between mathematics, computer science and medicine

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Combinatorial group testing is one of the mathematical disciplines with a wide range of applications in various fields. Testing methods have been developed for medical purposes, specifically to detect the spread of syphilis among American soldiers. Therefore, it is not surprising that considerations about using these methods are relevant even in the current covid-19 pandemic. The paper will introduce a project, which teaches how abstract theoretical mathematical theory can be applied to tests with specific real-world parameters. Mathematical methods correspond to the knowledge of high school students. The lesson uses actual data based on specific parameters of the genetic laboratory, which performs PCR testing of students in the Czech Republic. It is based on the experience gained from discussions on possible improvements and streamlining of the pooling methods used.

Usage CASIO Classwiz calculators in education the future primary-school teachers in mathematics

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Joint work with R. Krpec, T. Barot, J. Guncaga

Calculators can simplify the teachers' role in their Mathematics classrooms under a crucial condition: They are applied in accordance with the principles of active learning. It means that their function remains supportive and does not become the goal of learning. Activities performed with their use would therefore concentrate on analyzing the properties of the objects and intensifying the students' comprehension. At the same time, technology can enrich education by introducing realistic problems. Most textbook problems use simplified parameters (small integers). These are remnants from the times when all calculations were done manually. Using more complex numbers would slow down the classroom processes. Now, the problems can use data which are closer to those used in science, technology and economics. In this paper, we present the results of qualitative research, the research problem of which was the implementation of the CASIO Classwiz calculator into the teaching of mathematical subjects for future teachers at the Faculty of Education of the University of Ostrava and Comenius University in Bratislava.

CAS used as tools in STEAM

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The concept of function is already present in pre-university education, but instead of knowing the concept accurately, at the beginning of their university and college studies, students enrolled in STEAM courses identify the function with the concept of real function, and many people only know about the graphical picture of it, including polynomial functions, and possibly basic trigonometric functions. An overview of the concept of function and clarification of basic concepts in universities is also important, because it is seen by students in a wide variety of forms and often in different approaches in their later studies. By using computer algebra software, not only it will be easier to visualize functions, but we can also gain a deeper understanding of the concept by not using the "black box mathematics " way, i.e. focusing only on the advantages of clickable mathematics, but also on the so-called principle of "white box mathematics", notions in the sense used by Buchberger [1990] , i.e. we use this powerful tools to "walk around" the concepts we have learned and understood over and over again and to seek a more precise outline and a deeper understanding of the concept, to understand the limits and possible contradictions of the application of the concept. Computer algebra is a tool, a fantastically powerful tool, but only to the extent that it is consciously used to interpret the computer's "responses" correctly and to consciously find the methods that are most appropriate for a specific task, technical application, and in the computer program we need to "correctly" ask our questions , "translate " into the language of mathematics the task to be solved in real life, or the task to be fulfilled in the technical question and finally the result obtained is properly interpreted and "reversed" into the language of real life or technical application. The principle of white box mathematics means that computer algebra helps those who use it not only to help them solve tasks, but also to further develop them, to raise new questions and, ultimately, to a higher level of understanding and application of mathematics, see Buchberger [2002].

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Developing algorithmic thinking within STEM education for the preparation of future primary school teachers using robotic toys

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The development of algorithmic thinking for children in primary education is usually a task within the subjects of mathematics, informatics, but also involves other science subjects. With programmable robotic toys such as BeeBot, Botley, etc. children can be taught algorithmic thinking already in kindergarten. However, teachers must have sufficient knowledge of how to use these digital tools in STEM education to successfully integrate them at all levels of education. At Comenius University, we carried out research with future primary education teachers and examined their uses of robots in classrooms for a semester. We will report on the influence of robotic toys on teachers' and students' their motivation, creativities in a constructivist educational environment.

Use of augmented reality in STEAM education at primary school

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Nowadays, children are constantly surrounded by mobile technologies. Children use them from an early age. Some teachers perceive them negatively, but they can assist in teaching if we use them appropriately. This work presents findings of some possibilities for using mobile technologies, especially the augmented reality application Quiver usable in teaching in primary education. The method used for this research is quality research observation and we used semistructured interviews with children. The aim of the unstructured observation were proceedings of the teaching process via his/her teaching, the teacher began to consciously support the pupils' digital literacy development through augmented reality. The results of the pre-research form the base for the direction of the further study concerning the augmented reality in teaching practice. The paper focuses on the educational potential of augmented reality for support in primary education. The chapter presents the opportunities that augmented reality brings to primary education. The author gives a brief overview of selected application suitable for primary school children. She then focuses on application Quiver, which used in a research project in Primary school. The research shows that children are more motivate or not with application with augmented reality and if their estimation improves. The research aim is to create a model for didactic support to develop digital literacy of children in primary education through augmented reality in STEAM education. The main aim is to find out the impact and benefits of augmented reality on development of childrens' digital literacy and imagination. In the research I observed pupils in the digital environment of Quiver application. This application helps pupils visualize mathematical objects and many other things that are difficult to imagine. The author observed students' behaviour during using application and whether they were motivated by their work.

Usage of online platforms in education of mathematics in Transcarpathia at the beginning of quarantine

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Distance learning and e-learning as concepts have been in our minds for a long time. In March 2020, they suddenly gained great importance due to the introduction of quarantine and were immediately put into practice. It had to be applied in the everyday lives of teachers and students with surprising speed.
The goal of this research is to assess and demonstrate how teachers overcome the difficulties of mathematics education in distance learning. For this purpose, a month later after the beginning of distance education, I conducted a questionnaire survey among 20 teachers of mathematics in Transcarpathia with several different work experiences and who teach in several educational institutions. They were asked how education went on during quarantine, how they chose the platforms and methods needed to hold their lessons, what the checking and testing process was, what advantages and disadvantages they faced in distance learning.

Prospective elementary teachers' first experiences on designing Geogebra learning activities: Potentials and challenges

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Joint work with N. Hermita, D. Dahnilsyah

The integration of technology in learning is increasingly popular and inevitable, especially during the Covid-19 pandemic. In mathematics education, dynamic mathematics software, such as GeoGebra, has been used as a pedagogical tool in learning mathematics from primary school to university, thus supporting teachers' competencies on using technologies as instructional mathematical learning becomes crucial. Therefore, the present study explores prospective elementary teachers' first experiences designing GeoGebra learning instruction. Thirty-eight prospective elementary teachers work in small groups of 2-4 members to discuss and design learning activities based on GeoGebra. We present some examples of their projects aiming to explore some potentials and challenges from integrating GeoGebra as a pedagogical tool to support students' learning.

Forming the concept of function at the intersection of mathematics and physics

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The recent school reform in Poland has introduced some significant changes. The phenomenon of lack of correlation between mathematics and physics teaching in the context of concept of function which has been relocated to the secondary school level of mathematics, but is needed in the teaching of kinematics at the elementary school level of physics was the subject of our analysis in the article (Sajka,Rosiek, 2019). We posed two constructive recommendations on how to get out of this impasse. First was that mathematics teachers should teach the needed propaedeutic of the notion of function before (or at the same time) physics education starts in primary school (7th grade in Poland). This should be done purposefully during mathematics classes, with extreme care taken in regard to choosing the proper examples, including those based on movement analysis, and not only on statistical analysis, as recommended by the Polish Ministry of Education. Secondly, the optimal solution seems to be cross-curricular teaching, so that physics and mathematics teachers can introduce mutually-related issues, both emphasizing what is important from the standpoint of their science. This solution is in line with idea of STEM education - an issue which has recently been explored by researchers. In this presentation we would like to demonstrate chosen results of our research on attempts to implement these recommendations, combining our first and second conclusion. Our research involved a teacher teaching both physics and mathematics. In the presentation we will demonstrate the results of a study conducted in the methodology of a pedagogical experiment, which was carried out in the context of remote learning using ICT (e.g. Tracker software) and applying physical experiments presented remotely. Literature Sajka, M., Rosiek, R. (2019). Struggling with physics and mathematics curricula based on the notion of function in the context of the educational reform in Poland, AIP Conference Proceedings 2152, 030029; doi.org/10.1063/1.5124773

Mathematical Problem Solving with Technology: a Case Study with Pre-Service Teachers

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In educational environments, technology is present as a resource that facilitates teaching and learning. In the teaching and learning of mathematics, at all levels of education, it needs to integrate not only technology, but also the establishment of links with other areas of knowledge [1], namely with the sciences in general. The advantage of using technology in the teaching of mathematics and its effects on professional development, namely in basic and secondary education, it is well studied [4]. Several authors [2,3] are unanimous in stating that the teaching of mathematics, supported by activities supported by technology, favors the development of positive attitudes that will lead to better learning and a greater taste for this science. Even in higher education, modern education in science, technology, engineering and mathematics (STEM) faces fundamental challenges.
The objective of the present study is to analyse, the learning strategy with the use of technologies in mathematical activity, analyse mathematical activities and ways of thinking in Higher Education. The research methodology adopted consists of a case study, relating to a group of pre-service teachers in a public Higher Education. A qualitative approach was adopted through the interpretation of data collected through the activities on GeoGebra Classroom, brief questionnaire and conducting individual interviews, to triangular data interpretation.
It is concluded that the technologies have a significant participation in the educational environment and favour teaching and learning. The teacher, when perfecting his pedagogical practice, will be able to insert the technological tools in teaching and learning, to improve the interaction with students and favour the improvement of learning with the modelled use of technology in the classroom.
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Using neural networks in intelligent recognition problems of manuscripts

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Joint work with Zh. Zhunussova, Ye. Akkazin, Ye. Ashimov

One of the areas of artificial intelligence is machine learning which plays an important role in the field of IT. The amount of digital data we use in everyday life is growing, therefore there is a need for automatic analysis of smart data for further development of technological progress. The active development of machine learning has led to the widespread use of artificial neural networks. Neural networks based on the biological structure of the human brain are several times more efficient than other learning algorithms due to their high level of computational capabilities.
The application of neural networks to the problems of intelligent recognition of manuscripts in English and

French are discussed in this article. Handwriting recognition is one of the most important reports when processing documents. One of the most common machine learning methods in solving this problem is neural networks. Among the artificial neural networks, the analysis of the use of Convolution Neural Networks (CNN), LSTM (Long Short-Term Memory), MDLSTM (Multidimensional Long Short-Term Memory), MTRNN (Multidimensional Recurrent Neural Network) in intelligent recognition of manuscripts is carried out. The article describes how the IAM database was used for recognizing manuscripts in English, the RIMES database for recognizing French manuscripts. In addition, various approaches and achievements of recent years in the field of intelligent recognition of manuscripts are described, a review of the models of recognition of manuscripts related to Cyrillic graphics is conducted, and the results are analyzed.

The most common mathematical mistakes in the teaching of science subjects at secondary schools

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Mathematics is one of the subjects with the greatest overlap in other fields, especially natural sciences. Unfortunately, it is still taught separately and without sufficient connection to its use, for example, in physics, chemistry, or other areas. As a secondary effect of the project focused on creating and piloting problem tasks in the field of chemistry and physics, based on inquiry-based learning, we observed the most common mathematical mistakes and errors that students make. A total of 40 problem tasks on topics: water, air, colours and temperature from the physical and chemical point of view were created in cooperation with secondary-school teachers of physics and chemistry. Each task was verified on one class of the involved secondary school. By involving teachers in the work, it was also possible to monitor whether these mistakes arise from students' misunderstandings of mathematics or whether mistakes are transmitted from their teachers by misinterpretations of mathematics and thus give room for bad habits. The paper will present the most common mistakes and errors repeated across all tasks and compare their occurrence in teachers who have or do not have mathematics as a second subject. The mistakes and errors will also be explained from the mathematical point of view and a proposal will be outlined on how to innovate the teaching of mathematics at secondary schools. This would lead to a correct understanding of these areas and elimination of errors.

Teaching Mathematics to Enginering Students - Challenge and Adventure

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It has been proved by the everyday evidence that implementation of active learning methods in teaching increases enthusiasm for both learners and facilitators. And even more, active learning also improves learners' perception and attitude towards studied subject, strengthens their motivation and raises their interest and involvement into the entire learning process and desire to acquire new knowledge. These are all critical attitudes in establishing an active learning environment, in mathematical subjects in particular. The Fourth Industrial Revolution era with emerging 4D industry is imposing new challenges to the whole society and human activities, education at the first place. Teaching mathematics to engineering students becomes a challenge, as the roles of teachers in this new framework are changing accordingly. Universities are requested to provide their students with high scientific and soft-skill competencies needed for their future careers. Teachers must be prepared to work within international and multicultural teams practicing group collaboration, rather than work on individual basis, disconnected from everyone else in the world. Giving our students a stimulating learning experience in various different learning environments and introducing innovative ways of teaching focused on active learning is at the heart of the Education Strategy of our Age. A short analysis of the results of experiment carried out at the Faculty of Mechanical Engineering STU within the DRIVE MATH project activities will be presented in the talk. Project partners cooperated on examination of best strategies for implementation of active learning methods, innovative teaching strategies and adaptation of course curricula emphasizing the problem-based-learning approach, learning by doing (hands-on), and application of the eduScrum as pedagogical approach promoting active learning in engineering mathematics courses. The main goals of the experiment were to find out: - abilities of students to solve mathematical problems independently, and within a small stable group throughout the semester,

- attitudes and opinion of students on this form of teaching scenario, - opinion of students on introduction of applied mathematical problems to the work in groups. The main reason of the didactic experiment was to understand priorities of young people who begin as freshmen at the technical university with diverse study strategies, very different level of knowledge and learning skills, and uncertain motivation. University educators seek to improve their teaching methods in order to meet expectations of newcomers, but sometimes their efforts are directed in wrong way, missing the target group of students. Their needs differ from general conviction how they are used to work, learn and acquire new knowledge and information, due to many factors that influence everyday life and cause inevitable generation change. The results from a feedback questionnaire for students after completing the course revealed differences of their perceptions with respect to eduScrum and traditional methods. Students feel like discussing and solving problems in group context enhanced in the eduScrum environment is more stimulating then traditional individual learning strategies. Summarizing experiment outcomes, we have found implementation of active learning methods into basic courses of mathematics in STEM as a real challenge and adventure enriching both students and teachers.

Small Private Online Course (SPOC) for teaching modern mathematics education with technology in higher education during coronavirus pandemic

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Joint work with Jianlan Tang, Mario Jose Divan

The coronavirus began to spread at the end of 2019. The Chinese government carried out lockdown steps for all cities in China, stopping all people's activities for 5 months. When students cannot go to university to study, many universities take advantage of the massive Open Online Course (MOOC) and Small Private Online Course (SPOC) to support students still studying at home. This study aimed to analyze the effectiveness of SPOC and the problems during the implementation of SPOC during the coronavirus pandemic in higher education. The method in this re-search is descriptive qualitative. The sample was taken by using purposive sampling technique at Guangxi Normal University with a sample of 27 master's degree students who takes the subject of "modern mathematics education with technology" course. The research data is the data on students' activeness accessing SPOC on the zhihuishu platform and the results of the students' final project to make mathematics learning media using dynamic mathematics software. The results showed the effectiveness of SPOC on student learning activeness using the Zhihuishi platform reached 81.5

Use of robotic toys for the development of digital and mathematical literacy in preschool education

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Joint work with L. Korenova

In the Czech and Slovak Republics, the content of education at all levels is currently being innovated to support the better development of thinking in the field of IT. Informatics is integrated into all educational subjects. In connection with this reform, kindergartens and institutions that prepare future kindergarten teachers are responding to the innovation of educational programs. The comprehensive methodology of teaching mathematics in kindergartens already includes the methodology of working with robotic toys. This occurs in many didactic environments, both arithmetic and geometric. We will present activities related to these environments, show their use in kindergarten and think about their implementation by some teachers. At the beginning of 2021, we conducted research into the use of robotic toys in kindergartens. We wondered if and how they use robotic toys in their kindergarten. In this paper, we present the analysis and results of this survey. We will also present the results of qualitative research that monitored the activities of children and the work of teachers with robotic toys (bee-boot, code-a-pillar, robotic mouse,

etc.).

Challenges to the development of effective creativity

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Joint work with V. Mityushev, Ye. Ashimov, M. Rahmani, H. Noori

One of the key problems for students, especially from developing countries during pandemic is a lack of the electronic materials. Obviously, there are a lot of problems arise during education in the online regime. First of all, it is a weak Internet connection and combined with the lack of a appropriate equipment. Even having a higher quality computer they could not setup a program. It is connected to the both their knowledge and the specialty. For example, the students biology, chemistry, philology and others are not taught to the skills of a programmer. In general, they are users as ordinary people. But the real situation concerning pandemic is required its own rules. And these rules for all of the students in a group, do not depend on a country. That is why the math teachers have to look for an alternative method to make a proper decision for a stated problem under supervision of a diploma work. Modern computer programs improve the visualization tool which helps to select an optimal model for the considered problem. Especially, this concerns students of the specialties "Mathematics" and "Automation and control". We pay attention to the textbook [1]. It plays the role of a guidance for teachers as well as for students. The textbook establishes the general principles and methods of mathematical modeling. At the beginning a simple mathematical model is considered. The model is explained by hand calculations as well as applying the packages Mathematica and Matlab. There are examples. The textbook is useful for beginners as well as for higher qualified specialists.

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Session 3: Complex Analysis and Partial Differential Equations

Organizers: Sergei Rogosin, Ahmet Okay Celebi and Carmen Judith Vanegas

On a normalization of the Wiener–Hopf factorization for matrix functions

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Joint work with V. Adukov

We study the problem of a normalization for the Wiener–Hopf factorization of matrix functions. The Wiener–Hopf factorization

$$A(t) = A_-(t)D(t)A_+(t)$$

is called canonically normalized if the factors $A_\pm(t)$ are uniquely determined and the Wiener–Hopf factorization generates the Birkhoff factorization $A(t) = PD(t)P^{-1}B_-(t)B_+(t)$, where P is a permutation matrix. We consider in detail the case of 2×2 matrix functions with distinct partial indices. It turns out that there are two types of the normalization. The first type exists if for $A(t)$ the inequality $\left(A_-(\infty)\right)_{11} \neq 0$ is fulfilled. If $\left(A_-(\infty)\right)_{21} \neq 0$, then we have the second type of the normalization. For these types of the normalization the canonical forms of $A_-(t)$ and $B_-(t)$ are found. We prove that the types of the normalization and the factors $A_\pm(t)$ for the canonically normalized factorization are stable under a small perturbation of $A(t)$ preserving the partial indices. Similar results are obtained for $n \times n$ matrix functions in the stable case.

This work was supported by RFBR grant no. 20-41-740024.

Solution of Mixed Boundary Integral Equation via Adjoint Generalize Neumann Kernel on Multiply connected Domains.

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We present a unified boundary integral method for approximating the solution of the mixed boundary value problem from any bounded multiply connected domain. The mixed problem which reformulated in the form of Riemann–Hilbert(briefly, RH) problem which leads to a uniquely solvable Fredholm equation of the second kind. We present a uniquely solvable boundary integral equation with the adjoint generalized Neumann kernel for solving two-dimensional Laplace problem in bounded multiply connected regions with mixed boundary condition. The performance of the method is illustrated by several examples for regions with smooth boundaries and with piecewise smooth boundaries

Neumann-type problem for higher-order Poisson equation in Clifford analysis

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Joint work with A. Okay Celebi

Cauchy-Pompeiu type integral representation formulas and Neumann functions are given in the case of complex Clifford algebra $\mathbb{C}_m$. Neumann-type boundary value problem for higher-order Poisson equation is studied for $\mathbb{C}_m$-valued functions in $\mathbb{R}^m$.

Fractional-differential model of the viscoelastic periodontal ligament

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The periodontal ligament is a complex component of the tooth root supporting apparatus. It's a thin layer of soft connective tissue located between the hard surfaces of the tooth root and the dental alveolar bone.

One of the approaches to analytical modeling of the viscoelastic periodontal ligament behavior is the usage of a fractional kernel. The aim of this study is the assessment of the parameters of the Rabontov's fractional relaxation kernel during the tipping of the asymmetric tooth root.
The assessment of the relaxation kernel parameters was carried out on the basis of experimental dependences (R.J. Pryputniewicz and C.J. Burstone, 1979) of the tooth root displacements in the periodontal ligament under the action of a concentrated load. It was appeared that for any load, the simultaneous change of the parameter $\nu_\sigma = \frac{E_\infty - E_0}{E_0}$ and the fractional parameter γ enables to assign different time intervals for the phase transition of the periodontal ligament and the maximum magnitude of the tooth root displacements in the periodontal ligament. The maximal displacement magnitude and angle of rotation for different load values can be assign by changing the parameter ν_σ. An increase of the fractional parameter leads to an increase of the duration of the phase transition and the magnitude of the tooth root maximum displacement (for constant values of the parameters ν_σ and the retardation time τ_σ).
Study was supported by project Ph20R-083 of the Belarusian Republican Found for Fundamental Research.

Analysis Results for Dynamic Contact Problem with Friction in Thermo-Viscoelasticity

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Joint work with M. Bouallala and EL-H. Essoufi

We present a mathematical model which describes the dynamic frictional contact between a thermo-viscoelastic body and a conductive foundation. The contact is modeled using the normal compliance condition, the quasistatic version of Coulomb's law of fry friction. We derive the weak formulation and we prove the existence and uniqueness result. The proofs are based on the theory of first-order and second-order evolution inequalities and Banach fixed point theorem. We introduce a new problem on perturbation of the contact boundary condition and we establish its continuous dependence result

The Schwarz problem for higher order equations in a ring domain

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Joint work with Pelin Ayse Gökgöz

In this presentation the results, obtained in the Proceedings of 12. ISAAC Congress, will be extended to the Schwarz problem for higher order equations in a ring domain. The statement of the problem has as slight difference compared with the articles of Vaitekhovich. After the preliminaries we obtain the relevant integral representation which will lead to the solutions of higher order differential equations. Existence and uniqueness of the solutions for a second order linear equation is obtained using Fredholm alternative.

On One Tension-compression Oscillation Problem of Hierarchical Model for Kelvin-Voigt Plates with Variable Thickness

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The tension-compression oscillation problem is investigated in the zero approximation of governing system for Kelvin-Voigt plates with variable thickness which were derived by G. Jaiani [1], using I. Vekua's [2], [3] dimension reduction method.

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Convolution equations on the interval [-1,1]

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We will show what is in common between the celebrated Prandtl equation

$$\mathbf{P}u(x) = \frac{a(t)}{1-t^2}u(t) - \frac{1}{2\pi}\int_{-1}^{1}\frac{u'(\tau)d\tau}{\tau - t} = f(t), \qquad t \in \mathcal{J} := [-1,1], \tag{1}$$

the Singular Tricomi equation

$$\mathbf{T}v(x) = c_0 v(x) + \frac{c_1}{\pi i}\int_{-1}^{1}\frac{v(y)dy}{y-x} + \frac{c_2}{\pi i}\int_{-1}^{1}\frac{v(y)dy}{1-xy} = g(x), \qquad x \in \mathcal{J}, \tag{2}$$

the Lavrent'ev-Bitsadze equation

$$\mathbf{LB}\varphi(t) = c_0\varphi(t) + \frac{c_1}{\pi}\int_0^1\left[\frac{1}{\tau - t} + \frac{1-2\tau}{t+\tau-2t\tau}\right]\varphi(\tau)d\tau = h(t), \qquad x \in \mathcal{I} := (0,1) \tag{3}$$

and the Laplace-Beltrami equation on the unit sphere $\mathbb{S}^2 \subset \mathbb{R}^3$ written in spherical coordinates. These equations can be solved precisely, in closed form. For this we apply methods of convolution equations on the Abelian group $\mathcal{J} = [-1,1]$, equipped with the group operation $x +_{\mathcal{J}} y := \dfrac{x+y}{1+xy}, \quad x, y \in \mathcal{J}$, and the corresponding Fourier transform with the inverse. Equations (1)-(2) are solved in the Bessel potential spaces $\mathbb{H}^s_p(\mathcal{J}, d_{\mathcal{J}}x)$, $1 < p < \infty$, $s \in \mathbb{R}$, with the measure $d_{\mathcal{J}}x := (1-x^2)^{-1}dx$, based on the "weighted" derivatives $\mathcal{D}_{\mathcal{J}} := -(1-x^2)\dfrac{d}{dx}$.
These results extend those obtained earlier by Petrov V. E. and Suslina T.A.-Petrov V.E..

On local well-posedness and ill-posedness results for a coupled system of mkdv type equations

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Joint work with Xavier Carvajal

In the present work we study a dissipative versions of the Korteweg de Vries equation with rough initial data. By working in Bourgain's type spaces we prove local well posedness results in Sobolev spaces of negative order.

Acoustic propagation in a random saturated medium

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We study the problem of derivation of an effective model of acoustic wave propagation in a two-phase, non-periodic medium modeling a fine mixture of linear elastic solid and a viscous Newtonian fluid. Bone tissue is an important example of a composite material that can be modeled in this fashion. We extend known homogenization results for periodic geometries to the case a stationary random, scale-separated microstructure. The ratio ε of the macroscopic length scale and a typical size of the microstructural inhomogeneity is a small parameter of the problem. We employ stochastic two-scale convergence in the mean to pass to the limit $\varepsilon \to 0$ in the governing equations. The effective model is a biphasic phase viscoelastic material with long time history dependence.

Rearrangement universality of the prime numbers' power series in a complex field

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Joint work with G. Chelidze, V. Tarieladze

We obtain that for any complex number $s, 0 < Re(s) \leq 1$, the prime numbers' power series $\sum_n \frac{(-1)^{n-1}}{p_n^s}$ is universal in $\mathbb{C}$, i.e. its sum range under the rearrangements is the whole complex number field $\mathbb{C}$. As a consequence we get that the Dirichlet series $\sum_n \frac{(-1)^{n-1}}{n^s}$ is universal as well.

Solution of the Kirsch problem for the elastic materials with voids in the case of approximation N = 1 of Vekua's theory

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We consider a boundary value problem for an infinite plate with a circular hole. The plate is the elastic material with voids. The hole is free from stresses, while unilateral tensile stresses act at infinity. The state of plate equilibrium is described by the system of differential equations that is derived from three-imensional equations of equilibrium of an elastic material with voids (Cowin-Nunziato model) by Vekua's reduction method. its general solution is represented by means of analytic functions of a complex variable and solutions of Helmholtz equations. The problem is solved analytically by the method of the theory of functions of a complex variable. This is a joint work with R. Janjgava, T. Kasrashvili and M. Narmania.

N-th Approximation of Hierarchical Models for Cusped Kelvin-Voigt Plates with Variable Thickness

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Using I. Vekua's [1], [2] dimension reduction method, governing systems are derived and in the Nth approximation boundary value problems are set for Kelvin-Voigt plates with variable thickness. The ways of investigation of boundary value problems are indicated. In addition tension-compression and bending problems are investigated in the zeroth approximation of hierarchical models.

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On a three dimensional continuous octonion wavelet transform

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Joint work with Eusebio Alberto Ariza García

Wavelets and wavelets transforms have been studied for its many applications in areas like image processing and signal analysis in general. Because of this, the wavelet transform has been generalized to the context of complex and hypercomplex algebras. In this talk we introduce a three dimensional continuous octonion wavelet transform and give an admissibility condition to define octonion mother wavelets using a three dimensional octonion Fourier transform. We also give some of its main algebraic properties and give an inversion formula.

Integration over non-rectifiable curves and Riemann boundary value problem

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The talk is devoted to broad results obtained in connection with the classical problem of complex analysis - the Riemann boundary value problem - using integration generalized to non-rectifiable curves. Both simple and well-studied cases will be considered, as well as situations complicated by geometric and other features of arcs. The presentation will show the main results of recent years in this area, as well as several open problems.

Boundary Value Problems with displacement for the generalized analytic vectors

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Joint work with George Akhalaia, George Makatsaria

The present work deals with the linear conjugation type boundary value problems with displacement for the generalized analytic vectors. The solvability conditions of the problems and the index formulas are obtained.

Acknowledgment. The work is supported by the Shota Rustaveli National Science Foundation (SRNSF grant # FR 17-96).

On space-periodic problems for Stokes and Navier-Stokes equations

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First, the uniqueness and existence of solution to stationary anisotropic (linear) Stokes system in a compressible framework on n-dimensional torus are analysed in a range of periodic Sobolev (Bessel-potential) spaces. By employing the Leray-Schauder fixed point theorem, the linear results are used to show existence of solution to the stationary anisotropic (non-linear) Navier-Stokes incompressible system on torus in a periodic Sobolev space. Then the solution regularity for stationary anisotropic Navier-Stokes system on torus are proved. Some of these results are also extended to the corresponding non-stationary problems.

Separability and compactness, estimates for the eigenvalues and singular numbers (s-numbers) of a resolvent of a class of singular parabolic operators

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Joint work with M.M. Muratbekov, S. Igisinov

In this paper we study the singular parabolic operator

$$Lu = \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + q(x)u$$

initially defined on $C_{0,\pi}^{\infty}(\bar{\Omega})$ where $\bar{\Omega} = \{(t,x) : -\pi \leq t \leq \pi, -\infty < x < \infty\}$. $C_{0,\pi}^{\infty}(\bar{\Omega})$ is the set, which consist of infinitely differentiable finite functions with respect to the x and satisfying the condition

$$u(-\pi, x) = u(\pi, x).$$

We assume that the coefficients of L are continuous functions in $\mathbb{R} = (-\infty, \infty)$ and a strongly growing functions at infinity.

The operator L admits closure in $L_2(\Omega)$ and the closure we also denote by L.
In the paper, we have proved that there exists a bounded inverse operator and found a condition on $q(x)$ that ensures the existence of the estimate

$$\|\frac{\partial u}{\partial t}\|_{L_2(\Omega)} + \|\frac{\partial^2 u}{\partial x^2}\|_{L_2(\Omega)} + \|q(x)u\|_{L_2(\Omega)} \leq c(\|Lu\|_{L_2(\Omega)} + \|u\|_{L_2(\Omega)}),$$

with some restrictions on the coefficients, in addition to the above conditions; the compactness of the resolvent is proved; two-sided estimates of singular numbers (s-numbers) are obtained and an example is given of how above estimates will make it possible to find estimates for the eigenvalues of the operator under consideration.
Example. $q(x) = e^{1000|x|}, -\infty < x < \infty$.

Carathéodory balls and proper holomorphic maps on multiply-connected planar domains

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Joint work with Chiu Chak Tang and Jonathan Tsai

We establish the existence of disconnected open balls and the inequivalence of closed balls and the closure of open balls under the Carathéodory metric in some planar domains of finite connectivity greater than 2. This resolves a problem posed by Jarnicki, Pflug and Vigué in 1992. A corresponding result for some higher dimensional pseudoconvex domains is also obtained. Our results follows from an explicit characterization of proper holomorphic maps from a non-degenerate finitely-connected planar domain onto the unit disk which answers a question posed by Schmieder in 2005. This is analogous to Fatou's famous result that proper holomorphic maps of the unit disk onto itself are finite Blashcke products. A parameter space for proper holomorphic maps is also determined which extends a result of Grunsky in 1941.

Contributions of early works in PDE to the foundations and applications of Relevant Arithmetic

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$R^{\#}$ is Peano arithmetic with the first-order relevant system R as underlying logic. R is classical logic minus the irrelevant classical logic axiom that says that from a false antecedent anything can follow. R. K. Meyer proved that $R^{\#}$ being relevant Peano arithmetic formulated relevantly in the classical vocabulary, every theorem in the classical vocabulary of Peano arithmetic is also a theorem of $R^{\#}$.
This short way of presenting the system skips all rather involved technical details. The point of interest is that this result was seen as a interesting attempt at supporting, if not resurrecting, Hilbert's program of proving the consistency of mathematics by "finitary" means. A problem, though, was that H. Friedman provided a counterexample showing that the program was failing. Still, various extensions to the original system appeared to fix it anyway.
In this pretty historical contribution, we show to what extent some early contributions to the solution of systems of ODE and PDE in analysis, staring with Cauchy, Peano, Méray, Riquier, Janet (but not only; Sylvester, Hilbert and Invariant theory had their role in the picture) provided the basis of some powerful finiteness principles that not only underlie the solutions to various decision problems in logic and in algebra, this is a fact, but also support various daring intuitions. Examples of facts are the theory of WQO or, in computer algebra the so-called Gröbner basis.
Among the applications of $R^{\#}$ in number theory that were considered, a test case -Meyer's original intuition and dream- was an investigation of Fermat's LT (why not?). Wiles having found a proof, even though his proof seemed to be out of reach of PA, there was still Meyer's original intuition that some elementary proof must exist. An intuition shared by H. Friedman with his conjecture of FLT provable in EFA and some other logicians or number theorists hoping for appropriate extensions of PA like $R^{\#}$.

Factorization of the matrix coefficient corresponding to R-linear cojugation problem

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Joint work with L.Primachuk, M.Dubatovskaya

R-linear conjugation problem is reduced equivaqlently to the vector-matrix boundary value problem. We propose a method of factorization of the corresponding matrix coefficient. The method consists of two-fold application of G.N.Chebotarev approach.
At the first stage we transform the coefficient to the triangular form. Then the canonical matrix of the vector-matrix boundary value problem is determined by using G.N.Chebotarev approach. Thus the partial indices are defined and the solbability of R-linear conjugation problem is completely described.
The method is illustrated by the explicit calculation in the case of polynomial and rational coefficients of the starting problem.
Acknowledgement. The work is partially supported by the State Programm of the Scientific Investigations of the Republic of Belarus through the grant 1.7.01.04.

The Kippenhahn curve of some structured matrices

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Joint work with As per literature cited

The numerical range $W(A)$ (a.k.a. the field of values, or the Hausdorff set) of an n-by-n matrix A is defined as the image of the unit sphere of $\mathbb{C}^n$ under the mapping $f_A\colon x \mapsto x^*Ax$. It is a compact subset of $\mathbb{C}$, which is also convex due to the celebrated *Toeplitz-Hausdorff theorem*. Moreover, $W(A)$ is the convex hull of a certain algebraic curve $C(A)$ of class n, thus called the *numerical range generating curve* of A [6]. This provides an insight into the *Elliptical range theorem*: for $n = 2$ the numerical range is an elliptical disk (degenerating into the line segment connecting the eigenvalues of A when A is normal).
As n increases, there is more variety in possible shapes of $W(A)$. Surprisingly though, for some classes of matrices $W(A)$ stays elliptical (or ends up being the convex hull of a small, compared to n, number of ellipses). The state of the matter, as of the beginning of the century, has been described in [3]. It became clear later that the phenomenon at hand is caused by $C(A)$ consisting of several components, the "exposed" of them being ellipses.
In this talk, we describe several such classes. It is based on [4, 5, 2, 1], and some work in progress.

Acknowledgment. The work is partially supported by Faculty Research funding from the Division of Science and Mathematics, New York University Abu Dhabi.

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Laplace-Beltrami equation on hypersurfaces and Γ-convergence

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Joint work with T. Buchukuri and R. Duduchava

Let us consider heat conduction by an "isotropic" media, governed by the Laplace equation with the classical Dirichlet-Neumann mixed boundary conditions on the boundary in the layer domain $\Omega^\varepsilon := \mathcal{C} \times (-\varepsilon, \varepsilon)$ of a thickness 2ε. More precisely we impose zero Dirichlet and non-zero Neumann data on the corresponding parts of the boundary

$$\begin{array}{ll} \Delta_{\Omega^\varepsilon} T(x,t) = f(x,t), & (x,t) \in \mathcal{C} \times (-\varepsilon, \varepsilon), \\ T^+(x,t) = 0, & (x,t) \in \partial\mathcal{C} \times (-\varepsilon, \varepsilon), \\ \pm(\partial_t T)^+(x, \pm\varepsilon) = q(x, \pm\varepsilon), & x \in \mathcal{C}, \end{array}$$

where $\pm\partial_t = \partial_\nu$ represents the normal derivative on the surfaces $\mathcal{C} \times \{\pm\varepsilon\}$. Here $\mathcal{C} \subset \mathcal{S}$ is a smooth subsurface of a closed hypersurface $\mathcal{S}$ with smooth nonempty boundary $\partial\mathcal{C}$.
The suggested approach is based on the fact that the Laplace operator $\Delta_{\Omega^\varepsilon} = \partial_1^2 + \partial_2^2 + \partial_3^2$ is represented as the sum of the Laplace-Beltrami operator on the mid-surface and the square of the transversal derivative:

$$\Delta_{\Omega^\varepsilon} T = \sum_{j=1}^{4} \mathcal{D}_j^2 T = \Delta_{\mathcal{C}} T + \partial_t^2 T.$$

In the report we will review what happens with the above mentioned mixed boundary value problem when the thickness of the layer converges to zero in the sense of Γ-convergence. It is proved that the limit coincides with the Dirichlet BVP for the Laplace-Beltrami equation, which is described explicitly. It is shown how the Neumann boundary conditions from the initial BVP transform during the Γ-limit and wanders to the right hand side of the limit BVP. For this we apply the variational formulation and the calculus of Günter's tangential differential operators on a hypersurface and layers, which allow global representation of basic differential operators and of corresponding boundary value problems in terms of the standard Euclidean coordinates of the ambient space $\mathbb{R}^n$.
A similar results on Γ-limits of BVP for the Laplace equation, but for a plate, with a different approach and for different boundary conditions, was obtained in [1].

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Second order differential operators associated to the space of holomorphic functions

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Joint work with Gian Rossodivita

Let $\mathcal{F}$ be a given differential operator, then a function space $\mathcal{X}$ is called an associated space to $\mathcal{F}$ if $\mathcal{F}$ transforms $\mathcal{X}$ into itself. In this talk we show the construction of all operators of second order with complex coefficients that are associated with the space of holomorphic functions. As an application the solubility of initial value problems involving these operators is shown.

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Constructional methods for solutions of a free boundary problem model of cancer and related inverse problems

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In this talk we discuss a free boundary problem model of cancer. We will consider construction of solutions for the free boundary problems raised from the model. We will also discuss some inverse problems raised from the free boundary model of ductal carcinoma in situ.
We use some transformation and heat potential theory to establish the integral form of solution and proved the existence and uniqueness of solution to the related problem. Then we consider the inverse problem of determining the potential function of model from moving boundary information. Numerical simulations are presented to demonstrate the validity and applicability of solutions.

Session 4: Complex Variables and Potential Theory

Organizers: Tahir Aliyev Azeroglu, Massimo Lanza de Cristoforis, Anatoly Golberg and Sergiy Plaksa

Quasimöbius homeomorphisms between domains of Riemannian manifolds

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Joint work with A. Golberg

Let $\omega(t)$ be a distortion function. A topological embedding $f : X \to Y$ of a metric space is called ω-*quasimöbius (ω-QM)* [1], if for any four distinct points $x_1, x_2, x_3, x_4 \in X$,

$$\frac{d'(f(x_1), f(x_2)) \cdot d'(f(x_3), f(x_4))}{d'(f(x_1), f(x_3)) \cdot d'(f(x_2), f(x_4))} \le \omega\left(\frac{d(x_1, x_2) \cdot d(x_3, x_4)}{d(x_1, x_3) \cdot d(x_2, x_4)}\right).$$

The significance of quasimöbius mappings is that this class of mappings naturally lies between quasisymmetry and quasiconformality. For definitions of quasisymmetry and metric quasiconformality see also [1].
Theorem 1. *Let D and D' be domains in Riemannian manifolds $\mathbb{M}$ and $\mathbb{M}'$. Then the following statements hold.*

1. *If $f : D \to D^{'}$ is ω-QM, then f is a K-qc mapping.*
2. *If f is a K-qc mapping, then f is a locally ω-QM embedding on D.*

Theorem 2. *Let D and D' be domains on bounded c-quasiconvex Riemannian manifolds $\mathbb{M}$ and $\mathbb{M}'$, $z_l, z_2, z_3 \in \mathbb{M}$ and $d(z_i, z_j) \ge d(\mathbb{M})/\lambda$, $d'(f(z_i), f(z_j)) \ge d'(\mathbb{M}')/\lambda$ for $i \ne j$ and $\lambda > 0$. An embedding $f : D \to D'$ is ω-quasimöbius if and only if f is a weakly H-quasisymmetric embedding.*

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A singularly perturbed transmission problem for the Helmholtz equation

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Joint work with M. Lanza De Cristoforis

Let Ω^i, Ω^o be bounded open domains of $\mathbb{R}^n$ that contain the origin. Let $\Omega(\epsilon)$ be the annular domain obtained by removing from Ω^o the cavity $\epsilon\Omega^i$ for a small value of a parameter $\epsilon > 0$. Then we consider a linear transmission problem for the Helmholtz equation in the pair of domains $\epsilon\Omega^i$ and $\Omega(\epsilon)$ with Neumann boundary conditions on $\partial\Omega^o$. Under appropriate conditions on the wave numbers in $\epsilon\Omega^i$ and $\Omega(\epsilon)$ and on the parameters involved in the transmission conditions on the boundary of the cavity, the transmission problem has a unique solution $(u^i(\epsilon, \cdot), u^o(\epsilon, \cdot))$ for small small values of $\epsilon > 0$. Here $u^i(\epsilon, \cdot)$ solves the Helmholtz equation in $\epsilon\Omega^i$ and $u^o(\epsilon, \cdot)$ solves the Helmholtz equation in $\Omega(\epsilon)$. Then we analyze the behavior of $(u^i(\epsilon, \cdot), u^o(\epsilon, \cdot))$ as ϵ tends to zero by an approach that is alternative to that of asymptotic expansions.

Estimates of products of inner radii of pairwise non-overlapping domains

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Joint work with Alexander Bakhtin

We consider problems on extremal decomposition of the complex plane and propose an approach for obtaining an effective upper estimates for the products of inner radii of mutually non-overlapping domains with fixed poles corresponding quadratic differentials on radial systems of points. We establish conditions under which in the proven results the structure of points and domains is irrelevant.

Proved estimates of functionals have made it possible to find some enhanced solutions in an open extremal problems on mutually non-overlapping domains. In particular, an open problem of finding the maximum of product of inner radii of two domains relative to the points of a unit circle on the degree γ of the inner radius of the domain relative to the origin at arbitrary $\gamma \in (0, 2]$, provided that all three domains are mutually non-overlapping domains is solved. And it is generalized for the case of two an arbitrary points on the complex plane.

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Fractional versions of complex d-bar derivatives

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Joint work with C. Bouzouina

In complex analysis, the d-bar derivative provides a way of studying functions which are not necessarily holomorphic: it is equal to zero at a point if and only if the function is complex differentiable at that point. The Pompeiu formula is a generalisation of Cauchy's theorem to non-holomorphic functions using the d-bar derivative; the d-bar formalism can be used for solving partial differential equations; the theory of pseudoanalytic functions relates strongly to the d-bar derivative; and differential equations posed using d-bar derivatives have even found real-world applications.
Amidst the recent rise in popularity of fractional calculus, in which all manner of integro-differential operators are generalised to fractional orders, it is surprising how little use has been made of fractional ideas in complex analysis. Until recently, nobody had thought to define a fractional version of the complex d-bar derivative.
Here, I will present two closely related definitions of the fractional complex partial derivatives with respect to z and $\overline{z}$. These definitions are motivated by proving how they combine standard properties of fractional derivatives with standard properties of complex partial derivatives, thus demonstrating them as the natural choices for defining this notion. Properties explored include the Leibniz rule and composition properties, with pointers towards future directions of study such as fractional polyanalytic and pseudoanalytic functions.

Coefficients of quasiconformality of geodesic rings

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Joint work with E. Afanasjeva

The classical Riemann mapping theorem states that any simply connected domain in the complex plane $\mathbb{C}$ with more than two boundary points is conformally equivalent to the unit disk. By Liouville's theorem for dimensions $n \geq 3$, a smooth enough conformal mapping $f : D \to D'$ where $D, D' \subset \mathbb{R}^n$ is of the form $f = g|D$ for some Möbius transformation g. Thus the only domains conformally equivalent to the unit ball are the balls and half spaces of $\mathbb{R}^n$, $n \geq 3$.
The (inner and outer) coefficients of quasiconformality of domains in $\mathbb{R}^3$ were introduced by Gehring and Väisälä as the suprema of ratios between the moduli of arc families Γ^* and Γ in the image and the inverse image. Then the corresponding coefficients for a domain $D \subset \mathbb{R}^3$ are defined as the infima over all homeomorphisms of D onto the unit ball $\mathbb{B}^3$. The existence of the extremal mappings (for which the corresponding infimum is attained) follows from the lower semicontinuity of distortion coefficients.
The next essential step in studying extremal mappings in higher dimensions was done by Cabiria Andreian Cazacu. In her work, the conformal moduli of arc (curve) families were extended to the families of k-dimensional surfaces in $\mathbb{R}^n$, $k = 1, \ldots, n-1$.
In 1991, Golberg and Kud'yavin applied the above ideas and techniques to the class of mappings quasiconformal in the mean and spherical rings.
In this talk, we extend the above results to the Riemannian manifolds and geodesic rings. The monotonicity of the mean coefficients by real parameters and some illustrating examples are presented.

The generalized biharmonic equation with non-zero simple characteristics and monogenic functions taking values in commutative complex algebras of the second rank

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Among all two-dimensional algebras of the second rank with unity e over the field of complex numbers $\mathbb{C}$, we found a semi-simple algebra $\mathbb{B}_0 := \{c_1 e + c_2\omega : c_k \in \mathbb{C}, k = 1, 2\}$, $\omega^2 = e$, containing bases $\{e_1, e_2\}$, such that $\mathbb{B}_0$-valued "analytic" functions $\Phi(xe_1 + ye_2)$ (x, y are real variables) satisfy the fourth order homogeneous partial differential equation of the form:

$$\left(b_1\frac{\partial^4}{\partial y^4} + b_2\frac{\partial^4}{\partial x\partial y^3} + b_3\frac{\partial^4}{\partial x^2\partial y^2} + b_4\frac{\partial^4}{\partial x^3\partial y} + b_5\frac{\partial^4}{\partial x^4}\right)u(x, y) = 0, \tag{1}$$

where complex coefficients $b_k \in \mathbb{C}$, $k = \overline{1, 5}$, $b_5 \neq 0$, such than the Eq. of characteristics

$$l(s) := b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0, s \in \mathbb{C}, \tag{2}$$

has four has four pairwise different roots (each root is a simple root) roots.
A set of pairs $(\{e_1, e_2\}, \Phi)$ is described in the explicit form.
Particular cases and connections are considered in [1-3].

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Bounded composition operators on functional quasi-Banach spaces and stability of dynamical systems

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We investigate the boundedness of composition operators defined on a quasi-Banach space continuously included in the space of smooth functions on a manifold. We prove that the boundedness of composition operators strongly limits the behavior of the original map, and it provides us an effective method to investigate properties of composition operators via the theory of dynamical system. As a result, we prove that only affine maps can induce a bounded composition operator on any Banach space continuously included in the space of entire functions on the complex plane, in particular, on any reproducing kernel Hilbert space (RKHS) composed of entire functions of one-variable We also prove any polynomial automorphisms except affine transforms cannot induce a bounded composition operator on a quasi-Banach space composed of entire functions in the two-dimensional complex affine space under several mild conditions.

On asymptotically sharp Bernstein- and Markov-type inequalities

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In this talk, we discuss asymptotically sharp Bernstein- and Markov-type inequalities and demonstrate how the search for asymptotically sharp forms leads naturally to application of potential theory and geometric function theory. Some recent results will be presented too.

The results are based on joint works with Vilmos Totik and Béla Nagy.

Kelvin-Möbius-Invariant Harmonic Function Spaces on the Real Unit Ball

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Joint work with A. Ersin Üreyen

We define Kelvin-Möbius transforms as compositions with real Möbius maps followed by Kelvin transforms to preserve harmonicity. We determine the harmonic function spaces on the unit ball of $\mathbb{R}^n$ that are invariant under the action of these transforms. For each n, we identify the maximal and minimal invariant Banach spaces, the unique invariant Hilbert space, and all invariant Bergman-Besov spaces. There are essential differences between dimensions $n \geq 3$ and $n = 2$. The case $n = 2$ is similar to the holomorphic version, Kelvin transform is not needed, invariant spaces are defined with seminorms, and the counterparts of diagonal Besov spaces and the Bloch space are obtained. For $n \geq 3$, invariant spaces are defined with genuine norms, there is a whole family of invariant weighted Bergman spaces as well as Besov spaces, and a unique invariant harmonic Hardy space exists.

The Fox-Wright function near the singularity and the branch cut

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Joint work with Elena Prilepkina

The Fox-Wright function is a further extension of the generalized hypergeometric function obtained by introducing arbitrary positive scaling factors into the arguments of the gamma functions in each term of the series. Its importance comes mostly from its role in fractional calculus although other interesting applications also exist. If the sums of the scaling factors in the top and bottom parameters are equal, the series defining the Fox-Wright function has a positive and finite radius of convergence. It was demonstrated by Braaksma in 1964 that the Fox-Wright function can then be extended to a holomorphic function in the complex plane cut along a ray from the positive point on the boundary of the disk of convergence to the point at infinity. In the talk I will discuss the behavior of the Fox-Wright function in the neighborhood of this positive singular point. Under certain restrictions a convergent expansion with recursively computed coefficients will be presented completely characterizing this behavior. Finally, a formula for the jump and the average value of the Fox-Wright function on the banks of the branch cut will be given in terms of Fox's H function.

On the asymptotic behavior of ring Q-homeomorphisms

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Joint work with R. Salimov

Let Γ be a family of curves γ in $\mathbb{R}^n$, $n \geqslant 2$. A Borel measurable function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, (abbr. $\rho \in \operatorname{adm}\Gamma$), if

$$\int\limits_{\gamma} \rho(x)\, ds \ \geqslant\ 1$$

for any curve $\gamma \in \Gamma$. Let $p \in (1, \infty)$. The quantity

$$M_p(\Gamma) \ =\ \inf_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)$$

is called p*–modulus* of the family Γ.

For arbitrary sets E, F and G of $\mathbb{R}^n$ we denote by $\Delta(E,F,G)$ a set of all continuous curves $\gamma:[a,b]\to\mathbb{R}^n$, that connect E and F in G, i.e., such that $\gamma(a)\in E$, $\gamma(b)\in F$ and $\gamma(t)\in G$ for $a<t<b$.
Let D be a domain in $\mathbb{R}^n$, $n\geqslant 2$, $x_0\in D$ and $d_0=\mathrm{dist}(x_0,\partial D)$. Set

$$\mathbb{A}(x_0,r_1,r_2)=\{x\in\mathbb{R}^n : r_1<|x-x_0|<r_2\}\,,$$

$$S_i=S(x_0,r_i)=\{x\in\mathbb{R}^n : |x-x_0|=r_i\}\,,\quad i=1,\,2\,.$$

Let a function $Q:D\to[0,\infty]$ be Lebesgue measurable. We say that a homeomorphism $f:D\to\mathbb{R}^n$ is ring Q-homeomorphism with respect to p-modulus at $x_0\in D$, if the relation

$$M_p(\Delta(fS_1,fS_2,fD))\;\leqslant\;\int\limits_{\mathbb{A}} Q(x)\,\eta^p(|x-x_0|)\,dm(x)$$

holds for any ring $\mathbb{A}=\mathbb{A}(x_0,r_1,r_2)$, $0<r_1<r_2<d_0$, $d_0=\mathrm{dist}(x_0,\partial D)$, and for any measurable function $\eta:(r_1,r_2)\to[0,\infty]$ such that

$$\int\limits_{r_1}^{r_2}\eta(r)\,dr=1\,.$$

Denote by ω_{n-1} the area of the unit sphere $\mathbb{S}^{n-1}=\{x\in\mathbb{R}^n : |x|=1\}$ in $\mathbb{R}^n$ and by $q_{x_0}(r)=\frac{1}{\omega_{n-1}\,r^{n-1}}\int\limits_{S(x_0,r)}Q(x)\,d\mathcal{A}$ the integral mean over the sphere $S(x_0,r)=\{x\in\mathbb{R}^n : |x-x_0|=r\}$, here $d\mathcal{A}$ is the element of the surface area. Let $L(x_0,f,R)=\sup\limits_{|x-x_0|\leqslant R}|f(x)-f(x_0)|$.

Theorem. *Suppose that $f:\mathbb{R}^n\to\mathbb{R}^n$ is a ring Q-homeomorphism with respect to p-modulus at a point x_0 with $p>n$ where x_0 is some point in $\mathbb{R}^n$. Then for all numbers $r_0>0$ the estimate*

$$\varliminf_{R\to\infty}\left(L(x_0,f,R)\left(\int\limits_{r_0}^{R}\frac{dt}{t^{\frac{n-1}{p-1}}\,q_{x_0}^{\frac{1}{p-1}}(t)}\right)^{-\frac{p-1}{p-n}}\right)\geqslant\left(\frac{p-n}{p-1}\right)^{\frac{p-1}{p-n}}>0$$

holds.

Extremal results and related problems associated with the Loewner differential equation in $\mathbb{C}^n$ (in memory of Gabriela Kohr)

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Joint work with Ian Graham (Toronto), Hidetaka Hamada (Fukuoka) and Gabriela Kohr (Cluj-Napoca)

In the first part of this talk we survey various results related to Loewner chains, the generalized Loewner differential equation, and Herglotz vector fields on the Euclidean unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. Extremal problems and related results for the family $S^0_A(\mathbb{B}^n)$ of univalent mappings with A-parametric representation on $\mathbb{B}^n$ will be also discussed, where $A\in L(\mathbb{C}^n)$ such that $k_+(A)<2m(A)$. Here $k_+(A)$ is the Lyapunov index of the operator A and $m(A)=\min_{\|z\|=1}\Re\langle A(z),z\rangle$. Open problems and conjectures will be also considered.

Remarks on a T1 Theorem for singular integral operators on upper Ahlfors regular sets and applications to potential theory

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This talk concerns a few remarks on a T1 Theorem for singular integral operators with a generalized Calderon Zygmund kernel acting in Hölder spaces on subsets of the Euclidean space that satisfy an upper Ahlfors regularity condition. Then we also point out some application to potential theory.

Perturbation of the effective conductivity of a periodic composite

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Joint work with P. Musolino

In this talk we consider the effective conductivity of a two-phase periodic composite material obtained by introducing into an infinite homogeneous matrix a periodic set of inclusions of a different material. Roughly speaking, the effective conductivity can be thought of as the conductivity of a homogeneous material whose global behavior as a conductor is 'equivalent' to that of the composite. By exploiting a potential theoretic approach, we show that the effective conductivity depends real analytically upon perturbation of the shape of the inclusions, of the periodicity structure, and of the conductivity of each material.

Asymptotic behavior of u-capacities and singular perturbations for the Dirichlet-Laplacian

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Joint work with Laura Abatangelo, Virginie Bonnaillie-Noël, Corentin Léna

In this talk we study the asymptotic behavior of u-capacities of small sets and its application to the analysis of the eigenvalues of the Dirichlet-Laplacian on a bounded planar domain with a small hole. More precisely, we consider two (sufficiently regular) bounded open connected sets Ω and ω of $\mathbb{R}^2$, containing the origin. First, if ε is positive and small enough and if u is a function defined on Ω, we compute an asymptotic expansion of the u-capacity $\mathrm{Cap}_\Omega(\varepsilon\omega, u)$ as $\varepsilon \to 0$. As a byproduct, we compute an asymptotic expansion for the N-th eigenvalues of the Dirichlet-Laplacian in the perforated set $\Omega \setminus (\varepsilon\overline{\omega})$ for ε close to 0. Such formula shows explicitly the dependence of the asymptotic expansion on the behavior of the corresponding eigenfunction near 0 and on the shape ω of the hole.

Carathéodory's Inequality for Driving Point Impedance Functions on the Right Half Plane

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Joint work with Timur DÜZENLI(Amasya University)

In this talk, a boundary version of the Carathéodory's inequality for positive real functions is investigated. As engineering application of the analysis, driving point impedance functions of circuit theory are considered since they are actually positive real functions. Assuming that the driving point impedance function $Z(s)$ has the structure of $Z(s) = \frac{A}{2} + c_1(s-1) + c_2(s-1)^2 + ...$ with $0 < \Re Z(s) \leq A$ for $\Re s \geq 0$, where it is an analytic function defined in the right half of the s-plane, novel inequalities for $|Z'(0)|$ have been derived. Here, it is also assumed that $Z(s)$ is also analytic at the boundary point $s = 0$ on the imaginary axis. Finally, the two generic driving point impedance functions have been obtained by sharpness analysis of the presented inequalities and their frequency characteristics have been investigated for different parameter values.

On analytical conditions of local lineal convexity generalized to commutative algebras

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We consider a commutative and associative algebra $\mathcal{A}$ with identity over the field of real numbers. Let $\mathrm{dim}\mathcal{A} = n$ and let elements $\{e_k\}_{k=1}^n$ be a basis of $\mathcal{A}$ such that there exist the inverse elements e_k^{-1}, $k = \overline{1,n}$. Let $\Gamma^p = (\gamma_{lk}^p)$, $p = \overline{1,n}$, be matrixes of structure constants of $\mathcal{A}$ with respect to the basis, i.

e. $e_k e_l = \sum\limits_{p=1}^{n} \gamma_{lk}^p e^p,\ k,l = \overline{1,n}$. We suppose that among matrixes Γ^p there is at least one that is not degenerate.
The notion of lineally convex domains in the finite-dimensional complex space $\mathbb{C}^n$ and some of their properties are generalized to the finite-dimensional space $\mathcal{A}^n$, $n \geqslant 2$, that is the Cartesian product of n algebras $\mathcal{A}$. Namely, a domain in $\mathcal{A}^n$ is said to be ***(locally)*** $\mathcal{A}$***-lineally convex*** if for any boundary point $\boldsymbol{w}$ of the domain there exists a hyperplane in $\mathcal{A}^n$ passing through $\boldsymbol{w}$ but not intersecting the domain (in some neighborhood of $\boldsymbol{w}$).
There are obtained the separate necessary and sufficient conditions of the local $\mathcal{A}$-lineal convexity of domains with smooth boundary. The conditions are obtained in terms of nonnegativity and positivity of the differential of the second order of a real function defining the domain, respectively. Moreover, the sign of the differential is determined on the boundary of the domain and on the vectors of the hyperplane tangent to the domain. These conditions are a generalization of well-known conditions of the local linear convexity of a domain with a smooth boundary in $\mathbb{C}^n$ obtained by B. Zinoviev. Similar conditions where also obtained for non-commutative Clifford algebras and generalized quaternions algebras.

Integral formulas of the Hilbert, Schwarz and Poisson type for the β-analytic function theory

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Joint work with Juan Bory-Reyes, Ricardo Abreu-Blaya, Baruch Schneider

In this talk we introduce some analogues of the Hilbert formulas on the unit circle and on the upper half-plane for the theory of solutions of a special case of the Beltrami equation in $\mathbb{C}$ to be referred as β-analytic functions. Furthermore, we present the corresponding Schwarz and Poisson formulas.

On Menchoff-Trokhymchuk theorem in a three-dimensional commutative algebra

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Joint work with M. Tkachuk

The aim of our work is to weaken the conditions of monogenity for functions that take values in a concrete three-dimensional commutati- ve algebra over the field of complex numbers. The monogenity of the function is understood as a combination of its continuity and the exi- stence of the Gâteaux derivative. We proved an analog of Menchoff–Trokhymchuk theorem that takes into account the algebraic structure of algebra.

Modules of families of curves in solving some extremal problems of potential theory

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Let Γ be a family of curves in $\mathbb{R}^d$. We will assume that each curve $\gamma \in \Gamma$ is a union of a countable number of open arcs, closed arcs or closed curves and is locally rectifiable. *The* p*-modulus* or just *modulus* of the family Γ is defined as the quantity

$$M_p(\Gamma) = \inf \int_{\mathbb{R}^d} \rho^p dx,$$

where infumum is taken over all Borel functions $\rho : \mathbb{R}^d \to [0, \infty]$ such that $\int_\gamma \rho ds \geq 1$ holds for each curve $\gamma \in \Gamma$. It is said that the family Γ_2 is longer than the family Γ_1, if each curve $\gamma \in \Gamma_2$ has a sub-curve belonging to Γ_1 (in this case, the inequality $M(\Gamma_1) \geq M(\Gamma_2)$ holds). In this talk, we discuss some extremal

problems of potential theory, the proof of which is based on the method of moduli of families of curves. Namely, the problems of extremal partitioning for the p-harmonic radius [1] and optimal estimates for the discrete Green's energy of the domain of rotation [2]. To prove the results, we use known facts about the moduli of families of curves, and also construct some new transformations.
For example, let L be some hyperplane that divides $\mathbb{R}^d$ into two closed half-spaces Λ^+ and Λ^-. For any set $A \subset \mathbb{R}^d$ we denote $A^+ = A \cap \Lambda^+$, $A^- = A \cap \Lambda^-$. The set symmetric to A with respect to L is indicated by the symbol A^*.
Suppose that the hyperplane L intersects some bounded Jordan measurable domain $G \subset \mathbb{R}^d$, $(G^-)^* \subset G^+$ and curves from the families $\Gamma = \{\gamma\}$, $\Gamma_0 = \{\gamma_0\}$, $\Gamma_1 = \{\gamma_1\}$, $\Gamma_2 = \{\gamma_2\}$ satisfy the conditions: $\gamma \subset \overline{G^+}$, $\gamma_0 = \gamma \cup (\gamma \cap \overline{(G^-)^*})^*$, $\gamma_1 \subset \overline{G^+ \setminus (G^-)^*}$, $\gamma_2 \subset \overline{G}$. If Γ is longer than Γ_1 and Γ_0 is longer than Γ_2, then

$$2M_p(\Gamma) \leq M_p(\Gamma_1) + M_p(\Gamma_2).$$

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On a quadrature formula for the direct value of the double layer potential

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Joint work with P.A. Krutitskii, V.V. Kolybasova

The double layer potential is used in the numerical solution of boundary value problems for the Laplace and Helmholtz equations. With the help of potentials, boundary value problems can be reduced to integral equations, which are then solved numerically. For the numerical solution of integral equations, it is necessary to have quadrature formulas that calculate with good accuracy the direct values of the potentials on the surface, where the potential density is given. Engineering calculations use standard quadrature formulas for the potentials, but their accuracy is poor. An improved quadrature formula for the direct value of the simple layer potential and for the direct value of the normal derivative of the simple layer potential were proposed by the authors before. In this paper, an improved quadrature formula is derived for the direct value of the double layer potential. The improved formula gives significantly higher accuracy than the standard one, which is confirmed by numerical tests.
Testing the improved and standard quadrature formulas was carried out in the case when the surface is a sphere of unit radius. The calculation results show that the calculation error using the improved quadrature formula is several times less than the calculation error using the standard quadrature formula. Thus, the improved quadrature formula provides a much higher accuracy in calculating the direct value of the double layer potential. The improved quadrature formula can find application in the numerical solution of boundary integral equations arising in the process of solving boundary value problems for the Laplace and Helmholtz equations by the method of potentials.

Conformal mapping of multiply connected domains onto lemniscatic domains and the computation of the logarithmic capacity

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Joint work with Jörg Liesen, Mohamed M. S. Nasser

The computation of the logarithmic capacity of a compact set $E \subseteq \mathbb{C}$ is a notoriously difficult task. If $E^c = \widehat{\mathbb{C}} \setminus E$ is simply connected, the capacity of E can be obtained from the Riemann mapping from E^c onto the exterior of the unit disk. Many of the known explicit formulas for the capacity can be derived with this approach.

In this talk we consider the case that E^c is ℓ-connected. In a paper from 1956, Walsh proved the existence of a conformal map from a multiply connected domain to a lemniscatic domain. This map is a direct generalization of the Riemann map from simply to multiply connected domains. Walsh's map also readily yields the logarithmic capacity of E, the Green function with pole at infinity of E^c, and the asymptotic convergence factor of E. We will discuss Walsh's conformal map and a numerical method for its computation, which relies on solving boundary integral equations with the Neumann kernel. This method also leads to a method to compute the logarithmic capacity.

Problem on extremal decomposition of the complex plane with free poles

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Let $\mathbb{N}$ and $\mathbb{R}$ be the sets of natural and real numbers, respectively, $\mathbb{C}$ be the complex plane, $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be the Riemann sphere, and $r(B,a)$ be the inner radius of the domain $B \in \overline{\mathbb{C}}$ with respect to the point $a \in B$.
Consider the following problem which was formulated in 1994.

Problem. Consider the product $I_n(\gamma) = r^{\gamma}(B_0, 0) \prod\limits_{k=1}^{n} r(B_k, a_k)$, where B_0, B_1,...,B_n $(n \geq 2)$ are pairwise non-overlapping domains in $\overline{\mathbb{C}}$ and $a_0 = 0$ and $|a_k| = 1$ for $k = \overline{1,n}$, and $0 < \gamma \leqslant n$. Show that it attains its maximum at a configuration of domains B_k and points a_k possessing rotational n-symmetry.
This problem has a solution only if $\gamma \leqslant n$ as soon as $\gamma = n + \epsilon, \epsilon > 0$, the problem has no solution. Currently it still unsolved in general, only partial results are known.
The following theorem holds.
Theorem. Let $n \in \mathbb{N}$ and $n \geqslant 2$. Then for any $\beta \in (0; \frac{1}{2}]$ there exists $n_0(\beta)$ such that for all $n \geqslant n_0(\beta)$ and for all $\gamma \in (1, n^{\beta}]$ and for any different points of a unit circle and for any different system of non-overlapping domains B_k, such that $a_k \in B_k \subset \overline{\mathbb{C}}$ for $k = \overline{1,n}$, and $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, the following inequality holds

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k) \leqslant \left(\frac{4}{n}\right)^n \frac{\left(\frac{4\gamma}{n^2}\right)^{\frac{\gamma}{n}}}{\left(1 - \frac{\gamma}{n^2}\right)^{n+\frac{\gamma}{n}}} \left(\frac{1 - \frac{\sqrt{\gamma}}{n}}{1 + \frac{\sqrt{\gamma}}{n}}\right)^{2\sqrt{\gamma}}. \quad (1)$$

Equality is attained if a_k and B_k for $k = \overline{0,n}$, are, respectively, poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{(n^2 - \gamma)w^n + \gamma}{w^2(w^n - 1)^2}\, dw^2.$$

Session 5: Constructive Methods in the Theory of Composite and Porous Media

Organizers: Vladimir Mityushev, Natalia Rylko and Piotr Drygaś

ESTIMATIONS OF THE EFFECTIVE HEAT CONDUCTIVITY OF A COMPOSITE WITH HEXAGONAL LATTICE OF PERFECTLY CONDUCTING CIRCULAR INCLUSIONS

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Joint work with G. A. Starushenko, S. A. Kvitka

The problem of thermal conductivity for a composite of a hexagonal structure with perfectly conducting circular inclusions is considered in non-dilute case. Namely, we suppose $a = r/l \leq 1/\sqrt{3}$, where r is the radius of inclusion and l is the distance between opposite sides of periodic cell of lattice. The use of the lubrication theory [2] (thin layer asymptotics [6]) and homogenization procedure [5] leads to the following expression of the effective thermal conductivity at $\lambda \to \infty$ (parameter λ characterizes the contrast of matrix and inclusions properties) [4]:

$$q = F_1(a) + F_2(a), \tag{1}$$

where

$$F_1(a) = 1 + \frac{2\sqrt{3}a^2}{\sqrt{1-a^2}}\left(\arctan\frac{\sqrt{3}}{3\sqrt{1-a^2}} + \frac{2}{3}\arctan\sqrt{\frac{1-a}{1+a}}\right) - \frac{\sqrt{3}\pi a^2}{6},$$

$$F_2(a) = \frac{\sqrt{3}a^2}{\sqrt{1-a^2}}\left(\frac{2}{3}\arctan\frac{a}{\sqrt{1-a^2}} - \arctan\frac{\sqrt{3}}{3\sqrt{1-a^2}}\right).$$

Approximation (1) does not give the correct asymptotics for $a \to 1,\ \lambda \to \infty$. Improving of it is achieved using the Padé approximation. Expanding the function F_2 into a power series and rearranging the first 12 terms into a fractionally rational function according to the Padé scheme one obtains

$$F_2(a) \approx q_{[2/10]} = F_3(a). \tag{2}$$

New expression for the effective thermal conductivity is:

$$q + 1 = F_1(a) + F_3(a) \tag{3}$$

gives a good approximation for any inclusion size $0 \leq a < 1$ and has the asymptotics

$$q_1 \to \frac{\sqrt{3}\pi}{\sqrt{1-a^2}} - 6.345 \text{ at } a \to 1,$$

whose main term coincides with the well-known expression [1].
The approximate solutions q [4] and give, respectively, the upper and lower estimates of the reduced parameter and the gap between them is small for any size of the inclusions $0 \leq a < 1,\ a \to 1.$

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Optimal design problem for three disks on torus

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Joint work with V. Mityushev, K. Dosmagulova, Zh. Zhunussova

The talk is devoted to recently established connection between the packing problem of disks on torus and the effective conductivity of composites with circular inclusions [1]. The packing problem is usually investigated by geometrical arguments, the conductivity problem by means of elliptic functions. An algorithm is developed in order to determine the optimal location of three disks on torus formed by the hexagonal lattice. The corresponding minimization function is constructed in terms of expressions consisting of elliptic functions with unknown arguments. The numerically found roots coincide with the previously established optimal points by a pure geometrical study.

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Fractional-differential model of the viscoelastic periodontal ligament

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The periodontal ligament is a complex component of the tooth root supporting apparatus. It's a thin layer of soft connective tissue located between the hard surfaces of the tooth root and the dental alveolar bone. One of the approaches to analytical modeling of the viscoelastic periodontal ligament behavior is the usage of a fractional kernel. The aim of this study is the assessment of the parameters of the Rabontov's fractional relaxation kernel during the tipping of the asymmetric tooth root.
The assessment of the relaxation kernel parameters was carried out on the basis of experimental dependences (R.J. Pryputniewicz and C.J. Burstone, 1979) of the tooth root displacements in the periodontal ligament under the action of a concentrated load. It was appeared that for any load, the simultaneous change of the parameter $\nu_\sigma = \frac{E_\infty - E_0}{E_0}$ and the fractional parameter γ enables to assign different time intervals for the phase transition of the periodontal ligament and the maximum magnitude of the tooth root displacements in the periodontal ligament. The maximal displacement magnitude and angle of rotation for different load values can be assign by changing the parameter ν_σ. An increase of the fractional parameter leads to an increase of the duration of the phase transition and the magnitude of the tooth root maximum displacement (for constant values of the parameters ν_σ and the retardation time τ_σ).
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Random elastic composites with circular inclusions

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Joint work with S. Gluzman, V. Mityushev, W. Nawalaniec

Consider 2D multi-phase random composites with different circular inclusions. A finite number n of inclusions on the infinite plane forms a cluster. The corresponding boundary value problem for Muskhelishvili's potentials is reduced to a system of functional equations. Solution to the functional equations can be obtained by a method of successive approximations or by the Taylor expansion of the unknown analytic functions. Next, the local stress-strain fields are calculated and the averaged elastic constants are obtained in symbolic form. An extension of Maxwell's approach and other various self-consisting cluster methods from single- to n- inclusions problems is developed. An uncertainty when the number of elements n in a cluster tends to infinity is analyzed by means of the conditionally convergent series. Application of the Eisenstein summation yields new analytical approximate formulas for the effective constants for macroscopically isotropic random 2D composites.

Monodromy of Pfaffian equations for group-valued functions on Riemann surfaces

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We consider a system of differential equations of Pfaffian type

$$Df = \omega \tag{1}$$

on a Riemann surface and develop for it an analog of the monodromy theory of Fuchsian systems of differential equations. Namely, let X be a compact Riemann surface and G a compact Lie group. Let f_0 be a solution of system (1) in a neighborhood $U \subset X$ of the point z_0. After continuation of f_0 along a path circling around a singular point z_i system (1) determines a monodromy representation $\rho : \pi_1(X \setminus S, z_0) \to G_{\mathbb{C}}$.
For such system, it is possible to formulate Riemann-Hilbert monodromy problem as follows: whether, for a given homomorphism: $\rho : \pi_1(X \setminus S) \to G$, there exists a G-system monodromy of which coincides with ρ. It is known that solution of this problem depends on group G.
The following theorem holds.
Theorem. 1) If $\pi_1(M)$ is a free group and $G_{\mathbb{C}}$ is connected, then ρ is a monodromy homomorphism for a system (1).
2) If $\pi_1(M)$ is a free abelian group and G is a connected compact Lie group with torsion free cohomology, and if $Im\rho \subset G$, then ρ is a monodromy homomorphism for some system of type (1).
Above theorem is a solution of the Riemann-Hilbert problem for holomorphic systems of type (1).
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Application of the metamaterial based approach to detect cancer

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Early detection of a tumor makes it more probable that the patient will, finally, beat the cancer and recover. The main goal of broadly defined cancer diagnostics is to determine whether a patient has a tumor, where it is located, and what is its histological type and severity. Here, we present, in the first time to our knowledge, the studies on metamaterial properties of biological tissues to identify healthy and cancerous areas in the brain tissue. The results show, that the metamaterial properties strongly differ depending on the tissue type, if it is healthy or unhealthy one. The obtained effective permittivity values were dependent on various factors, like the amount of different cell types in the sample and their distribution. Based on these findings, the identification of the cancer affected areas based on their effective medium properties was performed. These results proved the metamaterial model capability in recognition of the cancer affected areas. The presented approach can have a significant impact on the development of methodological approaches toward precise identification of pathological tissues and would allow for more effective detection of cancer-related changes. We suggest the highly effective recognition of the cancerous specimens in stained brain biopsies.

Analogues the Kolosov-Muskhelishvili formulas for isotropic materials with double voids

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Analogues of the well-known Kolosov-Muskhelishvili formulas for homogeneous equations of statics in the case of elastic materials with double voids are obtained. It is shown that in this theory the displacement and stress vector components are represented by two analytic functions of a complex variable and two solutions of Helmholtz equations. The constructed general solution enables one to solve analytically a sufficiently wide class of plane boundary value problems of the elastic equilibrium with double voids.

On Conformal Modulus of Quadrilaterals

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In the talk we prove the theorems that emphasize geometric properties of a convex quadrilateral to construct Schwarz-Christoffel mapping function and find conformal modulus.

Theorem 1. If we have g function

$$g(x) = \int_{-\infty}^{x} (1-\zeta)^{\tau_1 - 1} \left(1 - \frac{\zeta}{1+\theta}\right)^{\tau_2 - 1} \left(1 - \frac{\zeta}{1+\theta+r\theta}\right)^{\tau_3 - 1} d\zeta \qquad (1)$$

where $\pi\tau_j$ is inner angle of a convex quadrilateral, then

$$l_2(r,\theta,\tau) = |g(1+\theta) - g(1)| \quad l_3(r,\theta,\tau) = |g(1+\theta+r\theta) - g(1+\theta)|$$

$$l_4(r,\theta,\tau) = |g(+\infty) - g(1+\theta+r\theta)|$$

can be expressed by Γ and ${}_2F_1$.

Theorem 2. Ratio of a convex quadrilateral defined by g function from (1) is

$$\varphi(r;\tau) = r^{1-\tau_1} \frac{\Gamma(\tau_2)\Gamma(\tau_3+\tau_4)}{\Gamma(\tau_4)\Gamma(\tau_2+\tau_3)} \frac{{}_2F_1(\tau_2, 1-\tau_1, \tau_2+\tau_3, -r)}{{}_2F_1(\tau_4, 1-\tau_1, \tau_3+\tau_4, -1/r)}$$

and if $\tau = (0.5, 0.5, 0.5)$, which is same as quadrilateral is rectagel, then

$$\varphi(r;\tau) = \frac{\mathcal{K}'(k)}{\mathcal{K}(k)}, \quad k = \sqrt{\frac{1}{1+r}}.$$

Dimension reduction in the periodicity cell problem for plate reinforced by a unidirectional system of fibers

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Joint work with S.I. Rakin

We consider a plate reinforced by a periodic system of parallel isotropic fibers. The structure of such a plate is invariant with respect to translation in the direction along the fibers. There is a reason to look for a two-dimensional model for this plate.
An attempt of dimension reduction in a “model” problem of bending the fiber-reinforced plate was done by Grigoluk with coauthors by using the double periodic function of complex variables. The “model” because it corresponds to the bending of a plate of “infinite” thickness. It may be useful to compute the strain-stress state (SSS) inside the plate but not near the free surfaces.
We use the homogenization theory as the starting point of the research, namely, we investigate the periodicity cell problem for plates. The characteristic features of plate PC are the free surfaces and the bending/torsion modes of deformation. These features drastically distinguish the plates from the solids.
We reduce the original periodicity cell problem to several 2-D boundary-value problems: problem for Laplace equation, problem for Poisson equation, planar elasticity problems. The first two problems correspond to anti-plane elasticity.
We present numerical solutions to the 2-D problems and discuss the following items: inhomogeneity of the local stress-strain state in the fibers and the matrix, and its relation to the multi-continuum behavior of high-contrast composites predicted by Panasenko; the boundary layers near the top and the bottom surfaces of the plate; the densely packed fibers.

Structural sums on plane hexagonal torus

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Joint work with V. Mityushev

The subject of the talk is the relationship between the problem of packing discs on a torus and the effective conductivity of composites with circular inclusions. This problem has been identified quite recently [1]. Typically, the packing problem is examined using geometric arguments, and the conductivity problem through elliptical functions. On this basis, an algorithm was developed to determine the optimal position of the three disks on the torus formed by the hexagonal lattice. Based on it, an appropriate minimization function was constructed in the form of expressions, which consist of elliptic functions with unknown arguments. The numerical solutions that have been achieved match with previously determined optimal points that have been established by purely geometric research.

[1] Mityushev, V.; Rylko, N. Optimal distribution of the non-overlapping conducting disks. Multiscale Model. Simul., 10, 180–190, 2012.

Quantitive analysis of the nano-TiC particles distributions in composites structure

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Joint work with N. Rylko, O. Afanasievaa, S. Gluzman, E. Olejnik, A. Wojcik, W. Maziarz

We propose an effective method of multiscale analysis of composite materials with a complex structure. The complexity of the composite structure results from the reinforcing particles being pushed into the interdendritic space during the manufacturing process. In such situations, classic image analysis methods are ineffective because they are not able to resolve the high heterogeneity of structure and problems with phase separation. The proposed methodology is a multiscale procedure that combines image analysis, statistics, Fourier analysis and analytical RVE methods. The qualitative relationship between the structure image and macro-properties of the nano composite is implemented directly on the 2D structural image?s pixel level. The proposed method establishes a foundation for building a database for the classification of composite structures.

Effective properties of random composites

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Joint work with N. Rylko

Various analytic formulas for random composites were deduced by means of EMA (effective medium approximation). Other terms like self-consistent method, Mori-Tanaka approach, Bruggeman's model, differential scheme and so on are also used. In many cases, a formula was derived having used physical arguments and mathematical manipulations with the further declaration that this formula is universal or valid at least for a wide class of composites. Put aside empiricism and analyze EMA. Follow one of such a work written many years ago or yesterday. Do not pay attention on the advanced computer simulations and mathematical manipulations. Just trace a line of arguments to check whether the lines hold for a class of composites. For instance, consider the conductivity of the hexagonal array of perfectly conducting disks embedded in medium of the normalized unit conductivity. The effective conductivity can be estimated by the Clausius-Mossotti approximation (CMA) equivalent to Maxwell's formula analyzed in [1]: $\sigma_e(f) = \frac{1+\varrho f}{1-\varrho f} + O(f^7)$, where f denotes the concentration of disks and the contrast parameter $\varrho = 1$. Take a random location of disks. The same formula (CMA) is valid up to $O(f^3)$ for any macroscopically isotropic composite. The term $O(f^3)$ depending on location of inclusions is exactly written in [2]. Therefore, any universal formula for the effective conductivity of a random composite cannot exceed the precision $O(f^2)$ without additional assumptions on location of inclusions. The analysis of precision [1,2] explains the seeming contradiction between various formulas derived in the framework of EMA for the same problem.

[1] S. Gluzman, V. Mityushev, W. Nawalaniec, Computational Analysis of Structured Media, Elsevier, London, 2018.
[2] V. Mityushev, N. Rylko, Maxwell's approach to effective conductivity and its limitations, Q. J. Mech. Appl. Math. 66 (2) (2013) 241–251.

Simulating local fields in carbon nanotube reinforced composites for infinite chan-

nel domains

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Joint work with El Mostafa Kalmoun and Vladimir Mityushev

We consider the steady heat conduction problem within a thermal isotropic and homogeneous infinite strip composite reinforced by non-overlapping and randomly distributed carbon nanotubes (CNTs). We treat the CNTs as thin elliptic inclusions and assume their temperature distribution to be fixed to an undetermined constant value along each ellipse. We consider the infinite strip $(x, y) \in (-\infty, \infty) \times (0, 1)$ and we assume that the temperature is 1 on the line $y = 1$ and -1 on the line $y = 0$. The equations for the temperature distribution are governed by the two-dimensional Laplace equation with Dirichlet boundary condition. This boundary value problem is solved using the boundary integral equation with the generalized Neumann kernel. We illustrate the performance of the proposed method through several numerical examples.

Mixed boundary-transmission problems for composite layered elastic structures containing interfacial cracks

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Joint work with M. Mrevlishvili

We investigate mixed type boundary-transmission problems of the generalized thermo-electro-magneto elasticity (GTEME) theory for complex elastic anisotropic layered structures containing interfacial cracks. This type of problems are described mathematically by systems of partial differential equations with appropriate transmission and boundary conditions for six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function). We apply the potential method and the theory of pseudodifferential equations and prove uniqueness and existence theorems of solutions to different type mixed boundary-transmission problems in appropriate Sobolev spaces. We analyze smoothness properties of solutions near the edges of interfacial cracks and near the curves where different type boundary conditions collide.

Energy of Graphs and Distributions of Disks Packings

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Joint work with W. Nawalaniec, V. Mityushev

The effective conductivity of disk packings embedded in a two-dimensional doubly periodic cell can be determined via the extension of the theory of structural approximations. The method is applied to the multiple distributions of disks with a varying degree of clusterization which can be separated into three distinct classes. The obtained results show a clear connection between the effective conductivity of packings and their geometrical characterization by clusterization degree.

Application of statistical moments to cloudiness

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Joint work with Adam Krezel, Natalia Rylko

Atmospheric cloud detection is a basic problem in contemporary satellite-based remote sensing. From a mathematical viewpoint, detection had to answer the question of how wide the range of patterns observed in the atmosphere can be assigned to a homogenous group. Mathematical models of the patterns are based on differential equations of selected structures visible during the radiation passage through the atmosphere. Such structures can be regarded as resulting from solving various partial differential equations. This work describes a method, based on the theory of statistical moments with invariants, developed to optimally represent cloud structures on satellite radiation maps. Strictly speaking, the method limits uncertainties arising during the identification of atmospheric objects, which are difficult, or impossible, to distinguish with conventional methods. Differential equations-based solutions were demonstrated to be amenable to approximation by physical functions, for example, equations describing the solar radiation transmission through the atmosphere. The shape analysis was conducted on a 2D plane by developing a finite difference technique for certain boundary conditions. This study applies a geometric method which can be referred to the structural approximation approach. The theory of moments is used to approximate the shape of each cloud as an ellipse. Next, the set of clouds (ellipses) is considered as a plane object having its "total" shape as the shape of the considered multiply connected domain. Thus, the shape of each cloud is approximated by a simple curve and the shape of the joint set of clouds considered as a two-phase two-dimensional medium. Two-phase media are traditionally investigated by the theory of spatial correlation functions which theoretically completely describe them. However, computational restrictions lead to the application only an autocorrelation function. We apply the moment theory and structural analysis to propose a classification of the sets of clouds. At the beginning we approximate each separate cloud by an ellipse and its set of moments $\{M_{mn}\}$ written as a vector. Next, the vector of structural sums $\{e_{mn}\}$ is assigned to the considered set of clouds. Therefore, we consider a set of clouds as a two-dimensional geometrical object, as a multiply connected domain on the macroscopic level and as an elliptical approximation assigned to a separate cloud. The atmosphere energy level (e.g., the amount of radiation reaching the sea surface) is described by transmission equations with appropriate initial and boundary conditions. The shape functions approximate locations of spectrally and spatially defined areas with ellipses embedded in a plane. Any function continuously differentiable within a closed smooth connected domain can be approximated by a function of the special type called the packing function. It is a slight modification of the class of piecewise constant and linear functions with a few variables. Therefore, radiation relationships can meet approximation conditions. This is why we are confident that the method can be used to approximate the radiation function with the packing functions. In addition to cloud approximations, ellipses form a graph related to the Voronoi diagram. Such a graph can be regarded as a discrete model of separate clouds.

Locally anisotropic field in polydispersed two-dimensional composites

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The local anisotropy in composites obtained as a result of Friction Stir Processing is investigated. The traces of inhomogeneity follow the trajectory of mechanical stirring, hence, produce the local anisotropy. This effect is invisible by means of direct observations and can be established by the combination of digital processing of pictures and of the theory of *analytical Representative Volume Element*. First, we set up the local inhomogeneity by estimation of the local particles concentration in the host medium by using image analysis. Next, the preliminary estimation of anisotropy is performed in terms of the global anisotropy vector value introduced earlier for monodispersed composites. The observed heterogeneous structures lead to the introduction of generalized anisotropy vector κ more suitable for comrehensive study of polydispersed inhomogeneous composites. The new value κ accumulates the geometrical information which can be alternatively obtained from the 2-point correlation functions. Therefore, it properly describes the anisotropy of conductive fields in composites modeling thermal and electric conductivity, diffusion and elastic antiplane deformation.

On electromagnetic wave equations for nonhomegenous medium

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We consider a system of Maxwell's equations for nonhomegenous medkum without currents and charges

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \qquad \nabla \times \boldsymbol{H} = -\frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}$$
$$\nabla \cdot \boldsymbol{D} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0 \tag{1}$$

or

$$\epsilon_{sab} E_{b,a} = -\frac{\mu}{c}\dot{H}_s \qquad \epsilon_{sab} H_{b,a} = -\frac{\varepsilon}{c}\dot{E}_s$$
$$(\varepsilon E)_{a,a} = 0 \qquad (\mu H)_{a,a} = 0 \tag{2}$$

The generally accepted notation is used here.
From now on, we assume that the medium has a microperiodic structure with an elementary cell. After applying standard asymptotic homogenisation [1, 2], we get wave equations. For example, for the electric field $\boldsymbol{E}$ we have

$$\frac{\partial^2 E_k^{(0)}}{\partial x_a \partial x_a} = \frac{\langle \varepsilon \rangle}{\langle (1/\mu \rangle c}\, \ddot{E}_k \tag{3}$$

Parentheses $\langle (\cdot) \rangle$ denote the mean over the elementary cell. An analogous equation for the $\boldsymbol{H}$ field is obtained.

[1] Sanchez-Palencia E.: *Non-homogeneous media and vibration theory*, Springer Verlag, Berlin, Heidelberg, New York 1980
[2] S. Bytner, B. Gambin, Homogenization of heterogeneous magnetoelastic medium, *Archives of Mechanics* **45** (2) 223–233 (1993).

Sensitivity analysis of Maxwell's equations in a cavity

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Joint work with P.D. Lamberti, P. Luzzini

In this talk we consider a second order differential eigenvalue problem arising from the theory of electromagnetism, closely related to the time-harmonic Maxwell's equations. It involves the $\operatorname{curl}\operatorname{curl}$ operator in a bounded domain $\Omega \subset \mathbb{R}^3$ of the three-dimensional Euclidean space, subject to the perfect conductor boundary conditions. If ϵ and μ denote the electric permittivity and magnetic permeability of the material filling the cavity, the eigenproblem reads as follows

$$\begin{cases} \operatorname{curl} \mu^{-1} \operatorname{curl} E = \lambda \epsilon E, & \text{in } \Omega, \\ \operatorname{div} \epsilon E = 0, & \text{in } \Omega, \\ E \times \nu = 0, & \text{on } \partial\Omega, \end{cases}$$

in the unknowns λ the eigenvalue and E the eigenvector, where ν denotes the outer unit normal to the boundary.
We present new results dealing with eigenvalue optimization upon perturbations of the parameters involved in the differential equations, providing also Hadamard-type formulas for the Fréchet derivatives of the eigenvalues. The parameters considered can be the shape of the region Ω or, in the more general case of inhomogeneous anisotropic media, the electric permittivity ϵ of the medium inside the cavity.
To conclude, we briefly discuss a boundary homogenization problem regarding the spectral stability for a family of oscillating domains converging to a limit set.

Session 6: Function spaces and their applications to nonlinear evolution equations

Organizers: Baoxiang Wang and Mitsuru Sugimoto

The nonlinear Brascamp–Lieb inequality

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Joint work with J. Bennett, S. Buschenhenke, M. Cowling, T. Flock

We prove a nonlinear variant of the Brascamp–Lieb inequality. The argument is based on induction-on-scales using gaussians, and a key component of the proof is a quantitative version of Lieb's theorem concerning gaussian near-maximisers to the classical Brascamp–Lieb inequality. As an application, we will present some multilinear convolution estimates of the type which have recently appeared in the theory of Zakharov systems.

The renormalized solution to reaction-cross-diffusion equations

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Joint work with Ansgar Jüngel

The global-in-time existence and weak-strong uniqueness of renormalized solutions to reaction-cross-diffusion systems for an arbitrary number of variables in bounded domains with no-flux boundary conditions is proved. The cross diffusion part describes the segregation of population species and is a generalization of the Shigesada-Kawasaki-Teramoto model. The diffusion matrix is not diagonal and generally neither symmetric nor positive semi-definite, but the diffusion operator possesses a formal gradient-flow or entropy structure. The reaction part includes reversible reactions of mass-action kinetics and does not obey any growth condition.

Hardy-Sobolev space and application in PDE

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Joint work with Kuijie Li, Chunyan Huang, Liang Song

Hardy space is of fundamental importance in analysis. It appears naturally in many occasions as a replacement of the Lebesgue space L1. The Hardy space shares the same scaling as L1, but behaves better in boundedness of many operators, especially for critical problems, due to the subtle structure (e.g. cancellation). We will talk about two applications of the Hardy Sobolev space (function and its derivatives are in Hardy space). One is the local well-posedness of the Euler equation in the critical Hardy Sobolev space. The other one is the decay of the solutions for Schrodinger equation. We will also give an alternative proof of the boundedness of the Schrodinger propagator in Hardy space.

Global solutions for Euler–Poisson system.

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Joint work with Yan Guo, Jingjun Zhang

In this report, we will introduce some results about global solutions with small initial data for Euler–Poisson system and some two fluid system in plasma physics. Especially, we will show that smooth solutions with small amplitude to the 1D Euler–Poisson system for electrons persist forever with no shock formation.

Well-posedness of Linearized Incompressible Ideal MHD with Closed Free Surfaces

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Joint work with Tao Luo

In this talk, I review some results of free boundary problem of incompressible ideal MHD in a bounded domain with closed free surfaces based on the joint works with Prof. T. Luo, especially the well-posedness for the linearized system. We expressed the magnetic field in terms of the velocity field and the deformation tensors in the Lagrangian coordinates, and substituted the magnetic field into the momentum equation to get an equation of the velocity in which the initial magnetic field serves only as a parameter. Then, we linearized this equation with respect to the position vector field whose time derivative was the velocity, and obtained the local-in-time well-posedness of the solution by using energy estimates of the tangential derivatives and the curl with the help of Lie derivatives and the smooth-out approximation.

Well-posedness for the initial boundary problem of the derivative nonlinear Schrödinger equation on the half-line

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In this talk, the global well-posedness of initial boundary of the derivative nonlinear Schrödinger equation on the half-line with initial data satisfying $\|u_0\| < \sqrt{2\pi}$

$$u_t - iu_{xx} = (|u|^2 u)_x, \;\; x \geq 0$$

is considered.
Moreover, the inviscid limit behavior of the one-dimensional Ginzburg-Landau(GGL) equation on the half-line

$$u_t - (\varepsilon + i)u_{xx} + (|u|^2 u)_x = 0, \;\; \varepsilon > 0, \; x \geq 0$$

can be considered. If $\varepsilon \to 0$, the solution of the generalized Ginzburg-Landau(GGL) equation on the half-line converges the solution of the Schrödinger equations with derivative on the half-line.

Some bilinear estimates and an application to SQG

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We consider the validity of several bilinear estimates for functions defined on the half space with the homogeneous Dirichlet boundary condition. As an application, the well-posedness for the surface quasi-geostrophic equations is shown.

Construction of solutions to Schrödinger equations via wave packet transform

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In this talk, we construct the solutions to Schrödinger equations with time dependent potentials by time slicing approximation with approximate solutions via wave packet transform.

Boundedness of bilinear pseudo-differential operators of $S_{0,0}$-type in Wiener amalgam spaces and in Lebesgue spaces

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Joint work with Akihiko Miyachi and Naohito Tomita

In this talk, we extend and improve several known results about the boundedness of the bilinear pseudo-differential operators. First, we consider symbol classes of $S_{0,0}$-type that generalize the bilinear Hörmander class $BS^m_{0,0}(\mathbb{R}^n)$ and characterize those classes for which the corresponding bilinear operators are bounded in Wiener amalgam spaces. Secondly, using the results for Wiener amalgam spaces, we prove sharp results that certain L^q or $L^{q,\infty}$ integrability of the symbol and its derivatives with respect to the frequency variables implies boundedness of the corresponding bilinear operator in Lebesgue spaces.

Well-posedness of fractional NLS and semi-relativistic equations with Hartree type nonlinearity

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We consider the Cauchy problem of fractional NLS and semi-relativistic equations with Hartree type nonlinearity. Under some angular regularity assumption, we show the small data global well-posedness and scattering. The key tools are the U^p, V^p space introduced by Koch-Tataru and classical L^2-bilinear transversal estimates.

Well-posedness for the kinetic derivative nonlinear Schrödinger equation on the torus

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Joint work with Y. Tsutsumi

We consider the following nonlinear Schrödinger equation (NLS) with a nonlocal derivative nonlinear term:

$$\partial_t u = i\partial_x^2 u + \partial_x \left[\alpha|u|^2 u + \beta\mathcal{H}(|u|^2)u\right] \qquad \text{on } \mathbb{R} \text{ or } \mathbb{T} = \mathbb{R}/2\pi\mathbb{Z},$$

where α, β are real constants and $\mathcal{H}$ denotes the Hilbert transform. This equation was introduced as a model of propagation of Alfvén waves in a plasma, and the nonlocal term represents the effect of the resonant interaction between the wave modulation and the ions. It is called the *kinetic* derivative NLS (KDNLS), as the collective motion of ions is modeled by the Vlasov equation rather than a fluid equation. Compared to the standard DNLS (the case $\beta = 0$), well-posedness for KDNLS (the case $\beta \neq 0$) is much less understood, especially in the periodic setting. One of the reasons is that the gauge transform technique, which is effective for DNLS to convert the derivative nonlinearity to a tamer one, cannot be directly adapted due to the presence of the Hilbert transform. However, KDNLS has dissipative nature when $\beta < 0$; in fact, a formal computation shows that the solution decreases its L^2 norm in this case. It turns out that stronger dissipation is available in the periodic case, as a parabolic term $\sim \beta(-\partial_x^2)^{1/2}u$ arises from the resonant nonlinear interactions. In this talk, we show some results on low regularity well-posedness for KDNLS on $\mathbb{T}$ with $\beta < 0$, by taking advantage of this dissipative structure.
This is a joint work with Yoshio Tsutsumi (Kyoto University).

Operating functions on $A^q_s(\mathbf{T})$

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Joint work with E. Sato

Let $A_s^q(\mathbf{T})$ denote the space of all Lebesgue integrable functions f on the torus $\mathbf{T}$ such that $\sum_{m\in\mathbf{Z}} |\widehat{f}(m)|^q \langle m\rangle^{sq} < \infty$, where $\{\widehat{f}(m)\}_{m\in\mathbf{Z}}$ denote the Fourier coefficients of f. We consider necessary and sufficient conditions for all functions $F \in A_\beta^1(\mathbf{T})$ to operate on all real-valued functions in $A_s^q(\mathbf{T})$. This talk is bases on joint work with Enji Sato (Yamagata University).

Wellposedness for the KdV hierarchy

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Joint work with F. Klaus, H. Koch

The KdV hierarchy is a hierarchy of integrable equations generalizing the KdV equation. Using the modified Muria transform, we first relate it to the Gardner hierarchy, and by exploiting the idea of approximate flow, we show that the whole hierarchy is wellposed for initial data in H^{-1}.

On boundedness of composition operators on Besov spaces in one dimension

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Joint work with M. Ikeda, I. Ishikawa

Let us define a composition operator T_ϕ with a symbol ϕ by

$$T_\phi(f)(x) := f(\phi(x)) \quad \text{for } x \in \mathbb{R}^d.$$

We consider the boundedness of the composition operator on the inhomogeneous Besov spaces $B_{p,q}^s(\mathbb{R}^d)$ with $s > 0$ and $1 \le p, q \le \infty$. The boundedness has been studied by several works, where necessary and sufficient conditions on ϕ for the boundedness are given in the lower order case $0 < s \le 1$ (see, e.g., [1], [2]). In this talk, we study the higher order case $s > 1$ in one dimension $d = 1$. Our purpose is to give a necessary and sufficient condition on ϕ for the boundedness to hold in the case.

[1] G. Bourdaud, *Changes of variable in Besov spaces. II*, Forum Math. **12** (2000), no. 5, 545–563.
[2] G. Bourdaud and W. Sickel, *Changes of variable in Besov spaces*, Math. Nachr. **198** (1999), 19–39.

Hörmander type multiplier theorems for bilinear operators

Naohito Tomita

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Joint work with A. Miyachi

In this talk, we consider the boundedness of bilinear Fourier multiplier operators. Our purpose is to weaken regularity conditions on multipliers to ensure the boundedness of the corresponding operators, and prove Hörmander type multiplier theorems for bilinear operators.

Fractional medians and its maximal function

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In this talk, we introduce a fractional median and its maximal function, and give the boundedness of the fractional maximal operator. The boundedness is related to restricted weak type estimates of the usual maximal function.

Global solutions for the relativistic Vlasov-Poisson equation

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In this talk, we will consider the relativistic Vlasov-Poisson equation for the plasma physics case. Our attention is restricted to the global existence theory. Both the small data case and the large data case will be introduced. In particular, we show that the solution exists globally for a class of arbitrary large localized initial data with certain symmetry property.

Local and global dynamics of stochastic Liouville equation.

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Joint work with T. Tadahiro, R. Tristan, N. Tzvetkov

In this talk, I will talk about stochastic hyperbolic partial differential equations (SPDEs) - the hyperbolic Liouville models arise in the context of Liouville quantum gravity. We consider stochastic damped wave equations of these models, also known as canonical stochastic quantisation equations. These dynamics are proposed to preserve their corresponding Gibbs measures. We construct global solutions to these equations and then show the invariance of the Liouville measure under the resulting dynamics. This is a series of joint works with Tadahiro Oh (Edinburgh) Tristan Robert (Rennes) and Nikolay Tzvetkov (Cergy-Pontoise).

Global Gevrey analyticity and decay for the compressible Navier-Stokes equations with capillarity

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We are concerned with a system of equations governing the evolution of isothermal, viscous and capillary compressible fluids, which can be used as a phase transition model. It is observed by the pointwise estimate that the linear third-order Korteweg tensor behaves like the heat diffusion of density fluctuation, which enables us to develop more general Lp energy methods. Consequently, we can establish the global existence of solutions in critical Lp spaces, which indicates the evolution of Gevrey analyticity. Moreover, the optimal time-decay estimates of Lq-Lr type are also available for large times if the low-frequency regularity assumption is reasonably strengthened.

Blowup dynamics for smooth equivariant solutions to energy critical Landau-Lifschitz flow

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Joint work with J. Xu

We study the energy critical equivariant Landau-Lifschitz flow with target manifold $\mathbb{S}^2$. We prove that there exists a codimension one smooth well-localized set of initial data which generates finite-time type II blowup solutions, and then give a precise description of the corresponding singularity formation. In our proof, both the Schrödinger part and the heat part play important roles in the construction of approximate solutions and the mixed Energy-Morawetz arguments. However, the blowup rate is independent of their coefficients.

Session 7: Generalized Functions and Applications

Organizers: Michael Kunzinger and Marko Nedeljkov

(w, c)-Almost periodic generalized functions

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Joint work with M. T. Khalladi

The aim of this work is to introduce and to study an algebra of (w,c)-almost periodic generalized functions containing the (w,c)-almost periodic functions as well as (w,c)-almost periodic distributions.
[1] C. Bouzar, and M. T. Khalladi, Almost periodic generalized functions, Novi Sad J. Math, Vol. 41, No. 1, (2011) 33-42.
[2] J. F. Colombeau, Elementary introduction to new generalized functions, North Holland, 1985.
[3] M. T. Khalladi, M. Kostić, A. Rahmani and D. Velinov, (w; c)–Almost periodic type functions and applications, Filomat, submitted.
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Waves in generalized fractional Zener model

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Joint work with S. Pilipović, D. Seleši

We use the Bochner–Schwartz theorem and obtain restrictions which follow from the dissipation inequality for the constitutive equation of the complex fractional Zener model. The restrictions are less restrictive than the ones used in the known literature. The main contribution is related to a stochastic excitation in the body force term. Moreover, in our model an arbitrary specification of the initial displacement and strain may lead to an additional term in the equation of motion. The main idea behind the fractional Zener model is to modify the wave equation for the case of viscoelastic media. This adaptation is carried out by modifying the constitutive equation since Hooke's law, while keeping unchanged the equation of motion and the equation providing the relation between strain and displacement of a deformable body. In this presentation we extend previous results three directions. Firstly we present a new proof for the restrictions on the constitutive equation that follow from the dissipation inequality. These restrictions on parameter values that keep the model thermodynamically admissible are derived by applying the Bochner-Schwartz theorem, and they turn out to be less restrictive than the ones obtained in the previous literature. The the restrictions obtained from dissipation condition turn out to be mathematically essential for the solvability of the equations (they are inevitable for the existence of the inverse Fourier transform). As a second generalization, we allow here for a stochastic excitation in the body force term as well as for stochastic initial data. As the third result, we show that an arbitrary specification of the initial displacement and strain leads to an additional term in the equation of motion.

Fractional short time Fourier transform of tempered distributions

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Joint work with S. Maksimovic, S. Pilipovic

We prove the continuity of the short-time fractional Fourier transform and the corresponding synthesis operator on the spaces rapidly decreasing functions on $\mathcal{R}$ and $\mathcal{R}^2$, respectively, and their duals - the spaces

of tempered distributions. We also provide Abelian and Tauberian type results relating the asymptotic behavior of distributions with the asymptotics of their fractional Fourier transform and short-time fractional Fourier transform.

On spaces of distributions equipped with the κ-topology

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Joint work with Eduard Nigsch, Norbert Ortner

The $\kappa(E, F)$-topology on E is the topology of uniform convergence on absolutely convex compact subsets of F. In the theory of distributions it has a number of interesting applications. For example, the space $\mathcal{B}_c$, which is used in the context of the integration of distributions, carries the topology $\kappa(\mathcal{D}_{L^\infty}, \mathcal{D}'_{L^1})$. We investigate the κ-topology on a number of distribution spaces, including L^p, $\mathcal{D}_{L^p}$ and $\mathcal{D}'_{L^p}$, and give descriptions in terms of simple seminorms.

The spaces $\mathcal{D}_{L^{\vec{p}}}$ and their duals

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Joint work with M. T. Khalladi

This work aims to introduce the mixed Lebesgue spaces $\mathcal{D}_{L^{\vec{q}}}$ and their duals $\mathcal{D}'_{L^{\vec{p}}}$, $\frac{1}{\vec{p}} + \frac{1}{\vec{q}} = 1$. The fundamental properties of these spaces are studied.

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An analysis of the fractional Zener wave equation

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Joint work with L. Oparnica

The fractional Zener wave equation is a generalization of the wave equation in the Zener model (also known as standard linear solid (SLS) model) of viscoelastic materials. In one space dimension, it is (after a suitable normalization) given by

$$\frac{\partial^2}{\partial t^2}u(x,t) = \mathcal{L}^{-1}_{s\to t}\left(\frac{1+s^\alpha}{1+\tau s^\alpha}\right) *_t \frac{\partial^2}{\partial x^2}u(x,t), \quad (1)$$

$\mathcal{L}^{-1}$ denoting the inverse Laplace transform. Here, τ is a constant $\in (0,1)$ depending on the material, and α is a parameter $\in (0,1]$, the case $\alpha = 1$ corresponding to the classical Zener model. In this talk, we will discuss the micro-local analysis of the fundamental solution of (1). We will also perform a qualitative analysis of some of its solutions, corresponding to various initial condition. This talk is based on collaborative work with Ljubica Oparnica.

Heat, Fractional Schrödinger and Klein-Gordon equation: hypoelliptic case.

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Joint work with M. Ruzhansky, N. Tokmagambetov

We will discuss the fractional Klein-Gordon, Schrödinger and heat equations for hypoelliptic operators, and prove that they have a very weak solution. Such analysis can be conveniently realised in the setting of graded Lie groups. We will show the uniqueness of the very weak solutions, and the consistency with the classical solutions.

Blow-up solutions of discrete semilinear Klein-Gordon equations on networks

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In this talk, we consider an initial-boundary value problem of the Klein-Gordon equation

$$\begin{cases} u_{tt} - \Delta_\omega u + \lambda u_t + \theta u = f(u), & \text{in } S \times (0, +\infty), \\ u = 0, & \text{on } \partial S \times [0, +\infty), \\ u(\cdot, 0) = u_0, u_t(\cdot, 0) = v_0, & \text{in } S, \end{cases}$$

where S is a network with boundary ∂S, $\lambda, \theta \geq 0$, u_0 and v_0 are real-valued functions. The main contribution of this work is to introduce a condition

$$(C)\ \alpha \int_0^u f(s)ds \leq uf(u) + \beta u^2 + \gamma, u \in \mathbb{R},$$

for some $\alpha > 2$ and $\beta, \gamma \geq 0$ with $0 \leq \beta \leq \frac{(\alpha-2)\lambda_0}{2}$, where λ_0 is the first eigenvalue of the discrete Laplacian Δ_ω and we use the concavity method to obtain blow-up solutions to above equations.

Sharp Weyl formula for operators on asymptotically Euclidean manifolds

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Joint work with Moritz Doll

We study the asymptotic behaviour of the eigenvalue counting function for self-adjoint, positive, elliptic linear operators, defined through classical weighted symbols of order $(1,1)$, on an asymptotically Euclidean manifold X. We first prove a two term Weyl formula, improving previously known remainder estimates. Subsequently, we show that, under a geometric assumption on the Hamiltonian flow at infinity, there is a refined Weyl asymptotics with three terms. Finally, we illustrate the results by analysing the operator $\mathrm{op}^w(q)$ on $\mathbb{R}^d$, with $q(x,\xi) = (1+|x|^2)(1+|\xi|^2)$. This is joint work with Moritz Doll.

The Borel-Ritt problem in ultraholomorphic classes

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We present recent results and some open problems concerning the Borel-Ritt problem in spaces of ultraholomorphic functions, both of Beurling and Roumieu type.

On hyperbolic equations with multiplicities and singular coefficients

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Joint work with Michael Ruzhansky, Costas Loizou

In this talk we discuss some recent well-posedness results obtained for hyperbolic equations with multiplicities. We distinguish between two cases: regular (smooth) coefficients and singular coefficients. We discuss different notions of solutions and the role played by the lower order terms.

The Picard-Lindelöf theorem for smooth PDE

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Joint work with Lorenzo Luperi Baglini

We first present the Banach fixed point theorem for a contraction P with loss of derivatives $L \in \mathbb{N}$, i.e. satisfying

$$\left\|P^{n+1}\left(x_0\right)-P^n\left(x_0\right)\right\|_i \leq \alpha_{i,n}\left\|P\left(x_0\right)-x_0\right\|_{i+nL}.$$

This result allow us to prove both a Picard-Lindelöf theorem for normal smooth PDE, i.e. of the form

$$\begin{cases}\partial_t^k y(t,x)=F\left[t,x,\left(\partial_x^a y\right)_{|a|\leq L},\left(\partial_t^b y\right)_{|b|<k}\right], \\ \partial_t^j y(t_0,x)=y_{0j}(x)\ j=0,\ldots,k-1,\end{cases}$$

and a very general inverse function theorem in Fréchet spaces. We compare these results with the classical counter-examples of Lewy and Mizohata. Finally, we see that for generalized smooth functions (in particular, for Colombeau generalized functions and hence for Schwartz distributions), this Picard-Lindelöf theorem implies that every Cauchy problem with a normal generalized PDE is well-posed in the Hadamard sense, but only if we allow for solutions defined in infinitesimal intervals in t. The aforementioned classical counter-examples (for existence) and the works of Tychonoff, de Giorgi et al. (for uniqueness), show that a better general result is not possible.

Stochastic parabolic equations with singularities

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Joint work with Tijana Levajković and Ljubica Oparnica

In this work, we study stochastic parabolic evolution problems of the type $\partial_t U = AU + qU + F$, which includes certain singularities. The focus is mainly on all possible singular behaviors of potential q, either in space and time or in random components, but also in investigation possibilities to allow irregular coefficients in the operator A. In the analysis of these evolution problems, we combine the chaos expansion method from the white noise analysis and the concept of very weak solutions. The notion of a stochastic very weak solution is introduced and the existence of a corresponding stochastic initial value problem is proved. The questions on the uniqueness of the stochastic very weak solution as well as its consistency to the stochastic weak solution are discussed.

Blow-up conditions to p-Laplace type nonlinear parabolic equations under nonlinear boundary conditions

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Joint work with S.-Y. Chung

In this talk, we study blow-up phenomena of the p-Laplace type nonlinear parabolic equations under nonlinear boundary conditions. Especially, we introduce new blow-up conditions which depend on the first Robin and Steklov eigenvalues to obtain the blow-up solutions. Therefore, the blow-up conditions depend on the shape of the domain and boundary conditions. In fact, it will be seen that our new conditions improve the conditions ever known so far.

Convolution Modules of Distributions and Fractional Initial Value Problems

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Joint work with R. Hilfer

Using simultaneous convolvability of multiple distributions a convolution module can be constructed for any given convolution semigroup of distributions. The obtained modules are maximal in a certain sense and their elements can be characterized by weighted integrability conditions of their convolution averages. The construction is applied to subalgebras of the quotient field of convolution operators generated by causal translation invariant fractional integrals and derivatives on the real line with real orders. It is then exemplified, how certain fractional initial value problems can be reinterpreted as translation invariant linear systems that are defined on enlarged domains of distributions.

Finite Time Stabilization of Nonautonomous First-Order Hyperbolic Systems

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Joint work with Natalya Lyulḱo

We address nonautonomous initial boundary value problems for decoupled linear first-order one-dimensional hyperbolic systems and describe the phenomenon of finite time stabilization. We provide sufficient and necessary conditions ensuring that solutions stabilize to zero in a finite time for any initial L^2-data. In the nonautonomous case we give a combinatorial criterion stating that the robust stabilization occurs if and only if the matrix of reflection boundary coefficients corresponds to a directed acyclic graph. An equivalent robust algebraic criterion is that the adjacency matrix of this graph is nilpotent. In the autonomous case we also provide a spectral stabilization criterion, which is nonrobust with respect to perturbations of the coefficients of the hyperbolic system.

Descent method based on second order shape derivatives

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Joint work with M. Vrdoljak

We consider optimal design problems for stationary diffusion in the case of two isotropic phases. Goal is to find an optimal distribution of the phases which maximizes the energy functional. Assuming that the interface between phases is regular, one deals with transmission problems for which first and second order shape derivatives are derived.
Considered numerical implementations consists of updating interface by an ascent vector which is constructed using shape derivatives. In this talk we present a new quasi-Newton approach for calculating the ascent vector using domain expression of second order shape derivative. The domain expression or distributed shape derivative seems more appropriate for numerical implementation since boundary representations introduce functions jumps. We also demonstrate heuristic for the step size which utilizes second order shape derivative.
Descent methods based on distributed first and second order shape derivatives are implemented and tested in classes of problems for which classical solutions exist and can be explicitly calculated from optimality conditions. We have observed a stable convergence of both descent methods with second order method converging in half as many steps.

Distributional and synthetic Ricci curvature bounds

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Joint work with M. Oberguggenberger, J.A. Vickers

There are two main approaches to defining Ricci curvature bounds for Riemannian or Lorentzian manifolds beyond the smooth setting. On the one hand, weak derivatives and the notion of positive distributions can be applied to metrics of low regularity. On the other hand, optimal transport theory gives a characterization of Ricci bounds via displacement convexity of an entropy functional, which carries over even to the general setting of metric measure spaces. Both approaches are compatible with the classical definition via the Ricci tensor (and hence with each other) in the case of a smooth metric. In this contribution we investigate whether this compatibility remains valid if the metric is less than twice continuously differentiable.

On some characteristic Cauchy problems

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Summarizing some previous studies, we show that a well defined generalized problem can be associated in the framework of $(\mathcal{C}, E, P)$-algebras with a ill posed (in Hadamard sense) characteristic one by means of generalized operators associated with stability properties of the algebra. Moreover, we give a meaning to the nonlinear characteristic Cauchy problem for the wave equation by replacing it by a family of non characteristic ones, this leads to a well formulated problem in an appropriate $(\mathcal{C}, E, P)$-algebrasalgebra. We prove existence of a solution and precise how it depends on the regularizing choice made.

Stochastic Semigroups within the Framework of Semigroup and Differential Classifications

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Joint work with V. Bovkun, U. Alekseeva

Introduction. A wide class of processes arising in various fields of natural science, economics and social phenomena can be mathematically described by stochastic differential equations (SDE). Recently, great interest in the problems, especially in financial mathematics and biology has led to significant advances in this area. The most studied is the class of diffusion SDEs with Wiener processes being the randomness sources. The solutions of such equations, due to the continuity properties of Wiener processes have continuous trajectories and are suitable for describing processes that do not have jumps. Simulation based on Levy and more general Levy type processes allows one to study along with continuous, jump processes. At the same time, both in applications and in fundamental science, often not the random process itself, defined by SDE or a set of properties, is needed, but its probability characteristic. The study of the relationship between SDEs and deterministic equations for probability characteristics is one of the main directions of stochastic analysis.
Main results and problems. The talk is devoted to solution properties of equations for probability characteristics, defined by stochastic Levy processes. It is shown that, in contrast to the PDEs for probability characteristics determined by Wiener processes, the equations determined by Levy processes are pseudo-differential.
The semigroup technique underlies the study of Cauchy problems for the obtained pseudo-differential equations. The central place is occupied by semigroups with kernels formed by transition probabilities of Markov processes and their important subclasses — Feller and Levy processes. The kernels are considered in spaces of tempered distributions.
The connection between the semigroup classification based on the spectral properties of generators, which are generally pseudo-differential operators, and the Gelfand-Shilov classification for differential systems based on generalized Fourier transform techniques, has been shown. The embedding scheme for stochastic semigroups has been constructed. The possibility of extending the Gelfand-Shilov classification to the class of systems with pseudo-differential operators with definition of correctness classes for the corresponding Cauchy problems in distribution spaces remains open.

Equivalent theorems concerning the convolution of Roumieu ultradistributions

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We use the approach to ultradifferentiable spaces of functions via weight matrices, introduced by A. Rainer and G. Schindl in [2] and developed in [3] and [4] and applied also by other authors (see e.g. the papers of A. Debrouwere and J. Vindas).
This approach allows us to prove certain equivalent results concerning various forms of the general convolution of Roumieu ultradistributions being extentions of theorems proved in [5].

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Maximal energy dissipation condition for shadow waves

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Joint work with Sanja Ruzicic

In the 1970s, C. Dafermos introduced the so-called maximal energy dissipation admissibility condition for weak solutions to conservation law systems. This condition is neither weaker nor stronger than the usual Lax entropy condition. It is based on the use of a single physical energy or entropy function instead of a set of all convex entropy – entropy flux pairs in the Lax condition. The idea is that a proper solution should maximize a general loss in energy (or maximize a rise of global entropy) in time.
Our first aim is to use that condition for eliminating non–physical shadow waves to some systems (that would imply the uniqueness of a solution to a Riemann problem). Systems considered here are the gas dynamics model with a nonpositive pressure (pressureless gas dynamics, Chaplygin, and generalized Chaplygin model). The main result is that we are able to obtain the uniqueness of solutions to all these systems. The uniqueness for the generalized Chaplygin gas is the completely new result that is not obtained by some other methods.
The second aim is to find a proper approximation of the initial data containing the Dirac delta function. We use a new condition, the backward energy condition, to achieve this goal. Again, we were able to choose a proper piecewise constant approximation for all the above systems.

Ultradistributional convolutor spaces

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Let E be a Banach space of tempered ultradistributions that is invariant under translation and modulation. We characterize exactly when a tempered ultradistribution f satisfies

$$f * \varphi \in E, \qquad \forall \varphi \in \mathcal{S}^{[M]}_{[A]}(\mathbb{R}^d),$$

where $\mathcal{S}^{[M]}_{[A]}(\mathbb{R}^d)$ is the Gelfand-Shilov space associated to the weight sequences M and A. This is done in a broad context, including both the quasianalytic and non-quasianalytic cases, where the main idea consists out of considering associated frequency spaces by applying tools from time-frequency analysis. As an application we discuss several extensions of convolution.

On various types of sequential solutions to partial differential equations

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The purpose of this talk is to compare various classes of generalized solutions to partial differential equations that are based on sequences or nets of smooth approximations. Starting from the general theory of *sequential solutions* of Rosinger, we pass by the concepts of *proper weak solutions* of Lindblad and *weak asymptotic solutions* of Maslov, Danilov, Omel'yanov, Shelkovich to arrive at the notions of *very weak solutions* of Garetto and Ruzhansky and *Colombeau solutions*. Rosinger's concepts of stability, exactness and generality admit a systematic view on the different approaches and also help in understanding what existence and uniqueness of solutions means. Finally, we shed some light on the role of factorizing with respect to various ideals.

Time frequency analysis within the spaces with the Hörmander metric

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Joint work with Bojan Prangoski

We analyze the short time Fourier transform $V_\phi f(X,\Xi) = \int_W e^{2\pi i[T,\Xi]} f(T)\phi_X(T)dT$ and the corresponding V_ϕ^* on $\mathcal{S}(W)$ and $\mathcal{S}'(W)$, where $W = V \times V'$ (dimension $2n$) is endowed with the symplectic form $[X,Y]$, the Hörmander metric $W \ni X \to g_X$ and ϕ_X a suitable family of smooth functions generalizing the partition of unity. Weighted p,q spaces, modulation type spaces as well as the action of ΨDO's between such spaces will be presented. We also analyze the interplay of ψDO and the translation-modulation package $\pi(X,\Xi)f(T) = f(g_X(T-X))e^{2\pi i T\Xi}|g_X|^{-1/2}$ over suitable test functions.

Characterisation of generalised modulation spaces via Gabor frames

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Joint work with A. Debrouwere

A Banach space F is said to be a translation-modulation invariant Banach space of distributions on $\mathbb{R}^{2n}$ (a TMIB space for short) if it satisfies the continuous and dense inclusions $\mathcal{S}(\mathbb{R}^{2n}) \hookrightarrow F \hookrightarrow \mathcal{S}'(\mathbb{R}^{2n})$ and if the translation and modulation operators act continuously on F and have polynomially bounded operator norms. We call the strong dual of such a space a dual translation-modulation invariant Banach space of distributions on $\mathbb{R}^{2n}$ (a DTMIB space for short). If F is a (D)TMIB space on $\mathbb{R}^{2n}$, then the modulation space $\mathcal{M}^F$ associated to F is defined as

$$\mathcal{M}^F = \{f \in \mathcal{S}'(\mathbb{R}^n) \,|\, V_g f \in F\}, \quad \text{with norm} \quad \|f\|_{\mathcal{M}^F} = \|V_g f\|_F,$$

where V_g is the short-time Fourier transform with window $g \in \mathcal{S}(\mathbb{R}^n)\backslash\{0\}$; it is know that these generalised modulation spaces satisfy all of the basic properties that the classical modulations spaces $M^{p,q}_\eta$ do. The main result of this talk is that $\mathcal{M}^F$, with F a (D)TMIB space, admits an atomic decomposition via Gabor expansions and that it is characterised by the summability properties of the Gabor coefficients. This generalises the fundamental results for the atomic decompositions of the classical modulation spaces $M^{p,q}_\eta$.

Solutions of hyperbolic stochastic PDEs on bounded and unbounded domains

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Joint work with Sandro Coriasco and Stevan Pilipović

We treat several classes of hyperbolic stochastic partial differential equations in the framework of white noise analysis, combined with Wiener-Ito chaos expansions and Fourier integral operator methods. The input data, boundary conditions and coefficients of the operators are assumed to be generalized stochastic processes that have both temporal and spatial dependence. We prove that the equations under consideration have unique solutions in the appropriate Sobolev-Kondratiev or weighted Sobolev-Kondratiev spaces. Moreover, an explicit chaos form of the solutions is obtained, useful for calculating expectations, variances and higher order moments of the solution.

Distributional convolutors for wavelet transform

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In this paper, we obtain a characterization of the space of convolutors in terms of the wavelet transform. An inversion formula for the wavelet transform of convolutors is also established. Further, Caldern-type reproducing formula is derived in distribution sense as an application of the same.

Characterization of the compactly supported dual windows of a Gabor frame

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Given a Gabor frame, it is well known that the canonical dual also has a Gabor structure, but not all the dual frames need to be Gabor frames. There exist characterizations of all the dual frames with a Gabor structure. In this talk we will present a contribution to the topic, which concerns a characterization of all the dual Gabor frames with compact support (when such ones exist). As an application of this characterization, we consider an iterative procedure for approximation of the canonical dual window via compactly supported dual windows. In certain cases, each iteration step of this procedure gives a dual window from certain modulation spaces or from the Schwartz space.

Spaces of generalized functions related to analytic pseudodifferential operators

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Joint work with J. Toft, P. Wahlberg

60 years ago Valentine Bargmann published a paper in which he introduced an integral transform that now bears his name. Bargmann's results gave a firm mathematical foundation to Vladimir Fock's study of canonical commutation relations of Schrödinger's approach to quantum mechanics. In fact, the Bargmann transform is a unitary map between $L^2(\mathbb{R}^d)$ and a space of analytic functions, nowadays called the Fock (or Bargmann) space. In a subsequent paper (1967) Bargmann considered tempered distributions, and more recently (2012, 2017) Joachim Toft studied the mapping properties of the Bargmann transform when acting on different families of test functions and their distribution spaces.
In the first part of the talk, we give an outline of Toft's fundamental results on the Bargmann transform images of Gelfand-Shilov and Pilipović spaces and their dual spaces.
These results are in the background of recent results on analytic pseudodifferential operators obtained together with J. Toft and P. Wahlberg. In the second part of the talk we will give a brief summary on these investigations.

c-almost periodic generalized functions

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Joint work with V. E. Fedorov, M. Kostic, S. Pilipovic

This talk is devoted to the classes of c-almost periodic type distributions and asymptotically c-almost periodic type distributions with values in complex Banach spaces. An interesting application in the study of existence of asymptotically c-almost periodic type solutions for a class of ordinary differential equations in the distributional spaces is provided.

Integration in generalized function algebras over generalized domains

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Joint work with M. Kunzinger, P. Giordano

Generalized functions in generalized function algebras of Colombeau type can be considered as pointwise functions on so-called generalized points. When doing analysis in these algebras, one naturally considers e.g. infinitely small or infinitely large intervals of generalized points. It is therefore natural to have a theory of integration on sets of generalized points. The classical definition of integral on generalized function algebras only considers an integral over a set of nongeneralized points in the domain. The natural definition on representatives that works for the latter integral, fails however to be independent of the chosen representative when integrating over a (so-called internal) set of generalized points. We show how to overcome this difficulty, and discuss the fundamental properties of this integral. Our integral allows integration over a large class of internal sets, and countable unions of them.

Optimal Control with Piecewise State-Differentiable Dynamics and Performance Index: Generalized Euler-Lagrange Equation

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A new spin is given on the classical optimal control problem with piecewise differentiable dynamics and performance index with respect to the state variables. While in each domain of differentiability, the necessary conditions for optimality are easily established, their interpretation at the boundaries between domains is not well-understood. In this paper, we show that in order to make sense of the Euler-Lagrange equation at this interface one needs to transcend the classical theory of Schwartz distributions and make suitable extensions to allow for the questionable behavior of impulses multiplied by discontinuities, and the notion of partial derivatives at a discontinuity. Such a theory has been developed, using notions from Nonstandard Analysis (NSA). We show by some simple examples the feasibility and utility of this development.

The nuclearity of Gelfand-Shilov spaces and kernel theorems

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Joint work with A. Debrouwere and L. Neyt

The purpose of this talk is to discuss nuclearity and kernel theorems in the context of global spaces of ultradifferentiable functions with rapid decay at infinity. We introduce general classes of Gelfand-Shilov spaces defined via weight matrices and weight function systems, and characterize when they are nuclear in terms of their defining weight systems. Our results might be regarded as counterparts of the classical nuclearity characterization for Köthe sequence spaces. Our general framework allows for a unified treatment of Gelfand-Shilov spaces defined via weight sequences and Beurling-Björck spaces described by means of weight functions (of Braun-Meise-Taylor type). Futhermore, our approach is stable under topological tensor products, hence covering anisotropic cases, and leading to new Schwartz-type kernel theorems.

Session 8: Harmonic Analysis and Partial Differential Equations

Organizers: Vladimir Georgiev, Michael Ruzhansky and Jens Wirth

On a non-local problem for the loaded parabolic-hyperbollic type equation with non-linear terms

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We consider a loaded parabolic-hyperbolic equation fractional order with nonlinear terms:

$$0=\begin{cases} u_{xx}-{}_CD_{ot}^{\alpha}u+a_1(x,\,t)\,u^{p_1}(x,\,t)+f_1(x,\,t;\,u(x,0)), \text{ for } t>0,\\ u_{xx}-u_{tt}+a_2(x,\,t)\,u^{p_2}(x,\,t)+f_2(x,\,t;\,u(x,\,0)), \text{ for } t<0, \end{cases} \tag{1}$$

where ${}_CD_{0t}^{\alpha}$ is Caputo differential operator, $a_i(x,\,t),\ f_i(x,\,t;\,u\,(x,\,0))$ are given functions, $p_i,\ \alpha$ are constants and $p_i>0,\ 0<\alpha<1,\ i=1,\,2.$
Let $I=\{(x,t);t=0,\,0<x<l\}\,,\Omega_1=\{(x,t):0<x<l,\,0<t<h\}\,,\Omega_2=\{(x,t):0<x+t<l,\,0<x-t<l,\ \ t<0\}$ and $\Omega=\Omega_1\cup I\cup\Omega_2$
Problem BV**.** It is required to find a solution $u\,(x,\,t)$ of the equation (1) with the following properties: 1) $u(x,t)\in C\left(\overline{\Omega}\right)\cap C^2(\Omega_2),\,u_{xx},{}_CD_{ot}^{\alpha}u\in C\left(\Omega_1\right),\,u_x\in C^1\left(\overline{\Omega}_1\setminus\{t=h\}\right)$; 2) $u(x,t)$ satisfy boundary value conditions:
$\alpha_1u(l,t)+\alpha_2u_x(l,t)=\varphi_1(t),\,\beta_1u(0,t)+\beta_2u_x(0,t)=\varphi_2(t),\,0\leq t<h;$
$\gamma_1u\left(\frac{x}{2},-\frac{x}{2}\right)+\gamma_2u\left(\frac{x+l}{2},\frac{x-l}{2}\right)=\psi_1(x),\ 0\leq x\leq l;$
and integral gluing condition:
$\lim\limits_{t\to+0}t^{1-\alpha}u_t(x,\,t)=\lambda_1(x)\,u_t(x,\,-0)+\lambda_2(x)\,u_x(x,\,-0)\,+\lambda_3(x)u\,(x,\,0)+\lambda_4(x),\ 0<x<1$, where $\psi(x)$, $\varphi_i(t)$ and $\lambda_k(x)\,(k=\overline{1,\,4})$ are given continuous functions, such that $\sum\limits_{k=1}^{3}\lambda_k^2(x)\neq0,\,\alpha_i,\,\beta_i,\,\gamma_i,\ (i=1,2)$ are given constants.
Unique solvability of solution of the formulated problem was proved by the method of successive approximations of factorial law for Volterra type nonlinear integral equations, under certain conditions to the given functions.

Uncertainty Principles for The Quaternion Linear Canonical Transform

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Joint work with N.Safouane, R.Daher, A. Abouelaz

The (right-sided) Quaternion Linear Canonical transform (QLCT) satisfies some uncertainty principles in a similar way to the Euclidean Fourier transform. The aim of this paper is to prove a quantitative uncertainty inequality about the essential supports of a nonzero function. We also extend signal recovery by using local uncertainty principle to the Quaternion Linear Canonical transform.

Quasi-Banach modulation spaces on LCA groups and pseudodifferential operators

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Joint work with Elena Cordero

We define the modulation spaces $M_m^{p,q}$ on a LCA group $\mathcal{G}$ for $0<p,q\leq\infty$. Connections and equivalences with the classical theory for $p,q\geq1$ by Feichtinger 1983 are investigated, moreover the Euclidean, discrete and compact cases are fully recovered. We then apply this theory to the study of boundedness properties

of pseudodifferential operators. Finally, decay properties for eigenfunctions of localization operators are investigated.

Boas-Type Theorems For The q-Bessel Fourier Transform

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Joint work with El Mehdi Loualid, Radouan Daher

In this paper, we give necessary and sufficient conditions in terms of $\mathcal{F}_{q,\nu}(f)$, the q-Bessel Fourier transform of f, to ensure that f belongs either to one of the generalized Lipschitz classes $H^m_{q,\alpha}$ and $h^m_{q,\alpha}$ for $\alpha > 0$.

The Fourier Chébli-Trimèche Transform On Boehmians

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Joint work with Radouan Daher

In this talk, we construct a Fourier Chébli-Trimèche transformable Boehmian space by proving all the required auxiliary results. Further, we extend the Fourier Chébli-Trimèche transform on these Boehmian spaces consistently and discuss their properties like linearity, injectivity, convolution theorem and continuity with respect to δ-convergence and Δ-convergence are obtained.

Pointwise convergence for the Schrödinger equation with orthonormal initial data

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Joint work with S. Lee, S. Nakamura

For the Schrödinger equation in one dimension, we establish some sharp maximal-in-time estimates associated with orthonormal systems of initial data. Such estimates will follow from maximal-in-space estimates for fractional Schrödinger equations and thus we also address an endpoint problem raised by R. Frank and J. Sabin in their work on Strichartz estimates for orthonormal systems of initial data. Our maximal-in-time estimates allow us to deduce certain pointwise convergence results associated with systems of infinitely many fermions.

Spectrally invariant algebras of pseudodifferential operators with ultradifferentiable orbits

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Many operator algebras occurring in the theory of pseudodifferential operators are spectrally invariant. A famous example is the algebra of smooth operators T such that the orbit

$$G \ni x \mapsto \pi(x)T\pi(x)^{-1} \in \mathcal{L}(H_\pi) \tag{O}$$

is smooth for the Heisenberg group $G = \mathbb{H}$ and the Schrödinger representation $\pi = \rho$ on $H_\pi = L^2(\mathbb{R}^n)$. This algebra corresponds to the symbols in $\mathcal{S}^0_{0,0}(\mathbb{R}^n \times \mathbb{R}^n)$ via the Kohn-Nirenberg quantization. On compact Lie groups G an analogue situation can be found with respect the left regular representation $\pi = L$ on $H_\pi = L^2(G)$ and the symbol space $S^0_{0,0}(G \times \widehat{G})$. If G is a torus and $\pi = L$, it is known that the algebra of operators with analytic orbit (O) corresponds to a space of analytic symbols.

We discuss operator algebras with generalized regularity conditions on the orbits (O) and search for analogues of the cases above. In the case of $G = \mathbb{H}$ with $\pi = \rho$ or general compact Lie groups G with $\pi = L$ we get spectrally invariant algebras of pseudodifferential operators with ultradifferentiable orbits, which correspond to spaces of ultradifferentiable symbols.

Global theory of subelliptic pseudo-differential operators and Fourier integral operators on compact Lie groups

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Joint work with Michael Ruzhansky

In this talk we will present the general aspects of the theory of global subelliptic pseudo-differential operators on compact Lie groups, developed recently by the author and M. Ruzhansky. More precisely, given a compact Lie group G, and the sub-Laplacian $\mathcal{L}$ associated to a system of vector fields $X = \{X_1, \cdots, X_k\}$ satisfying the Hörmander condition, we introduce a (subelliptic) pseudo-differential calculus associated to $\mathcal{L}$, based on the matrix-valued quantisation process developed by M. Ruzhansky and V. Turunen in their previous works. Singular kernels for this calculus, estimates of L^p-L^p, H^1-L^1, and L^∞-BMO type, Calderón-Vaillancourt theorem, heat traces, regularisation of traces, Dixmier traces, global functional calculus, subelliptic Hulanicki theorem, subelliptic Gårding inequalities, the global solvability of subelliptic pseudo-differential problems and the L^2-boundedness of global Fourier integral operators on compact Lie groups, are the topics that we will discuss.

Optimal well-posedness and forward self-similar solution for the Hardy-Hénon parabolic equation

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Joint work with M. Ikeda, K. Taniguchi

The Cauchy problem for the Hardy-Hénon parabolic equation is studied in the critical and subcritical regime in weighted Lebesgue spaces on the Euclidean space $\mathbb{R}^d$. Well-posedness for singular initial data and existence of non-radial forward self-similar solution of the problem are previously shown only for the Hardy and Fujita cases ($\gamma \leq 0$) in earlier works. The weighted spaces enable us to treat the potential $|x|^\gamma$ as an increase or decrease of the weight, thereby we can prove well-posedness to the problem for all γ with $-\min\{2, d\} < \gamma$ including the Hénon case ($\gamma > 0$). As a byproduct of the well-posedness, the self-similar solutions to the problem are also constructed for all γ without restrictions. A non-existence result of local solution for supercritical data is also shown. Therefore our critical exponent s_c turns out to be optimal in regards to the solvability. This talk is based on a joint work with M. Ikeda (Keio U./RIKEN) and K. Taniguchi (Tohoku U.)

Infinite energy solutions to supercritical problems

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Joint work with Lorenzo Pompili, Bonn University

We construct families of smooth solutions to the supercritical defocusing NLS in any space dimension, corresponding to large initial data of infinite energy. The initial data are obtained as n dimensional perturbations of lower dimensional data. A key tool for the proof are global Strichartz estimates for a Schrodinger equation with a large, time dependent potential. Similar results hold for supercritical defocusing nonlinear wave equations.

Lipschitz Conditions in Damek-Ricci Spaces

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Joint work with Salah El Ouadih

In this talk we extend classical Titchmarsh theorems on the Fourier-Helgason transform of Lipschitz functions to the setting of L^p-space on Damek-Ricci spaces. As consequences, quantitative Riemann-Lebesgue estimates are obtained and an integrability result for the Fourier-Helgason transform is developed extending ideas used by Titchmarsh in the one dimensional setting.

Spectral analysis of differential operator on compact Riemannian manifold

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In this article we consider the case of the differential operator defined on the set of real or complex functions of a compact Riemannian manifold. The Laplacian is naturally generalized to the case of complex-valued functions in such a way that its spectrum in the complex and real cases coincides. Studying works where a detailed study of the structure of spaces of eigenfunctions in both cases is carried out, we were able to show specific examples. The invariance of the Laplace operator with respect to isometries implies the coincidence of the spectra of the Laplace operators of isometric Riemannian manifolds.
Thus, the spectrum of the Laplacian is an isometric invariant. One of the earliest results concerning the solution of the inverse problem belongs to G. Weil, who used the theory of integral equations developed by D. Hilbert to show that the volume of a bounded domain in Euclidean space can be determined from the asymptotic behavior of the eigenvalues of the Dirichlet boundary value problem for the Laplace operator. This particular result shows that the spectrum of the Laplacian contains information about some isometric invariants of the manifold on which it is given.
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Lipschitz conditions for the discrete Fourier-Laplace transform associated with the Laplace-Beltrami operator on the sphere

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Joint work with Radouan Daher

Our aim in this talk is to prove an analog of the classical Titchmarsh theorem on the image under the discrete Fourier-Laplace transform of a set of functions satisfying a generalized Lipschitz condition in the space $L_p,\ 1 < p \leq 2$ on the sphere. More precisely, we give a Lipschitz-type condition on f in $L_p(\sigma^{m-1})$ for which its Fourier-Laplace series belongs to l^β for some values of β, where σ^{m-1} be the unit sphere in the space $\mathbb{R}^m,\ m \geq 3$. In the particular case, when $p = 2$, we provide equivalence theorem: we get a characterization of the space $Lip(\gamma, 2)$ of Lipschitz class functions by means of asymptotic estimate growth of the norm of their Fourier-Laplace series for $0 < \gamma < 1$. Furthermore, we introduce Laplace-Dini-Lipschitz class $LDLip(\gamma, \delta, p)$ and we obtain analogous of Titchmarsh's theorems in this occurence.

Smoothing effect and Strichartz estimates for some time-degenerate Schrödinger operators

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Joint work with G. Staffilani, M. Ruzhansky

In this talk we will first investigate the local smoothing effect (both homogeneous and inhomogeneous) for time-degenerate Schrödinger operators of the form

$$\mathcal{L}_{\alpha,c} = i\partial_t + t^\alpha \Delta_x + c(t,x) \cdot \nabla_x, \quad \alpha > 0,$$

where $c(t,x)$ satisfies suitable decay conditions. The local smoothing effect will then be used to prove local well-posedness results for the associated nonlinear Cauchy problem.
Afterwards we will show the validity of Strichartz estimates for a class of operators similar to the previous one, that is of the form

$$\mathcal{L}_b := i\partial_t + b'(t)\Delta_x,$$

with b' satisfying suitable conditions. An application of these estimates will give a (different) local well-posedness result for a semilinear Cauchy problem associated with $\mathcal{L}_b$.

Standing-wave solutions to one-dimensional non-linear Klein-Gordon equations

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In this talk we illustrate the existence of standing-wave solutions to the scalar non-linear Klein-Gordon equation in dimension one and the stability of the ground-state, the set which contains all the minima of the energy constrained to the manifold of the states sharing a fixed charge. For non-linearities which are combinations of two competing powers we prove that standing-waves in the ground-state are orbitally stable. We also show the existence of a degenerate minimum and the existence of two positive and radially symmetric minima having the same charge.

Generalized Sobolev-Morrey estimates for hypoelliptic operators on homogeneous groups

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Let $\mathbb{G} = \left(\mathbb{R}^N, \circ, \delta_\lambda\right)$ be a homogeneous group, Q is the homogeneous dimension of $\mathbb{G}$, $X_0, X_1, \ldots, X_m$ be left invariant real vector fields on $\mathbb{G}$ and satisfy Hörmander's rank condition on $\mathbb{R}^N$. Assume that $X_1, \ldots, X_m$ $(m \leq N-1)$ are homogeneous of degree one and X_0 is homogeneous of degree two with respect to the family of dilations $\left(\delta_\lambda\right)_{\lambda>0}$. Consider the following hypoelliptic operator $\mathcal{L} = \sum\limits_{i,j=1}^{m} a_{ij} X_i X_j + a_0 X_0$ with drift on $\mathbb{G}$, where (a_{ij}) is a $m \times m$ constant matrix satisfying the elliptic condition in $\mathbb{R}^m$ and $a_0 \neq 0$. In [1], for this class of operators, we obtain the generalized Sobolev-Morrey estimates by establishing boundedness of a large class of sublinear operators T_α, $\alpha \in [0, Q)$ generated by Calderón-Zygmund operators ($\alpha = 0$) and generated by fractional integral operator ($\alpha > 0$) on generalized Morrey spaces and proving interpolation results on generalized Sobolev-Morrey spaces on $\mathbb{G}$. The sublinear operators under consideration contain integral operators of harmonic analysis such as Hardy-Littlewood and fractional maximal operators, Calderón-Zygmund operators, fractional integral operators on homogeneous groups, etc.

Keywords: Hypoelliptic operators with drift, Homogeneous group, Fractional integral operator, Singular integral operators, Generalized Morrey space, Generalized Sobolev-Morrey estimates.

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On pointwise convergence of Jacobi-Dunkl series

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The purpose of this talk is to study the pointwise convergence of the Jacobi-Dunkl series. Indeed, we recall some properties of the Jacobi-Dunkl coefficients. Then, we establish a Dirichlet type theorem for expansions in term of Jacobi-Dunkl polynomials.

On the solvability of the problem of synthesizing distributed and boundary controls in the optimization of oscillation processes.

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Joint work with E.Abdyldaeva, A.Anarbekova

We study the solvability of the problem of synthesis of distributed and boundary controls in the optimization of oscillation processes described by partial integro-differential equations with the Fredholm integral operator. Functions of external and boundary actions are nonlinear with respect to the controls. For the Bellman functional, an integro-differential equation of a specific form is obtained and the structure of its solution is found, which allows this equation to be represented as a system of two equations of a simpler form. An algorithm for constructing a solution to the problem of synthesizing distributed and boundary controls described, and a procedure for finding the controls as a function (functional) of the state of the process is described.
Consider a controlled oscillatory process described by the boundary value problem

$$v_{tt} - Av = \lambda \int_0^T K(t,\tau)v(\tau,x)d\tau + f[t,x,u(t)], \quad x \in Q, \ \ 0 < t < T, \tag{1}$$

$$v(0,x) = \psi_1(x), \ \ v_t(0,x) = \psi_2(x), \ \ x \in Q, \tag{2}$$

$$\Gamma v(t,x) \equiv \sum_{i,k=1}^{n} a_{ik}(x)v_{x_k}(t,x)cos(\nu,x_i) + a(x)v(t,x) = p[t,x,\vartheta(t)],$$
$$x \in \gamma, 0 < t < T, \tag{3}$$

where A — is an elliptic operator

$$Av(t,x) = \sum_{i,k=1}^{n} (a_{ik}v_{x_k}(t,x))_{x_i} - c(x)v(t,x), \tag{4}$$

Q —is area of space R^n bounded by piecewise smooth curve γ ; $f[t,x,u(t)] \in H(Q_T)$, $\forall$ control $u(t) \in H(0,T)$, $p[t,x,\vartheta(t)] \in H(\gamma_T)$, $\forall$ of the boundary control $\vartheta(t) \in H(0,T)$; $H(Y)$ — is a Hilbert space of square-summable functions ; λ — is parameter; T — is fixed point in time; with respect to the function of external and boundary actions, we will assume that

$$f_u[t,x,u(t)] \neq 0, \ \ \forall (t,x) \in Q_T; \ \ p_\vartheta[t,x,\vartheta(t)] \neq 0, \ \ \forall (t,x) \in \gamma_T, \tag{5}$$

i.e., monotone with respect to the functional variable.
In the synthesis problem, it is required to find such controls $u^0(t) \in H(0,T)$ and $\vartheta^0(t) \in H(0,T)$ which minimizes the integral quadratic functional.

$$J[u(t),\vartheta(t)] = \int_Q \{[v(T,x) - \xi_1(x)]^2 + [v_t(T,x) - \xi_2(x)]^2\}dx$$
$$+ \int_0^T \{\alpha M^2[t,u(t)] + \beta N^2[t,\vartheta(t)]\}dt, \ \alpha,\beta > 0, \tag{6}$$

defined on the set of generalized solutions of the boundary value problem (1)–(5).
In this case, the desired controls $u^0(t)$ and $\vartheta^0(t)$ defined as a function (functional) of the state of the controlled process, i.e. as

$$u^0(t) = u[t,v(t,x),v_t(t,x)], \ \ (t,x) \in Q_T, \ Q_T = Q \times (0,T),$$

$$\vartheta^0(t) = \vartheta[t,v(t,x),v_t(t,x)], \ \ (t,x) \in \gamma_T, \ \gamma_T = \gamma \times (0,T).$$

The lifespan estimate for 1D semilinear wave equations with spatial weights and compactly supported data

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Joint work with K. Morisawa, H. Takamura

In this talk, I will discuss about initial value problems for semilinear wave equations with spatial weights in one space dimension. The lifespan estimates of classical solutions for compactly supported data are established in all the cases of polynomial weights. The results are classified into two cases according to the total integral of the initial speed.

On the effect of slowly decreasing initial data for nonlinear wave equations with damping and potential in the scaling critical regime

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Joint work with M. Kato

In this talk, I'd like to present a recent result on the Cauchy problem for the nonlinear wave equation with damping and potential:

$$(\partial_t^2 + D(r)\partial_t - \Delta + V(r))U = \pm|U|^{p-1}U \quad \text{in } (0,T)\times\mathbb{R}^3,$$

where $p > 1$. In my previous work obtained jointly with Prof. Georgiev and Prof. Wakasa, the coefficients of damping and potential terms are supposed to satisfy the relations: $V(r) = D(r)^2/4 - D(r)/2$ for $r > 0$, and $D(r) = 2/r$ for $r \geq 1$. Without such restrictions, upper bounds of the lifespan of the solution to the above problem are derived by recent works due to Dai, Kubo, and Sobajima, and also Lai, Liu, Tu, and Wang. However, those proofs are based on the test function method and require the compactness of the support of the initial data. Here, instead of removing the relation between the coefficients of damping and potential terms, we relax the assumption on the initial data at spatial infinity. Actually, we obtain upper bound of the lifespan for slowly decreasing initial data, which seem to be optimal in comparison with the free case. Moreover, we are able to broaden the choice of the damping coefficient as $D(r) = \mu/r$ for $\mu \geq 0$ and $r \geq 1$. The number μ affects on the shift of the critical exponent of the Strauss type.

Recent results on semilinear wave equations with space or time dependent damping

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Joint work with M. Y. Liu, N. M. Schiavone, H. Takamura, Z. H. Tu, K. Wakasa, C. B. Wang and Y. Zhou

In this talk I will present the progress on the small data Cauchy problem of semilinear wave equations with space or time dependent damping

$$\begin{cases} u_{tt} - \Delta u + \dfrac{\mu}{(1+t)^\alpha} u_t = |u|^p, & (t,x) \in [0,T)\times\mathbf{R}^n, \\ u(x,0) = \varepsilon f(x), \quad u_t(x,0) = \varepsilon g(x), & x \in \mathbf{R}^n, \end{cases} \tag{1}$$

or

$$\begin{cases} u_{tt} - \Delta u + \dfrac{\mu}{(1+|x|)^\beta} u_t = |u|^p, & (t,x) \in [0,T)\times\mathbf{R}^n, \\ u(x,0) = \varepsilon f(x), \quad u_t(x,0) = \varepsilon g(x), & x \in \mathbf{R}^n, \end{cases} \tag{2}$$

and will show some recent results.

Subelliptic problems with critical Sobolev exponent and Hardy potential on stratified groups

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We present some recent results about subelliptic equations with critical Sobolev exponent and Hardy potential on stratified Lie groups. Precisely, we provide existence, regularity and qualitative properties of solutions on the entire space and, by exploiting the obtained results on the limit problem, we get existence results for the related Brezis-Nirenberg type problem on bounded domains.

Asymptotic behavior for the nonlinear damped wave equation with variable coefficients

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In this work, we study the well-posedness of solutions, finite-time blow-up mechanism and asymptotic behavior of a nonlinear damped wave equation which arises from the propagation of elastic waves in the condensed matters. On the one hand, the well-posedness is intimately related to the parameter m, we construct exact counterexamples to show the finite-time blow-up phenomena, non-existence of the global solution with small initial data and prove the global existence of solutions with suitable m, etc. On the other hand, by the application of micro-local analysis, we explore the asymptotic energy estimates of the nonlinear equation which is influenced by two types of time-dependent oscillating coefficients on the principal Laplacian operator part. Furthermore, as an inverse problem, in order to demonstrate the optimality of the energy estimates, typical coefficients and initial Cauchy data will be constructed to show the lower bound of growth rate by the application of instability arguments.

Inverse Source Problems in Fractional Dual-Phase-Lag heat conduction

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Joint work with Marian Slodicka

Non-classical thermal models based on a non-Fourier type law have attracted a lot of interest in the past few decades. In this talk, I will discuss the fractional dual-phase-lag heat equation and the uniqueness of a solution to an associated inverse source problem. The constitutive relation

$$\left(1+\tau_q^\alpha \mathsf{D}_t^\alpha\right)\mathbf{q}(\mathbf{x},t) = -\mathbf{k}(\mathbf{x})\left(1+\tau_T^\beta \mathsf{D}_t^\beta\right)\nabla T(\mathbf{x},t), \quad 0<\alpha,\beta<1$$

will replace the classical Fourier law. It allows for two phase-lag parameters and involves fractional derivatives of the heat flux $\mathbf{q}$ and the temperature gradient ∇T. First, an introduction to the modeling part and fractional calculus will be given. Next, I will state and discuss our main uniqueness results of determining a space dependent source given the final time observation. Finally, a possible relaxation of the assumptions will be investigated in two modified models.

Parabolic nonsingular integral operator on generalized Orlicz-Morrey spaces

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We study the boundedness of the parabolic nonsingular integral operator

$$\mathcal{R}f(x) = \int_{D_+^{n+1}} \frac{|f(y)|}{\rho(\widetilde{x}-y)^{n+2}}\,dy.$$

on parabolic generalized Orlicz-Morrey spaces $M^{\Phi,\phi}(D_+^{n+1})$. Here ρ is a certain metric on $\mathbb{R}_+^{n+1}$ and $\widetilde{x} = (x'', -x_n, t) \in D_+^{n+1} = \mathbb{R}^{n-1} \times \mathbb{R}_+ \times \mathbb{R}_+$. The generalized Orlicz-Morrey space $M^{\Phi,\phi}(D_+^{n+1})$ is equipped with the norm

$$\|f\|_{M^{\Phi,\phi}(D_+^{n+1})} \equiv \sup_{x\in D_+^{n+1},\, r>0} \frac{1}{\phi(x,r)} \Phi^{-1}\left(\frac{1}{|E^+(x,r)|}\right) \|f\|_{L^\Phi(E^+(x,r))},$$

where $E^+(x,r) = \Big\{y \in \mathbb{R}^{n+1} : \frac{|x'-y'|^2}{r^2} + \frac{|t-\tau|^2}{r^4} < 1\Big\}$.
The operator $\mathcal{R}$ and its commutator appear in connection with boundary estimates for solutions to parabolic equations.
One of the main theorems states that if Φ satisfies the so-called Δ_2 and ∇_2 condition and, together with $\phi_1, \phi_2 : D_+^{n+1} \times \mathbb{R}_+ \to \mathbb{R}_+$, satisfies some integral inequality, then the operator $\mathcal{R}$ is bounded from $M^{\Phi,\phi_1}(D_+^{n+1})$ to $M^{\Phi,\phi}(D_+^{n+1})$.
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Extensions of the Calderón Problem for discontinuous complex conductivities

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In this talk we present the inverse conductivity problem for complex conductivities with jumps. Our first result concerns the uniqueness for the complex conductivity with a jump along a closed curve. With the tools obtained in these case we explain our result for the case of various curves of discontinuity. For the study of this problem we model it as an interior transmission problem. To treat this problem several new concepts are required, such as an adaptation of the notion of scattering data, and the definition of admissible points, which permit the enlargement of the set of CGO incident waves. This will allow us to prove the reconstruction of the conductivity. Given that this are early results in this direction, we also present some of the footwork necessary to proceed further in this direction. Future ideas are to expand this work into three dimensions by usage of quaternionic analysis, and if time permits some foundations for this extension are established.

Space-time fractional differential equations

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Joint work with D. Suragan and M. Ruzhansky

We present explicit solutions of a class of generalized space-time fractional Cauchy problems with time-variable coefficients [3, 4]. The representation of a solution involves kernels given by convergent infinite series of fractional integro-differential operators, which can be extensively and efficiently applied for analytic and computational goals [2]. We also study inverse Cauchy problems of finding time dependent coefficients for fractional wave and heat type equations, which involve the explicit representation of the solution of the direct Cauchy problem and a recent method to recover variable coefficients for the considered inverse problems [1].

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Global existence and blow-up of solutions to the nonlinear porous medium equation

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Joint work with Berikbol Torebek

In this talk, we present a global existence and blow-up phenomena in a finite time of the positive solution to the nonlinear porous medium equation under some new conditions.

Wavelet Transform of Dini Lipschitz Functions on the Quaternion Algebra

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Joint work with R. Daher, A. Achak, and A. Bouhlal

In this present work, we generalize Titchmarsh's theorem for the complex- or hypercomplex-valued functions. Firstly, we examine the order of magnitude of the windowed linear canonical transform (WLCT) of complex-valued functions that achieved certain Lipschitz conditions on $\mathbb{R}$. Secondly, we studied the order of magnitude of the 2-D continuous quaternion wavelet transform (CQWT) of certain quaternionic valued Lipschitz functions.

Global well-posedness for the non-linear Maxwell-Schrödinger system

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Joint work with P. Antonelli, P. Marcati

In this talk I will discuss the global well-posedness and the growth of Sobolev norms for the Maxwell-Schrödinger system with an additional pure-power non-linearity. The main ingredients are suitable a priori bounds, based on smoothing-Strichartz estimates for the Schrödinger equation, combined with the analysis of a modified energy functional.
I will also discuss the existence and stability of global, finite energy, weak solutions to a quantum hydrodynamic system, which is connected to the non-linear Maxwell-Schrödinger system by means of the Madelung transform.

A constructive approach to nonlinear damped wave equations

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In this talk, an constructive approach to the existence of solutions to damped wave equations will be explained. The key idea is the use of hypergeometric functions.

Representation formulae for the higher-order Steklov and L^{2^m}-Friedrichs inequalities

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Joint work with Tohru Ozawa

In this talk, we obtain remainder term representation formulae for the higher-order Steklov inequality for vector fields which imply short and direct proofs of the sharp (classical) Steklov inequalities. The obtained results directly imply sharp Steklov type inequalities for some vector fields satisfying Hörmander's condition, for example. We also give representation formulae for the L^{2^m}-Friedrichs inequalities for vector fields.

Global dynamics for the critical Hardy-Sobolev parabolic equation below the ground state

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Joint work with N. Chikami, M. Ikeda

We consider the Cauchy problem of the energy critical Hardy-Sobolev parabolic equation

$$\begin{cases} \partial_t u - \Delta u = |x|^{-\gamma}|u|^{2^*(\gamma)-2}u, & (t,x) \in (0,T) \times \mathbb{R}^d, \\ u(0) = u_0 \in \dot{H}^1(\mathbb{R}^d), \end{cases}$$

where $d \geq 3$, $T > 0$, $\gamma \in [0,2)$, and $2^*(\gamma)$ is the critical Hardy-Sobolev exponent, i.e.,

$$2^*(\gamma) := \frac{2(d-\gamma)}{d-2}.$$

Our aim is to give a necessary and sufficient condition on initial data below or at the ground state, under which the behavior of solutions is completely dichotomized. More precisely, the solution exists globally in time and its energy decays to zero in time, or it blows up in finite or infinite time. The proof of global existence is based on the linear profile decomposition, the perturbative result, and the rigidity argument. The proof of blow up result is based on Levine's concavity method with some modifications. The result on the dichotomy for the corresponding Dirichlet problem is also shown as a by-product via a comparison principle.

On Determination of Certain Conductivity Distribution via Partial Boundary Measurements

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We investigate an inverse problem to identify conductivity of special type via single Dirichlet-Neumann partial boundary measurement in the case of rectangle domain and conductivity depending on only one variable.

Time-frequency transforms in Euclidean spaces

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Joint work with V. Vuojamo, H. Orelma

Time-frequency analysis can be described as Fourier analysis simultaneously both in time and in frequency. A time-frequency transform is a sesquilinear mapping from a family of test functions in a Euclidean space to functions in the time-frequency plane. The class of time-frequency transforms is further restricted by imposing conditions stemming from basic transformations of signals and those which an idealized energy density should satisfy. We characterize the time-frequency transforms in terms of the corresponding pseudo-differential operator quantizations and integral kernel conditions.

Discrete time-dependent wave equations I. Semiclassical analysis

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In this paper we consider a semiclassical version of the wave equations with singular Hölder time-dependent propagation speeds on the lattice $\hbar\mathbb{Z}^n$. We allow the propagation speed to vanish leading to the weakly hyperbolic nature of the equations. Curiously, very much contrary to the Euclidean case considered by Colombini, de Giorgi and Spagnolo and by other authors, the Cauchy problem in this case is well-posed in $\ell^2(\hbar\mathbb{Z}^n)$. However, we also recover the well-posedness results in the intersection of certain Gevrey and Sobolev spaces in the limit of the semiclassical parameter $\hbar \to 0$.

Reconstruction of a solely time-dependent source in a time-fractional diffusion equation with non-smooth solutions

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Joint work with A. S. Hendy

In this talk, an inverse source problem (ISP) for a time fractional diffusion equation of order $\alpha \in (0, 1)$ where the coefficients of the elliptic operator are dependent on spatial and time variables is discussed. The missing solely time-dependent source is recovered from an additional integral measurement. First, the uniqueness of a solution to the ISP will be shown. Next, two numerical algorithms based on Rothe's method over uniform and graded grids will be proposed and the convergence of iterates towards the exact solution will be discussed. An essential feature of the fractional subdiffusion problem is that the solution lacks the smoothness near the initial time, although it would be smooth away from $t = 0$. Rothe's method on a uniform grid addresses the existence of a such a solution (non-smooth with t^γ term where $1 > \gamma > \alpha$) under low regularity assumptions, whilst Rothe's method over graded grids has the advantage to cope better with the behaviour at $t = 0$ (also here t^α is included in the class of admissible solutions) for the considered problems. This theoretical obtained results will be supported by numerical experiments.

On the solvability of some systems of integro-differential equations with anomalous diffusion in higher dimensions

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The work deals with the studies of the existence of solutions of a system of integro-differential equations in the case of the anomalous diffusion with the negative Laplace operator in a fractional power in $\mathbb{R}^d,\ d = 4, 5$. The proof of the existence of solutions is based on a fixed point technique. Solvability conditions for non Fredholm elliptic operators in unbounded domains are used.

Hypoelliptic Sobolev and Gagliardo-Nirenberg inequalities and ground states

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Joint work with M. Ruzhansky

In this talk we discuss the dependence of the best constants in Sobolev and Gagliardo-Nirenberg inequalities on general graded Lie groups, which includes the cases of $\mathbb{R}^n$, Heisenberg, and general stratified Lie groups. Moreover, we show that critical Gagliardo-Nirenberg and Trudinger-Moser inequalities are equivalent and give the relation between their best constants. The best constants are expressed in the variational form as well as in terms of the ground state solutions of the corresponding nonlinear subelliptic equations. If time permits, we will discuss versions of the above inequalities in the settings of general noncompact Lie groups.

Uniqueness and stability of traveling waves to the time-like extremal

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Joint work with Jianli Liu

There are a few results about the global stability of nontrivial solutions to quasilinear wave equations. In this paper we are concerned with the uniqueness and stability of traveling wave to the time-like extremal hypersurface in Minkowski space. Firstly, we can get the existence and uniqueness of traveling wave solutions to the time-like extremal hypersurface in $\mathbb{R}^{1+(n+1)}$, which can be considered as the generalized Bernstein theorem in Minkowski space. Furthermore, we also get the stability of traveling wave solutions with speed of light to time-like extremal hypersurface in 1 + (2 + 1) dimensional Minkowski space, which is corresponding with quasilinear wave equation in two dimensions.

Session 9: Integral Transforms and Reproducing Kernels

Organizer: Zouhaïr Mouayn

Generalized conditional expectations with an infinite dimensional conditioning function

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Let $C[0,T]$ denote a generalized analogue of Wiener space, the space of real-valued continuous functions on $[0,T]$ and let

$$Z_{\vec{e},\infty}(x) = (x(0), \int_0^T e_1(t)dx(t), \int_0^T e_2(t)dx(t), \ldots)$$

for $x \in C[0,T]$, where $e_1, e_2, \ldots$ are appropriate functions on $[0,T]$. In this talk, we derive an evaluation formula for calculating the conditional expectations of functions on $C[0,T]$ given $Z_{\vec{e},\infty}$ which has an initial weight and a kind of drift. In fact, we derive the formula for calculating Radon-Nikodym derivatives similar to the conditional expectations of functions on $C[0,T]$ given $Z_{\vec{e},\infty}$. As applications of the formula, we evaluate the derivatives of various functions containing the time integral which is interested in quantum mechanics, especially in Feynman integration theories.

Reproducing kernel Hilbert C*-module for data analysis

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Joint work with I. Ishikawa, M. Ikeda, F. Komura, T. Katsura, and Y. Kawahara

RKHM (Reproducing kernel Hilbert C*-module) is a generalization of RKHS (Reproducing kernel Hilbert space) and characterized by a C*-algebra-valued positive definite kernel and inner product induced by this kernel. Regarding RKHS, it has been actively researched for data analysis. RKHSs effectively handle nonlinearities in original data spaces. However, if the data space is a function space, RKHSs are not sufficient for capturing and extracting continuous behaviors of the data. Therefore, we consider using RKHMs instead of RKHSs for functional data. Since inner products in RKHMs are C*-algebra-valued, they capture more information about functions than complex-valued ones. We show some important properties available in RKHSs are also available in RKHMs. Then, we apply them to the kernel principal component analysis and show RKHMs can effectively extract continuous behaviors of functional data.

The affine ensemble: determinantal point processes associated with the $ax+b$ group

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Joint work with Luis Daniel Abreu, Peter Balazs

We introduce the affine ensemble, a class of determinantal point processes (DPP) in the half-plane $\mathbb{C}^+$ associated with the $ax+b$ (affine) group, depending on an admissible Hardy function g. We obtain the asymptotic behavior of the variance and the exact value of the asymptotic constant, non-asymptotic upper and lower bounds for the variance on a connected set $\Omega \subset \mathbb{C}^+$ ('observation window'), upper bounds for the corresponding polynomial and smooth linear statistics and for the entanglement entropy of the reduced state (the von Neumann entropy of the smooth restriction, in the Toeplitz quantization sense, of the affine ensemble to Ω). When g is a Cauchy function we obtain as special case the DPP associated to the weighted Bergman kernel. When g is chosen within a finite family whose Fourier transform are

Laguerre functions, we obtain the DPP associated to Comtet's hyperbolic Landau levels, modelled on the eigenspaces associated with the finite spectrum of the Maass Laplacian. The wave functions associated with these DPPs describe the quantum dynamics of a charged particle evolving on the open hyperbolic plane under the action of a constant magnetic field, with the number of eigenspaces dependent on the strength of the magnetic field.

A new product formula involving Bessel functions.

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Joint work with M.A Boubatra and M. Sifi

Since their discovery in 1732 by the mathematician Daniel Bernoulli, the Bessel functions developed by the astronomer Friedrich Wilhelm Bessel, have continued to serve as the basis for the description of several scientific phenomena. It is in this context that the mathematics relating to these functions continued to develop. In this talk I will present the results obtained recently in a jointly work with M.A Boubatra and M. Sifi when we considered the normalized Bessel function and we found an integral representation of the product of two mixed Bessel functions with index of step an integer. It has explicit kernel invoking Gegenbauer polynomials. This allows to establish a product formula for the generalized Hankel function which is the kernel of a generalized Fourier transform arising from the Dunkl theory. As application, we define and study a generalized translation operator and a generalized convolution structure.

On Fundamental Solutions of Higher-Order Space-Fractional Dirac equations

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Starting from the pseudo-differential decomposition $\mathbf{D} = (-\Delta)^{\frac{1}{2}}\mathcal{H}$ of the Dirac operator $\mathbf{D} = \sum_{j=1}^{n} \mathrm{e}_j \partial_{x_j}$ in terms of the fractional operator $(-\Delta)^{\frac{1}{2}}$ of order 1 and of the Riesz-Hilbert type operator $\mathcal{H}$ we will investigate the fundamental solutions of the space-fractional Dirac equation of Lévy-Feller type

$$\partial_t \Phi_\alpha(\mathbf{x}, t; \theta) = -(-\Delta)^{\frac{\alpha}{2}} \exp\left(\frac{i\pi\theta}{2}\mathcal{H}\right) \Phi_\alpha(\mathbf{x}, t; \theta)$$

involving the fractional Laplacian $-(-\Delta)^{\frac{\alpha}{2}}$ of order α, with $2m \leq \alpha < 2m+2$ ($m \in \mathbb{N}$), and the exponentiation operator $\exp\left(\frac{i\pi\theta}{2}\mathcal{H}\right)$ as the hypercomplex counterpart of the fractional Riesz-Hilbert transform carrying the *skewness parameter* θ, with values in the range $|\theta| \leq \min\{\alpha - 2m, 2m + 2 - \alpha\}$.
Such model problem permits us to obtain hypercomplex counterparts for the fundamental solutions of higher-order heat-type equations $\partial_t F_M(x,t) = \kappa_M (\partial_x)^M F_M(x,t)$ $(M = 2, 3, \ldots)$ in case where the even powers resp. odd powers $\mathbf{D}^{2m} = (-\Delta)^m$ $(M = \alpha = 2m)$ resp. $\mathbf{D}^{2m+1} = (-\Delta)^{m+\frac{1}{2}}\mathcal{H}$ $(M = \alpha = 2m+1)$ of $\mathbf{D}$ are being considered.

Expected number of zeros of random power series with dependent Gaussian coefficients

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Joint work with Tomoyuki Shirai

We consider a random power series with coefficients being a stationary, centered, complex Gaussian process $\Xi = \{\xi_n\}_{n\in\mathbb{Z}}$, whose covariance matrix is defined by $\gamma(k) = \mathbb{E}\left[\xi_n\overline{\xi_{n+k}}\right]$,

$$f(z) = \sum_{n=0}^{\infty} \xi_n z^n.$$

Its covariance matrix is given by

$$\mathbb{E}\left[f(z)\overline{f(w)}\right] = \frac{G_2(z,w)}{1 - z\overline{w}},$$

where $G_2(z,w) = 1 + G(z) + \overline{G(w)}$ and $G(z) = \sum_{k=1}^{\infty} \overline{\gamma(k)} z^k$. Peres and Virág showed that zeros of the random power series $f_{\mathrm{PV}}(z)$ with independent, identically distributed Gaussian coefficients form the determinantal point process associated with the Bergman kernel. In this talk, we derive the asymptotic behavior of the expected number of zeros $f(z)$ approaching to the circle of convergence and compare that of $f(z)$ with that of $f_{\mathrm{PV}}(z)$. We found that zeros of $G_2(z,z)$ give the negative contribution the expected number of zeros of $f(z)$. As a consequence, we showed that there are less zeros near the boundary of the circle of convergence in our dependent cases than the independent case of Peres-Virág. This talk is based on a joint work with Prof. Tomoyuki Shirai.

A Deep Result of Qi'an Guan for the Hardy H_2 reproducing kernels and New Type Isometrical Inequalities

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Q. Guan ([1]) derived the surprising identity of the conjugate analytic Hardy norm and the Bergman norm. By using the identity, we obtain a new type isometrical inequality:
Theorem: For any given $\epsilon > 0$ and for any fixed analytic function $f(z)$ on $D \cup \partial D$, there exists $r : (0 < r < 1)$ satisfying the inequality

$$\iint_D |f'(z)|^2 dxdy - \epsilon \leq \frac{1}{1-r} \iint_{\{e^{-2G(z,t)} \geq r\}} |f'(z)|^2 dxdy.$$

[1] Q. Guan, A proof of Saitoh's conjecture for conjugate Hardy H_2 kernels, J. Math. Soc. Japan, **71**, No. 4 (2019), 1173–1179. doi:10.2969/jmsj/80668066
[2] S. Saitoh, The Bergman norm and the Szegö norm, Trans. Amer. Math. Soc. **249** (1979)(2), 261–279.
[3] S. Saitoh, The Dirichlet norm and the norm of Szegö type, Trans. Amer. Math. Soc. **254**(1979), 355–364.

Koopman operators on reproducing kernel Hilbert spaces

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Joint work with M. Ikeda, I. Ishikawa

Koopman operators have been utilized in analysis of nonlinear dynamical systems.They have been getting popular in a number of fields in science these days including data science. Recent works have investigated merging the Koopman operator with another tool that is widely used in data science - reproducing kernel Hilbert spaces (RKHS). But combining both objects is not trivial since properties of the Koopman operator essentially depend on the choice of the function space it acts on. We present partially known results concerning the Koopman operator on general RKHS as well as stronger results in specific cases where the kernel of the RKHS is adapted to the dynamics. Finally we introduce extensions to reproducing kernel Banach spaces (RKBS).

Ridgelet transform on the matrix space

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Joint work with Isao Ishikawa, Masahiro Ikeda

A general ridgelet transform $R[f]$ is defined as a right inverse operator of an integral operator $S[\gamma]$ that formulates a certain form of neural networks. Namely, it satisfies $S[R[f]] = f$ for a given function f, meaning that a neural network $S[\gamma]$ represents a function f. It was first discovered by Murata, Candès and Rubin independently in the mid 1990s for the case $S[\gamma](\mathbf{x}) = \int_{\mathbb{R}^m \times \mathbb{R}} \gamma(\mathbf{a}, b)\sigma(\mathbf{a} \cdot \mathbf{x} - b)\mathrm{d}\mathbf{a}\mathrm{d}b$ with activation function $\sigma : \mathbb{R} \to \mathbb{R}$, which formulates a single fully-connected layer followed by element-wise nonlinearity. In this study, we present a new ridgelet transform for the rectangular-matrix case $S[\gamma](\mathbf{x}) = \int_{\mathbb{R}^{m \times k} \times \mathbb{R}^k} \gamma(A, \mathbf{b})\sigma(A\mathbf{x} - \mathbf{b})\mathrm{d}\mu(A, \mathbf{b})$ with activation function $\sigma : \mathbb{R}^k \to \mathbb{R}$, which formulates a single fully-connected hidden layer followed by group-wise nonlinearity, motivated by more modern network architectures in the deep learning age.

Session 10: Operator Theory and Harmonic Analysis

Organizers: Alexey Karapetyants and Vladislav Kravchenko

Maximal commutators and commutators of fractional integral operators in some vanishing Morrey (sub)spaces

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In this talk we consider some closed subspaces of Morrey spaces defined in terms of so called vanishing properties.
We show that such vanishing properties are preserved under the action of maximal commutators and commutators of fractional integral operators.

Approximation properties of pseudo-linear operators with discrete kernels and applications to image processing

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Joint work with T.Y. Gokcer

In the present study, we examine sampling-type discrete operators. By changing the algebraic structure of summation and multiplication, we obtain pseudo-linear form of these operators. Then we study the approximation properties of these operators under usual supremum norm. Rate of approximations by means of suitable Lipschitz classes of continuous functions are also obtained. Finally, we exemplify our kernels and give some applications of our study to the image processing.

Acknowledgements. This study is supported by TÜBİTAK 3501 Career Development Program, Project ID: 119F262.

A numerical study of quantum graphs equipped by one-dimensional Dirac operators

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Joint work with Prof. Dr. Vladimir S. Rabinovich

Let us consider periodic metric graphs Γ equipped by one-dimensional Dirac operators

$$\mathfrak{D}_{\mathrm{Q}} = \frac{1}{\mathrm{i}}\sigma_2 \frac{d}{dx} + \mathrm{Q}\,(x)\,,$$

with potentials Q defined by

$$\mathrm{Q}\,(x) = q_{\mathrm{el}}\,(x)\,\mathbb{I} + q_{\mathrm{am}}\,(x)\,\sigma_1 + (m + q_{\mathrm{sc}}\,(x))\,\sigma_3,$$

and certain conditions at the vertices of Γ. Here we denote by σ_i ($i = 1, 2, 3$) the Pauli matrices, q_{el} is an electrostatic potential, q_{am} is an anomalous magnetic moment, q_{sc} is a scalar potential, $\mathbb{I}$ is the identity 2×2-matrix, and $m \geq 0$ is the rest mass of a relativistic $1/2$-spin particle. Graphs are assumed periodic with respect to a group $\mathbb{G}$ isomorphic to $\mathbb{Z}^m$.
In this talk we consider the application of the spectral parameter power series method [1] for the description of the essential spectra of $\mathbb{G}$-periodic quantum graphs.

1. *Gutiérrez Jiménez N, Torba S M.* Spectral parameter power series representation for solutions of linear system of two first order differential equations. Appl. Math. Comput. 2020; **370**: 124911.

Boundedness of composition operators in holomorphic Hölder type spaces

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Joint work with Alexey Karapetyants, Joel Restrepo

A new characterization of boundedness of the composition operators in generalized Hölder spaces is obtained in terms that do not use the derivative of the composition operator, but using some averaging construction which represents certain integration of the modulus of continuity involving the symbol of the composition operator. This approach also allows to recover previously known results for the standard weights $\omega(t) = t^\alpha$ with $0 < \alpha < 1$. Certain further results on characterization of the same spaces are obtained as well.

Inverse Sturm-Liouville problem with analytical functions in the boundary condition and applications

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We consider the Sturm-Liouville eigenvalue problem

$$-y'' + q(x)y = \lambda y, \quad x \in (0, \pi),$$

$$y(0) = 0, \quad f_1(\lambda)y'(\pi) + f_2(\lambda)y(\pi) = 0,$$

where $q \in L_2(0, \pi)$ is a complex-valued function, called the potential, λ is the spectral parameter, $f_1(\lambda)$ and $f_2(\lambda)$ are entire analytic functions. The inverse spectral problem, which consists in recovering the potential q from a subspectrum $\{\lambda_n\}_{n=1}^{\infty}$, will be discussed. The necessary and sufficient conditions of the solution uniqueness, a constructive algorithm for the solution, global solvability, local solvability, and stability for this inverse problem have been obtained in [*N. Bondarenko*, Open Math. 18, no. 1, 512-528 (2020)] and [*N. Bondarenko*, Math. Meth. Appl. Sci. 43, no. 11, 7009-7021 (2020)]. Applications of these results to partial inverse problems on intervals and on metric graphs, and also to the inverse transmission eigenvalue problem will be discussed.

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Generalized Hankel convolutions and related shift operator

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We introduce some generalized convolutions related to the Hankel integral transform

$$h_\nu[f](s) = \tilde{f}_\nu(s) = \int_0^\infty f(t) j_\nu(st) t^{2\nu+1} dt, \quad \nu > -1/2.$$

Here the function

$$j_\nu(st) = \frac{2^\nu \Gamma(\nu+1)}{(st)^\nu} J_\nu(st) = \sum_{m=0}^{\infty} \frac{(-1)^m \Gamma(\nu+1)(st)^{2m}}{2^{2m} m! \Gamma(m+\nu+1)}$$

is associated with the Bessel function J_ν of the first kind of order ν.
The function $j_\nu(st)$ is the solution of the equation

$$\frac{d^2y}{dt^2} + \frac{2\nu+1}{t}\frac{dy}{dt} + s^2 y = 0$$

under conditions $y(0) = 1$ and $y'(0) = 0$.
The existance conditions of the received convolutions are found and the corresponding generalized shift operators are studied. The results are obtained by using the Kakichev approach. The properties and applications of generalized convolutions and the corresponding shift operators are considered.

Some regularity results for a class of elliptic equations with lower order terms

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Let us consider the Dirichlet problem

$$\begin{cases} \mathrm{div}(A(x,Du)) + b(x)|u(x)|^{p-2}u(x) = f(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

For $p = 2$ we establish higher differentiability of solutions to problem (1) under a Sobolev assumption on the partial map $x \to A(x,\xi)$.
Moreover, we will deal also with degenerate elliptic operator $A(x,\xi)$, with p-growth, $p \geq 2$, with respect to the gradient variable, obtaining, also in this case, boundedness and higher differentiability of the solution.
The novelty, in both cases, is that we take advantage from the regularizing effect of the lower order term, due to the interplay between $b(x)$ and $f(x)$.

[1] C. Capone, T. Radice, *Higher differentiability for solutions to a class of elliptic equations with lower order terms*, Journal of Elliptic and Parabolic Equations, 6 (2), (2020), 751–771.
[2] C. Capone, *On the regularity of solutions of degenerate PDEs with lower order terms*, submitted to Journal of Mathematical Analysis and Applications (2021).

Solvability on Riemannian Symmetric Space

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We study the existence for certain invariant linear differential operators, on simply connected Riemannian symmetric spaces $\mathcal{S}$. We prove that an invariant differential operator on $\mathcal{S}$ admits a fundamental solution if and only if its partial Fourier coefficients satisfy a condition of slow growth.

Nearly outer functions as extreme points in punctured Hardy spaces

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Our starting point is a theorem of de Leeuw and Rudin that describes the extreme points of the unit ball in the Hardy space H^1. We extend this result to subspaces of H^1 formed by functions with smaller spectra. More precisely, given a finite set E of positive integers, we prove a Rudin–de Leeuw type theorem for the unit ball of H^1_E, the space of functions $f \in H^1$ whose Fourier coefficients $\widehat{f}(k)$ vanish for all $k \in E$.

$L^p - L^q$ ESTIMATES OF BERGMAN PROJECTOR ON THE MINIMAL BALL

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Joint work with J. Gonessa

We study the $L^p - L^q$ boundedness of Bergman projector on the minimal ball. This improves the L^p-boundedness of Bergman projector on the minimal ball due to G. Mengotti and E.H. Youssfi, see **The weighted Bergman projection and related theory on the minimal ball and applications**, *Bull. sci. Math.* **123** (1999), 501-525.

Maximization of the solution of Poisson’s equation: Brezis-Galouet-Wainger type inequalities and applications

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Joint work with Prof. Hayk Mikayelyan

For the solution of the Poisson problem with an L^∞ right hand side

$$\begin{cases} -\Delta u(x) = f(x) & \text{in } D, \\ u = 0 & \text{on } \partial D, \end{cases}$$

we derive an optimal estimate of the form

$$\|u\|_\infty \leq \|f\|_\infty \sigma_D(\|f\|_1/\|f\|_\infty),$$

where σ_D is a modulus of continuity defined in the interval $[0,|D|]$ and depends only on the domain D. In the case when $f \geq 0$ in D the inequality is optimal for any domain and for any values of $\|f\|_1$ and $\|f\|_\infty$. We also show that

$$\sigma_D(t) \leq \sigma_B(t), \text{ for } t \in [0,|D|],$$

where B is a ball and $|B| = |D|$. Using this optimality property of σ, we derive Brezis-Galloute-Wainger type inequalities on the L^∞ norm of u in terms of the L^1 and L^∞ norms of f. The estimates have explicit coefficients depending on the space dimension n and turn to equality for a specific choice of u when the domain D is a ball. As an application we derive $L^\infty - L^1$ estimates on the $k-$th Laplace eigenfunction of the domain D.

A direct approach to asymptotic Bergman projections

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Joint work with Alix Deleporte, Johannes Sjostrand, Matthew Stone

In this talk, we shall be concerned with the semiclassical asymptotics for Bergman projections in exponentially weighted spaces of holomorphic functions, with strictly plurisubharmonic weights. Here, in the case of real analytic weights, a result due to O. Rouby, J. Sjöstrand, S. Vũ Ngoc, and to A. Deleporte, establishes that one can describe the Bergman projection up to an exponentially small error. We shall discuss a direct approach to the construction of asymptotic Bergman projections in the analytic case, developed with A. Deleporte and J. Sjöstrand, which allows us to give a simplified proof of this result. A recent extension of the approach to the case of smooth weights, carried out jointly with M. Stone, will also be mentioned.

On orthogonal polynomials and Weyl multipliers

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Given orthonormal system Φ and $1 < p < \infty$ we consider the numerical sequence

$$\mathcal{A}_n^p(\Phi) = \sup_{\{g_k\}} \frac{\left\|\max_{1\leq m\leq n} \left|\sum_{k=1}^m g_k(x)\right|\right\|_p}{\left\|\sum_{k=1}^n g_k(x)\right\|_p}, \quad n = 1,2,\ldots, \tag{1}$$

where the $\sup$ is taken over all sequences of non-overlapping Φ-polynomials g_k, $k = 1,2,\ldots,n$. For $p = 2$ the classical Menshov-Rademacher theorem implies the bound $\mathcal{A}_n^2(\Phi) \lesssim \log n$ for any orthogonal system. For particular orthogonal systems $\mathcal{A}_n^p(\Phi)$ may have a slower growth. If Φ is either a wavelet type system or a martingale difference we prove the sharp bound $\mathcal{A}_n^p(\Phi) \lesssim \sqrt{\log n}$ for any $1 < p < \infty$. We also establish that the Menshov-Rademacher logarithmic bound is the best possible in the case of trigonometric system. This results provide different corollaries, concerning the convergence properties of the mentioned orthogonal systems.

Dyadic analogue of the Paul Erdos problem

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Joint work with Professor V. Yu. Protasov

We are exploring the dyadic analogue of one of the Paul Erdos problem, namely, the existence of a probability density of a random variable (which is a power series), extended to a dyadic half-line. We consider the power series with coefficients being either zeroes or ones at the fixed point x of the $(0, 1)$ interval. The question is whether there is a density from $\mathbb{L}_1$? In classic case it is still an opened problem for x greater than one half (P. Erdos proved the non-existence of the density for lambdas equal to $\frac{1}{p}$, where p is the Pisot number). Moreover, we study the so called "dual problem". The same random variable, but the point x is fixed now ($x = \frac{1}{2}$) and the coefficients are integer and belong to $[0; N]$ segment for some natural N. Here we answer the same question and provide criteria of the existence of a density in terms of the solution of the refinement equation as well as in terms of the coefficients of a random variable.

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Algebras of convolution type operators with continuous data do not always contain all rank one operators

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Joint work with Eugene Shargorodsky (King´s College London)

Let $X(\mathbb{R})$ be a separable Banach function space such that the Hardy-Littlewood maximal operator is bounded on $X(\mathbb{R})$ and on its associate space $X'(\mathbb{R})$. The algebra $C_X(\dot{\mathbb{R}})$ of continuous Fourier multipliers on $X(\mathbb{R})$ is defined as the closure of the set of continuous functions of bounded variation on $\dot{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$ with respect to the multiplier norm. It was proved by C. Fernandes, Yu. Karlovich and the first author that if the space $X(\mathbb{R})$ is reflexive, then the ideal of compact operators is contained in the Banach algebra $\mathcal{A}_{X(\mathbb{R})}$ generated by all multiplication operators aI by continuous functions $a \in C(\dot{\mathbb{R}})$ and by all Fourier convolution operators $W^0(b)$ with symbols $b \in C_X(\dot{\mathbb{R}})$. We show that there are separable and non-reflexive Banach function spaces $X(\mathbb{R})$ such that the algebra $\mathcal{A}_{X(\mathbb{R})}$ does not contain all rank one operators. In particular, this happens in the case of the Lorentz spaces $L^{p,1}(\mathbb{R})$ with $1 < p < \infty$.

Trace inequalities for fractional integrals in mixed norm grand Lebesgue spaces

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D. Adams type trace inequalities for multiple fractional integral operators in grand Lebesgue spaces with mixed norms are established. Operators under consideration contain multiple fractional integrals defined on the product of quasi-metric measure spaces and one-sided multiple potentials. In the case when we deal with operators defined on bounded sets, the established conditions are simultaneously necessary and sufficient for appropriate trace inequalities. The derived results are new even for multiple Riesz potential operators defined on the product of Euclidean spaces.
The talk is based on the paper V. Kokilashvili and A. Meskhi, Trace inequalities for fractional integrals in mixed norm grand Lebesgue spaces, *Fractional Calculus and Applied Analysis*, **23**(2020), No. 5, 1451–1471.

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Extension and Factorization of Riesz Multimorphisms Between Pre-Riesz Spaces

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Joint work with A. Kalauch, Dresden TU

We show that an n-Riesz homomorphism on pre-Riesz spaces can be extended to an n-Riesz homomorphism on their Riesz completions. Moreover, we establish that an n-Riesz homomorphism on pre-Riesz spaces is the product of Riesz homomorphisms, if one considers the universal completion of the range space.

Steklov series and trace spaces for biharmonic boundary value problems

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Joint work with L. Provenzano

We consider boundary value problems of Dirichlet or Neumann type involving the biharmonic operator on Lipschitz domains of the n-dimensional Euclidean space. Following the approach introduced by G. Auchmuty for the Laplace operator, we discuss natural representations for the corresponding boundary operators and solutions by means of ‘Steklov series’ (Fourier expansions associated with appropriate Steklov eigenfunctions).

The differentiation operator: hypercyclicity of its eigenoperators.

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Joint work with Bensaid, Ikram; González, Manuel; Romero, Pilar

An operator T acting on a separable F-space X is called hypercyclic if there exists $f \in X$ such that the orbit $\{T^n f\}$ is dense in X. We will discuss when an operator that λ-commutes with the operator of differentiation on the space of entire functions is hypercyclic, extending results by G. Godefroy and J. H. Shapiro and R. M. Aron and D. Markose.

Spectral analysis of one-dimensional Dirac operators with regular and singular interactions

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Joint work with V. Barrera, V. Rabinovich

This work considers the one-dimensional stationary Dirac operators with regular and singular potentials, which arise in relativistic quantum mechanics for massive $1/2$-spin particles. Dirac operator is described by the following differential expression

$$\mathfrak{D}_{Q+Q_S} := J\frac{d}{dx} + Q\left(x\right) + Q_S\left(x\right),\ x \in \mathbb{R},$$

where $J = \frac{1}{\mathrm{i}}\sigma_2$, the regular potential is

$$Q\left(x\right) = q_{el}\left(x\right)\mathbb{I} + q_{am}\left(x\right)\sigma_1 + \left(m + q_{Sc}\left(x\right)\right)\sigma_3,$$

being $m \geq 0$ is the rest mass of the particle, q_{el} is an electrostatic potential, q_{am} is an anomalous magnetic moment, and q_{sc} is a scalar potential such that $q_{el}, q_{am}, q_{sc} \in L^{\infty}(\mathbb{R})$. By σ_i $(i = 1, 2, 3)$ we denote the Pauli's matrices. Moreover, the singular potential is given by

$$Q_S(x) = \sum_{j=1}^{N} A_j \delta(x - x_j),$$

where N is the number of point interactions, and A_j are strength matrices.
We associate to the Dirac operator $\mathfrak{D}_{Q+Q_S}$ an unbounded operator $\mathcal{D}_{Q,\Omega}$ in $L^2(\mathbb{R}, \mathbb{C}^2)$, being $\Omega = \{x_j\}_{j=1}^{N}$ the support of Q_S. Operator $\mathcal{D}_{Q,\Omega}$ is constructed upon the regular part of Dirac operator and certain boundary conditions at the points Ω, which depend on the matrices A_j.
This talk is devoted to the investigation of the discrete spectrum of operator $\mathcal{D}_{Q,\Omega}$ and the construction of eigenfunctions by means of an efficient numerical method known as SPPS (Spectral Parameter Power Series) method [1]. These results are expressed in the form of power series of the spectral parameter.

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Boundary-value-interface problems on polyhedral domains

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Joint work with Victor Nistor and Yajie Zhang

We study well-posedness and regularity for elliptic and parabolic mixed-boundary-value-interface problems on polyhedral domains in 2 and 3 space dimensions, using suitable weighted Sobolev spaces.

On the essential norm for multilinear operators

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We investigate multilinear variants of the quantities which measure the non-compactness of multilinear operators taking values in Banach spaces with the uniform approximation property. The derived results are applied to multilinear variants of the Hilbert and Riesz transforms, and Riesz potential operators on rearrangement invariant spaces. We derive lower estimates of the essential norm of these operators when the target space has bounded approximation property. The latter problem is studied, generally speaking, in the two–weighted setting. As one of the consequences of the derived results, we conclude that these operators are not compact multilinear operators.

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Integral operators on Orlicz-Morrey spaces

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Joint work with Ryutaro Arai, Ryota Kawasumi and Minglei Shi.

In this talk, I introduce some recent work on the boundedness of several integral operators on Orlicz-Morrey spaces. We consider the Hardy-Littlewood maximal, fractional maximal, Calderón-Zygmund and fractional integral operators and their generalizations. We also consider their commutators with functions in Campanato spaces. The Orlicz-Morrey spaces unify Orlicz and Morrey spaces, and the Campanato spaces unify BMO and Lipschitz spaces. Therefore, our results contain many previous results as corollaries.

Fractional Poincaré-Sobolev inequalities and Harmonic Analysis

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Joint work with Ritva Hurri-Syrjänen, Javier Martínez and Annti Vähäkangas

In this lecture we will discuss some recent results concerning fractional Poincaré and Poincaré-Sobolev inequalities with weights. These results improve some celebrated results by Bourgain-Brezis-Minorescu, Maz'ya-Shaponiskova unified by M. Milman. Our approach is based on methods from Harmonic Analysis. This is especially visible since there is intimate connection of this theory with the BMO space and its different variants.

New descriptions of harmonic Hardy spaces and applications

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Joint work with D. Suragan

We present equivalent descriptions of the harmonic Hardy spaces in the unit disc and in the upper half plane. Such descriptions are found as an application of the generalized Hadamard operator of M. M. Djrbashian (in the unit disc [1] and in the half plane [2]) of a standard function kernel. We also give some results on its inverse operator. We use the obtained results to show explicit solutions of some generalized integro–differential equations over the harmonic Hardy space. To prove our statements we mainly follow some ideas from [1, 2, 3].

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Interpolation of Morrey spaces

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Joint work with Denny Ivanal Hakim, Miestyslaw Mastylo

The goal of this talk is to consider interpolation of Morrey spaces especially complex interpolation of Morrey spaces. Given two arbitrary Morrey spaces, we consider their interpolation. We will also give complex interpolation of closed subspaces. These closed subspaces are related to various truncations and mollifications.

On some weighted inequality for singular integrals

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Joint work with Aline Bonami and Sandrine Grellier

We prove the boundedness of some singular integrals from a family of Orlicz-Musielak type spaces to weighted L^1 spaces. Our family of Orlicz-Musielak type functions generalizes the L/LogL function used in the definition of the space $H^{\log}$.

On weak convergence of shift operators to zero on rearrangement-invariant spaces

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Joint work with A. Karlovich

Let $\{h_n\}_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}^d$ tending to infinity and let $\{T_{h_n}\}$ be the corresponding sequence of shift operators given by $(T_{h_n}f)(x) = f(x - h_n)$ for $x \in \mathbb{R}^d$.
We prove that $\{T_{h_n}\}$ converges weakly to the zero operator as $n \to \infty$ on a separable rearrangement-invariant Banach function space $X(\mathbb{R}^d)$ if and only if its fundamental function φ_X satisfies $\varphi_X(t)/t \to 0$ as $t \to \infty$.
For a non-separable rearrangement-invariant Banach function space $X(\mathbb{R}^d)$, we show that $\{T_{h_n}\}$ does not converge weakly to the zero operator as $n \to \infty$ if
• $h_n = nh,\ h \in \mathbb{R}^d \setminus \{0\}$,
or
• $X(\mathbb{R}^d)$ is a Marcinkiewicz endpoint space $M_\varphi(\mathbb{R}^d)$ or an Orlicz space $L^\Phi(\mathbb{R}^d)$.

Generalized Bessel potential

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In this talk we give an information about generalized Bessel potential. A generalization is achieved by considering the singular Laplace-Bessel operator instead of the Laplace operator in the construction of the Bessel potential. First, we give some auxiliary statements as weighted plane wave relation and generalized Bochner formula. Next, we consider the Bessel kernel and prove some its properties. After that we present definition and properties of generalized Bessel potential. Finally we consider the generalized Bessel potential as a solution to the singular screened Poisson equation.

The Second Boundary Value Problem for Neutral Differential–Difference Equation

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Joint work with N.O.Ivanov

We consider the second boundary value problem for the second order neutral differential–difference equation with variable coefficients on the finite interval $(0, d)$. We assume that the difference operator contains only integer shifts of independent variable and that the Hermitian part of difference operator is positively definite operator. We have proved that the corresponding differential–difference operator is Fredholm operator of index zero. It is shown that smoothness of generalized solutions in the Sobolev space can be violated at the points of orbits for the ends of the interval $(0, d)$, generated by a group of shifts. In addition, smoothness of generalixed solutions holds on subintervals obtained by deleting the above mentioned orbits from the interval $(0, d)$. Finally we formulate the sufficient conditions for existence of generalized solutions from the Sobolev space $W_2^2(0, d)$ in the form of orthogonality of right side of the equation to a finite number of linearly independent functions in the space $L_2(0, d)$.
For the first boundary value problem similar results were obtained in [1].

Acknowledgement. This work is supported by the Ministry of Science and Higher Education of the Russian Federation: agreement no. 075-03-2020-223/3 (FSSF-2020-0018).
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On a Steklov eigenproblem for Maxwell's equations

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Joint work with P. D. Lamberti (University of Padova, Italy)

We discuss a Steklov-type problem for Maxwell's equations which is related to an interior Calderón operator and an appropriate Dirichlet-to-Neumann type map. The corresponding Neumann-to-Dirichlet map turns out to be compact and this provides a Fourier basis of Steklov eigenfunctions for the associated energy spaces. With an approach similar to that developed by G. Auchmuty for the Laplace operator, we provide natural spectral representations for the appropriate trace spaces, for the Calderón operator itself and for the solutions of the corresponding boundary value problems subject to electric or magnetic boundary conditions on a cavity.

Recent results on Bergman kernel estimates and Toeplitz operators in weighted Bergman-type spaces.

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Joint work with José Bonet, Raffael Hägger, Congwen Liu, Wolfgang Lusky and Jani Virtanen

We review recent results on the boundedness of Bergman projections and on the Bergman kernel estimates in weighted Bergman spaces A^p_v, $1 < p < \infty$, or H^∞_v, of the unit disc, where the weights are rapidly decreasing and the spaces are "large". We use a number of techniques based e.g. on estimates of the Taylor series. We also review some progress in the questions of boundedness, compactness, Fredholm and spectral properties of Toeplitz operators in Bergman spaces of several variables.
The results are contained in a series of papers published together with José Bonet, Raffael Hägger, Congwen Liu, Wolfgang Lusky and Jani Virtanen.

An analogue of Chernoff's theorem for the Heisenberg group and an uncertainty principle.

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Joint work with Pritam Ganguly

In this talk we discuss an analogue of Chernoff's theorem for the sublaplacian $\mathcal{L}$ on the Heisenberg group $\mathbb{H}^n$ and as an application we indicate how to prove an Ingham type uncertainty principle for the associated spectral projections.

Transmutation operators, Fourier-Legendre series and solution of direct and inverse Sturm-Liouville spectral problems

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Joint work with V. V. Kravchenko

We consider a differential equation

$$-y'' + q(x)y = \lambda y \tag{1}$$

on a finite interval $[0, b]$. Here $q \in L_2[0, b]$ is the potential (can be complex-valued) and $\lambda = \omega^2 \in \mathbb{C}$ is the spectral parameter.
It is known that arbitrary solution of equation (1) can be obtained by applying a transmutation operator to a solution of the simplest equation (having $q \equiv 0$). This transmutation operator can be taken in the form $Tu(x) = u(x) + \int_{-x}^{x} K(x, t)u(t)\,dt$, where the integral kernel K satisfies certain Goursat problem. We propose to expand the integral kernel K into a Fourier-Legendre series (with respect to the second

variable). As a result, we obtain new representations for fundamental system of solutions of (1),

$$c(\omega, x) = \cos \omega x + 2 \sum_{n=0}^{\infty} (-1)^n \beta_{2n}(x) j_{2n}(x), \tag{2}$$

$$s(\omega, x) = \sin \omega x + 2 \sum_{n=0}^{\infty} (-1)^n \beta_{2n+1}(x) j_{2n+1}(x), \tag{3}$$

where j_n are the spherical Bessel functions, as well as a simple recurrent procedure for computing the coefficients $\beta_n(x)$. Remarkable properties of the representations (2) and (3) are the existence of uniform (with respect to $\omega \in \mathbb{R}$) error bound for truncated sums and fast convergence for both small and large values of the spectral parameter ω. As a result, efficient numerical method for solution of direct spectral problems for equation (1) is proposed.
Moreover, we show that the Fourier-Legende expansion of the integral kernel K combined with the Gelfand-Levitan equation leads to efficient method of solution of various inverse spectral problems. It is worth to mention that only the first coefficient β_0 is sought for, not the whole integral kernel K, so the resulted truncated linear system contains only several equations.

On the Lipschitz and Dini-Lipschitz classes associated with the Fourier-Jacobi operator

Othman Tyr

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Joint work with R. Daher

The purpose of this work is to prove analogues of the classical Titchmarsh's theorem and Younis' theorem associated with the Fourier-Jacobi transform of a set of functions satisfying a generalized Lipschitz condition of a certain order in suitable weighted spaces $L^p([0, +\infty))$, $1 < p \leq 2$. For this purpose, we use a generalized translation operator defined by Flensted-Jensen and Koornwinder.

Convolution estimates for quasi-Orlicz spaces

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Joint work with J. Toft, E. Nabizadeh, S. Öztop

Let $(\Omega_j, \Sigma_j, \mu_j)$ be Borel measure spaces with $\Omega_j \subseteq \mathbb{R}^{d_j}$, $\Phi_{0,j}$ be Young functions, Φ_j be quasi-Young functios of order $r_0 \in (0, 1]$ given by $\Phi_j(t) = \Phi_{0,j}(t^{r_0})$, $t \geq 0$ and let ω be a moderate weight on $\mathbb{R}^{d_1+d_2}$.
In this talk first we deduce discrete convolution estimates between elements in discrete Orlicz space. Thereafter we focus on the semi-continuous convolution and prove corresponding estimates in convolutions between elements in $L^{\Phi_1,\Phi_2}_{(\omega)}(\mathbb{R}^{2d})$, and $\ell^{r_0}_{(v)}$. In the end we also deduce similar estimates for continuous convolutions are replaced by the Wiener spaces $W(L^{\Phi_1,\Phi_2}_{(\omega)})$ and $W(L^1, L^{r_0}_{(v)})$.
For $p \in (0, \infty]$, let $\Phi_p(t) = \frac{t^p}{p}$ when $p < \infty$, and set $\Phi_\infty(t) = 0$ when $0 \leq t \leq 1$ and $\Phi_\infty(t) = \infty$ when $t > 1$. Then it is well-known that $L^{\Phi_p,\Phi_q}_{(\omega)} = L^{p,q}_{(\omega)}$ with equality in quasi-norms. Hence the family of quasi-Orlicz spaces contain the usual Lebesgue spaces and mixed quasi-normed spaces of Lebesgue types.

Construction of transmutation operators for Sturm-Liouville equations in impedance form

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Joint work with Vladislav V. Kravchenko

The aim of this talk is to present a construction of an integro-differential transmutation operator for a Sturm-Liouville equation in the impedance form

$$\frac{d}{dx}\left(a^2(x)\frac{du(x)}{dx}\right)+\rho^2a^2(x)u(x)=0,\quad x\in(0,\ell),\tag{1}$$

where $\rho\in\mathbb{C}$ and $a\in W^{1,\infty}(0,\ell)$ is a complex-valued function called the *impedance function*. We suppose that a does not vanish in the whole interval. The transmutation operator admits the form

$$\mathbf{T}_a u(x)=u(x)-\int_{-x}^{x}K_a(x,t)u'(t)dt,$$

and has the property of transforming the solutions of the equation $y''+\rho^2y=0$ into solutions of (1) preserving the conditions at the origin. We show that the kernel $K_a(x,t)$ exists as the solution of a Goursat type problem together its regularity properties. Additionally, if $a\in C^1[-\ell,\ell]$, the operator $\mathbf{T}_a$ is bounded and invertible in $C^1[-\ell,\ell]$.
When $a\in W^{2,\infty}(0,\ell)$ is real-valued, the kernel $K_a(x,t)$ is used to solve the inverse Sturm-Liouville problem for (1). We show that $K_a(x,t)$ satisfies a Gelfand-Levitan equation similar to the one presented in [1]. We present a Fourier-Legendre expansion of $K_a(x,t)$ in terms of odd Legendre polynomials, and use it to deduce a procedure for solving the inverse problem of recovering the impedance function a from spectral data, through solving an infinite system of linear algebraic equations, thus extending the method from [2,3] onto the Sturm-Liouville equations in impedance form.

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Embedding theorems and generalized weighted composition operators on large Bergman spaces

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Joint work with H. Arroussi and J. Taskinen

We characterize boundedness, compactness and Schatten class membership of the generalized weighted composition operator between exponentially weighted Bergman spaces. We also obtain estimates for the norm of the derivatives of the corresponding reproducing kernel and describe bounded and compact differentiation operators, which generalize the embedding theorems of Luecking and the criteria for bounded and compact Carleson measures.

On the inverse scattering for hierarchies of evolution equations

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We introduce and discuss an inverse scattering approach to find solutions of hierarchies of evolution equations which include the well-known hierarchies of Korteweg-de Vries and Camassa-Holm. The method is based on the scattering theory for the Sturm-Liouville operator

$$L:\frac{1}{y}[-D^2+q],$$

with nontrivial potential q and density y. We make some concrete examples and provide a method to find solutions of the equations of the hierarchies which arise as isospectral flows of the pair (q, y).

Session 11: Operator Theory and Time-dependent PDEs

Organizer: Marcus Waurick

Time evolution of superoscillations

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Joint work with Yakir Aharonov, Fabrizio Colombo, Peter Schlosser

In physics, superoscillations are superpositions of plane waves which (locally) oscillate faster than each of its components. In mathematics, this phenomenon is described by a sequence of functions of the form

$$F_n(x) = \sum_{l=0}^{n} C_l(n) e^{ik_l(n)x} \to e^{ikx} \quad \text{as} \quad n \to \infty,$$

which are linear combinations of exponentials with frequencies $|k_l(n)| \leq 1$, but converge to some exponential with frequency $k > 1$. The topic of this talk is the persistence of superoscillations during the time evolution with respect to the Schrödinger equation. In other words, if $F_n(x)$ is the initial condition of the time dependent Schrödinger equation, does the resulting wave function $\Psi_n(t, x)$ have similar properties at some time $t > 0$? Until now only specific potentials were investigated, for which the corresponding Green's function is known explicitely. In this talk we present a unified approach to the time evolution of superoscillations that only assumes suitable estimates and regularity properties on the Green's function.

A homogenisation theory for a general class of high-contrast problems; asymptotics with error estimates

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Joint work with Ilia Kamotski, Valery Smyshlyaev

In this talk, we present a framework to study the asymptotic behaviour of (a large class of) periodic *non-uniformly elliptic* systems with respect to a (small period) parameter. We determine, under very few readily verifiable assumptions, the leading-order approximation of the solution and derive error estimates, uniform in with respect to the right-hand-side. Spectral asymptotics with error estimates directly follow.

Disjointness-preserving operators and isospectral Laplacians

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Joint work with Wolfgang Arendt

It has been known for decades that one cannot hear the shape of a drum, that is, that it is possible to find two Euclidean domains which are not congruent, but for which there exists a unitary operator intertwining the (Dirichlet or Neumann) Laplacians on these domains. However, these counterexample domains to Kac' famous question all have a special structure: they are composed of a certain number of copies of a given building block glued together in two different ways, such that the unitary intertwining operator acts as a sum of overlapping "local" isometries mapping the copies to each other.
We will explore and sketch a proof of a complementary positive statement: suppose an operator intertwining two appropriate realisations of the Laplacian on a pair of domains preserves disjoint supports of functions defined on those domains. Then, under additional assumptions on the operator far weaker than unitarity, the domains are congruent and the operator is, up to scaling, composition with an isometry. This is inspired by earlier results of Wolfgang Arendt, who proved similar statements for intertwining order isomorphisms.

A Compactness Result for the div-curl System with Inhomogeneous Mixed Boundary Conditions

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Joint work with Nathanael Skrepek

For a bounded Lipschitz domain with Lipschitz interface we show the following compactness theorem: Any L2-bounded sequence of vector fields with L2-bounded rotations and L2-bounded divergences as well as L2-bounded tangential traces on one part of the boundary and L2-bounded normal traces on the other part of the boundary, contains a strongly L2-convergent subsequence. This generalises recent results for homogeneous mixed boundary conditions by the first author and collaborators. As applications we present a related Friedrichs/Poincare type estimate, a div-curl lemma, and show that the Maxwell operator with mixed tangential and impedance boundary conditions (Robin type boundary conditions) has compact resolvents.

Closability and closure of time-variant DAE operators

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Joint work with Marcus Waurick

Given measurable matrix functions $E, A : [0,T] \to \mathbb{K}^{d\times d}$, we consider the corresponding differential-algebraic expression $\mathfrak{t}(x) = Ex' + Ax$ as a perturbation of the differential expression $\tau(x) = Ex'$. We analyze the maximal operator T_m associated with τ and show that only fairly strong conditions on E lead to its closability. To represent the closure T_E of T_m it is usually assumed that $E \in W^{1,\infty}$ and that $\operatorname{rank} E$ is constant. Here, we are able to represent T_E under the more natural condition that $\operatorname{rank} E$ is piecewise constant. In addition, we find a large class of matrix multiplication operators M_A such that the closure of $T_m + M_A$ coincides with $T_E + M_A$.

On Abstract Friedrichs Systems and their Use in Complex Media.

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Joint work with S. Trostorff, M. Waurick

We shall discuss a particular problem class, which is closely linked to the classical concept of Friedrichs systems. We shall re-consider Friedrichs systems from a operator theoretic perspective by initially studying operator equations of the form

$$(1 + A)\,U = F,$$

where A is maximal accretive, i.e. $A,\ A^*$ accretive.

Maximal Accretive Extensions. Beginning with the structural assumption that A is an extension of a skew-symmetric operator $\overset{\circ}{A}$, we are interested in describing maximal accretive extensions A of $\overset{\circ}{A}$. This amounts to construction of suitable boundary conditions, which will be formulated in the framework of boundary data spaces.

Theorem. The operator A_S is m-accretive iff

$$S = \ker\left(\iota_+^* - \mu\iota_-^*\right)$$

for some $\mu : \ker(1 + A) \to \ker(1 - A)$ such that

$$|\mu| \leq 1.$$

Complex materials can be addressed by combining such maximal accretive extensions with suitable material laws, which allows to go beyond the classical Friedrichs type systems framework. Such materials are distinguished by resulting in maximal accretive space-time equations. We shall illustrate the setting by inspecting its utility in the context of electrodynamics in metamaterials.

Extrapolation of input mappings for abstract evolution equations

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This talk deals with recent progress in the study of extremal estimates for evolution equations subject to input functions in time. More precisely, we consider input-to-state stability and variants thereof with respect to different norms and their mutual relations. A focus is laid on L^∞-norms. In particular, we will discuss new insights based on classical factorization results in operator theory exploiting the geometry of the underlying Banach space. Furthermore, we explain how such a-priori estimates for linear sub-problems apply to nonlinear situations.

Observability and Null-Controllability for Evolution Equations

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Given a homogeneous Cauchy problem $x'(t) = -Ax(t)$ for $t > 0$ and $x(0) = x_0 \in X$ in some Banach space X, on the one hand one can try to steer the system to some target state (for example 0) at finite time $T > 0$ via some inhomogeneity of the form $Bu(t)$ given by a control function $u : [0,T] \to U$ for some Banach space U and $B \in \mathcal{L}(U,X)$, while on the other hand one may try to get information on the final state $x(T)$ by just measuring a so-called observation function $y : [0,T] \to Y$ given by $y(t) := Cx(t)$ for some Banach space Y and $C \in \mathcal{L}(X,Y)$. It is well-known that these two properties, null-controllability and observability, are dual to each other. We will present recent results on sufficient criteria for observability as well as applications to Cauchy problems induced by elliptic differential operators. The talk is based on joint works with Clemens Bombach, Fabian Gabel, Dennis Gallaun and Martin Tautenhahn.

On periodical solutions of parabolic nonlocal problem

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Let $\Omega_T = (0,T) \times (0,2) \times (0,1)$. We consider the parabolic equation

$$\partial_t w(t,x) - \sum_{i=1,2} \partial_{x_i}(|\partial_{x_i} w(t,x)|^{p-2} \partial_{x_i} w(t,x)) = f(t,x), \tag{1}$$

where $(t,x) \in \Omega_T$, with nonlocal boundary condition

$$\left.\begin{array}{ll} w(t,x_1,0) = w(t,x_1,1) = 0 & (t \in (0,T), 0 \le x_1 \le 2), \\ w(t,0,x_2) = \gamma_1 w(t,1,x_2) & (t \in (0,T), 0 < x_2 < 1), \\ w(t,2,x_2) = \gamma_2 w(t,1,x_2) & (t \in (0,T), 0 < x_2 < 1), \end{array}\right\} \tag{2}$$

and

$$w(0,x) = w(T,x) \qquad (x = (x_1,x_2) \in Q = (0,2) \times (0,1)). \tag{3}$$

Here $f \in L_2(\Omega_T)$ for $p = 2$ and $f \in L_q(0,T; W_q^{-1}(Q))$ for $p \in (2,\infty)$, $1/p + 1/q = 1$.

Theorem. *Let $p = 2$ and $|\gamma_1 + \gamma_2| < 2$. Then for any $f \in L_2(\Omega_T)$ there exists a unique solution of problem (1)–(3) $w \in L_2(0,T; W_2^1(Q))$, $\partial_t w \in L_2(\Omega_T)$.*
Let $p \in (2,\infty)$ and $|\gamma_1 + \gamma_2| + p^{1/p} q^{1/q} |\gamma_1 - \gamma_2| < 2$. then for any $f \in L_q(0,T; W_q^{-1}(Q))$ there exists at least one solution of problem (1)–(3) $w \in L_p(0,T; W_p^1(Q))$, $\partial_t w \in L_q(0,T; W_q^{-1}(Q))$.

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A Structural Observation for port-Hamiltonian Systems

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Joint work with Rainer Picard, Bruce Watson, Marcus Waurick

We inspect the classical port-Hamiltonian differential operator on an interval and show that this operator can be transformed to the usual derivative via suitable congruence relations. In this way, we provide an alternative proof for the existence of a contraction semigroup to solve the corresponding evolution equation. Moreover, we are able to provide a characterisation result for the exponential stability of this semigroup, without imposing additional regularity constraints on the operators involved, which generalises the results existing in the literature.

Perturbations of periodic Sturm–Liouville operators

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Joint work with J. Behrndt (Graz), P. Schmitz (Ilmenau), and G. Teschl (Vienna).

The work by G.W. Hill in 1886 has led to the 'Hill's equation' for the linear second-order ordinary differential equation with periodic coefficients,

$$\frac{1}{r_0}\left(-\frac{\mathrm{d}}{\mathrm{d}x}p_0\frac{\mathrm{d}}{\mathrm{d}x}+q_0\right)y=\lambda y.$$

The above time-independent Schrödinger equation in one spatial dimension with a periodic potential is used within the description of certain effects of atomic nuclei in a crystal. Here the spectral parameter λ has a physical interpretation as the total energy of an electron, and the band structure of the essential spectrum to regions of admissible and forbidden energies. Moreover, impurities (i.e. perturbations) can lead to additional discrete energy levels in the forbidden regions (i.e. eigenvalues in the gap of the essential spectrum).

Here we investigate the change of the spectrum under L^1-assumptions on the differences of the coefficients. We describe the essential spectrum and the absolutely continuous spectrum of the perturbed operator. If a finite first moment condition holds for the differences of the coefficients, then at most finitely many eigenvalues appear in the spectral gaps. This observation extends a seminal result by the Ukrainian mathematician Rofe-Beketov from the 1960ies.

Homogenisation in electro- and magnetostatics

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In the talk we provide a method of discussing homogenisation problems for problems of the type $\operatorname{div}\varepsilon q=f$, $\operatorname{curl}q=g$ with ε being highly oscillatory on bounded weak Lipschitz domains with continuous boundary subject to suitable boundary conditions. The challenge lies in the fact that due to the topology of the underlying domain, depending on the boundary conditions, there is a finite-dimensional space of harmonic (Dirichlet/Neumann) fields in the intersection of the kernels of $\operatorname{div}\varepsilon$ and curl (with suitable boundary conditions). Thus, the above problem is well-posed only if additional conditions on the harmonic fields are given. Typically, the shape of these fields depends on the coefficients. We treat this problem by generalising and discussing the operator-theoretic notion of nonlocal H-convergence initially developed for the topologically trivial situation of vanishing harmonic fields. Furthermore, we would like to address so-called metamaterials so that we particularly allow for non-selfadjont ε introducing the need for generalised Helmholtz type decomposition results.

Session 12: Partial differential equations on curved spacetimes

Organizers: Anahit Galstyan, Makoto Nakamura and Karen Yagdjian

Numerical solutions of a semilinear Klein-Gordon equation inside a black hole embedded in an expanding universe

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Joint work with K. Yagdjian

Numerical simulations are presented for a a semilinear Klein-Gordon equation with singularity that models a black hole and with time dependent coefficient that models an expanding universe. The high performance parallel computations use the CuPy open-source array library accelerated with NVIDIA CUDA. The numerical solution is based on an explicit Runge-Kutta scheme for the temporal discretization and finite difference discretization in the three-dimensional space. The properties of solutions with compact support are examined numerically.

Asymptotics of the radiation field for the massless Dirac–Coulomb system

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Joint work with Robert Booth, Jesse Gell-Redman

We consider the long-time behavior of the massless Dirac equation coupled to a Coulomb potential. For nice enough initial data, we find a joint asymptotic expansion for solutions near null and future infinities and explicitly characterize the rates of the decay seen in the expansion. Some key new elements are propagation estimates near the singularity of the potential (building on work of the first author with Wunsch) and an explicit calculation with special functions to determine the rates of decay.

Critical Exponents for a class of semilinear evolution equations with scale-invariant time-dependent damping

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Joint work with Jorge Marques, Wanderley Nunes do Nascimento

In this talk we discuss the long time behavior for the Cauchy problem for σ-evolution equation with a scale-invariant time-dependent damping and power nonlinearity $|u|^p$,

$$u_{tt} + (-\Delta)^\sigma u + \frac{\mu}{1+t} u_t = |u|^p, \quad t \geq 0, \quad x \in \mathbb{R}^n, \tag{1}$$

where $\sigma \geq 1,\ \mu > 0$ and $p > 1.$ The critical exponent $p = p_c$ for the global (in time) existence of small data solutions to the Cauchy problem is related to the long time behavior of solutions, which changes accordingly $\mu \in (0, \mu_\sharp)$ or $\mu > \mu_\sharp$. Under the assumption of small initial data in $L^1 \cap L^2$, in the case $\sigma = 1$ we have a competition between the shifted Strauss exponent $p_0(n+\mu)$ and Fujita exponent: M. D'Abbicco (Math. Meth. Appl. Sci (2015), JDE (2021)) and M. Ikeda - M. Sobajima (Math. Ann. 2018)

$$p_c = \max\left\{ p_0(n+\mu), 1 + \frac{2\sigma}{n} \right\} = \begin{cases} p_0(n+\mu), & \mu \leq \frac{n^2+n+2}{n+2} \\ 1 + \frac{2}{n}, & \mu > \frac{n^2+n+2}{n+2}. \end{cases}$$

We also refer to recent papers of K. Tsutaya - Y. Wakasugi (J. Math. Phys., 2020) and A. Galstian - K. Yagdjian(Rev. Math. Phys., 2020) for blow up of solutions in the case that (1) has a polynomial decreasing

in time speed of propagation $a(t) = (1+t)^{-2\alpha}, \alpha \in (0,1)$.
The main goal of this talk is to explain that in the case $\sigma > 1$ in (1) a new effect appears, the critical exponent became a shift of Kato exponent for small values of μ whereas it is still of Fujita type for large values of μ

$$p_c = 1 + \max\left\{\frac{2\sigma}{[n-\sigma+\sigma\mu]_+}, \frac{2\sigma}{n}\right\} = \begin{cases} 1 + \frac{2\sigma}{[n-\sigma+\sigma\mu]_+}, & \mu \in (0,1) \\ 1 + \frac{2\sigma}{n}, & \mu > 1. \end{cases}$$

Fundamental solutions for the Dirac equation in curved spacetime

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Joint work with Karen Yagdjian

In this talk we present the fundamental solutions for the spin-1/2 fields propagating in the spacetimes with power type expansion/contraction and the fundamental solution of the Cauchy problem for the Dirac equation. The derivation of these fundamental solutions is based on formulas for the solutions to the generalized Euler-Poisson-Darboux equation, which are obtained by the integral transform approach.

[1] Karen Yagdjian, Anahit Galstian, *Fundamental solutions for the Dirac equation in curved spacetime and generalized Euler-Poisson-Darboux equation*, arXiv:2103.01405v2

Smooth Loops

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A loop is a rather general algebraic structure that has an identity element and division, but is not necessarily associative, and thus generalizes the notion of a group. A smooth loop is a manifold that is also a loop with smooth multiplication and division operations, and is hence a direct generalization of a Lie group. A key example of a non-associative smooth loop is the 7-dimensional sphere regarded as the loop of octonions of unit length. Given a smooth loop, the tangent space at identity then inherits an algebra structure that generalizes a Lie algebra structure. In this talk we will first overview the key properties of loops in general, and will then specialize to smooth loops, their associated tangent algebras, and will show the key differences and similarities with Lie theory, including a loop analog of the Maurer-Cartan equation and Darboux derivatives of maps from manifolds to smooth loops.

The continuity of the flow map with applications to Euler-Poisson equations

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Joint work with U. Brauer, Universidad Complutense Madrid.

Though there are numerous results for the existence and uniqueness of Euler–Poisson equations in various situations, surprisingly, the continues depending on the initial data has not been established so far. Recently we proved the continuity of the flow map for symmetric hyperbolic systems and we applied it to Euler–Poisson equations. Previously, the continuous depending on the initial data for symmetric hyperbolic was proved by Kato, our method is based upon a new type of energy estimates. This method will enable us to apply it for relativistic flows, including non–isentropic flows.

The large time asymptotics of nonlinear Schrödinger equations

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Joint work with A. Soffer

We consider the Schrödinger equation with a general interaction term, which is localized in space. Under the assumption of radial symmetry, and boundedness uniformly in $H^1(\mathbb{R}^3)$ of the solution, we prove it is asymptotic to a free wave and a weakly localized solution. General properties of the localized solutions are derived.

On the Cauchy problem for the semilinear Proca equations in the de Sitter spacetime

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The Cauchy problem for the semilinear Proca equations is considered in the de Sitter spacetime. The effects of the spatial expansion on the existence of solutions of the equations are considered. The global solutions for small data are shown based on the dissipative effect caused by the spatial expansion.

Decay of solutions of the wave equation in expanding cosmological spacetimes

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Joint work with Joao Costa, Pedro Oliveira, Flavio Rossetti, Amol Sasane,

We analyse the decay of sufficiently regular solutions of the wave equation is various expanding cosmological spacetimes. As illustrated by examples in closed form, important roles are played by the nature of future null infinity, the cosmological redshift and dispersion.

Blow-up results for semilinear wave equations in Einstein-de Sitter spacetime

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Joint work with Makram Hamouda, Mohamed Ali Hamza (Imam Abdulrahman Bin Faisal University, Dammam, KSA)

In this talk we present some blow-up results for the following semilinear damped wave equations in the generalized Einstein-de Sitter spacetime

$$\begin{cases} u_{tt} - t^{-2k}\Delta u + \mu t^{-1} u_t = f(u, u_t), & t > 1, x \in \mathbb{R}^n \\ u(1, x) = \varepsilon u_0(x), & x \in \mathbb{R}^n \\ u_t(1, x) = \varepsilon u_1(x), & x \in \mathbb{R}^n, \end{cases}$$

where $k \in (0, 1)$, $\mu \geqslant 0$ and the nonlinearity $f(u, u_t)$ is either $|u|^p$ or $|u_t|^p$. In particular, we will propose a Strauss-type exponent (a Glassey-type exponent, respectively) as critical exponent for the semilinear equation with power nonlinearity (nonlinearity of derivative type, respectively).

A new classification of semilinear damped wave equations by lifespan estimates

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Joint work with T.Imai, M.Kato, K.Wakasa

The initial value problem for semilinear wave equations with time-dependent damping sometimes appears as a reduced model of curved space-time setting. Its critical exponents of energy solutions of are major to classify the models into "heat-like", or "wave-like". But we point out that this criterion is not always appropriate. Our results [1,2] show that some prototype model has wave-type lifespan estimates even it

has Fujita exponent as a critical one. It means that we have to classify the models according not only to the critical exponent, but also to the lifespan estimate.
[1] Imai T., Kato M., Takamura H., Wakasa K., The lifespan of solutions of semilinear wave equations with the scale-invariant damping in two space dimensions, J. Differential Equations, 269(10) (2020), 8387-8424.
[2] Kato M., Takamura H., Wakasa K., The lifespan of solutions of semilinear wave equations with the scale-invariant damping in one space dimension, Differential Integral Equations, 32(11-12) (2019), 659-678.

Asymptotic expansion of solutions to elastic wave with structural damping

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In this talk, we consider the Cauchy problem for elastic waves with structural damping term. Based on the expression formula for the solutions, we obtain asymptotic profiles of solutions as $t \to \infty$. In other words, we prove approximation formulas of the solutions by the special functions, which shows the asymptotic profiles of solutions are classified into several patterns, depending on the parameters in the equation. As a consequence, we conclude the lower bounds of the norms of the solutions, which implies time decay estimates in the previous works are sharp for large t.

Numerical simulations of semi-linear Klein Gordon equations in the de Sitter spacetime with structure preserving scheme

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Joint work with Makoto Nakamura

The numerical simulations of the Cauchy problem for semi-linear Klein-Gordon equations in the de Sitter spacetime are considered. To perform hi-precision numerical simulations, we use the structure preserving scheme for the canonical formulation of the equations. The reliability of the simulations is confirmed by the preservation of the numerically the Hamiltonian of the equations.

Complex powers of the wave operator on asymptotically Minkowski spacetimes

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Joint work with N.V. Dang

The spectral theory of the Laplace–Beltrami operator on Riemannian manifolds is known to be intimately related to geometric invariants thanks to e.g. zeta functions and heat kernel expansions. On the other hand, despite strong motivation from physics, the case of Lorentzian manifolds has remained mysterious: elliptic theory no longer applies so something different is needed.
In this talk I will report on joint work on this problem with Nguyen Viet Dang, `arXiv:2012.00712`. We consider a class of spacetimes on which the wave operator $\Box_g$ is known to be essentially self-adjoint by a recent result of Vasy. Complex powers $(\Box_g - i\varepsilon)^{-\alpha}$ are defined by functional calculus, and we show that their trace density exists as a meromorphic function. We relate the poles to geometric quantities including the scalar curvature, proving therefore a Lorentzian analogue of a theorem attributed to Connes, Kastler and Kalau–Walze.
The main consequence is that Einstein equations can be derived from a spectral action.

Dirac equation in the de Sitter spacetime

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In the talk we present integral transform approach to solving Dirac equation in the de Sitter spacetime of cosmology. The retarded and advanced fundamental solutions to the Dirac operator and generalized Dirac operator as well as the fundamental solution to the Cauchy problem we write in an explicit form via the fundamental solution of the wave equation in the Minkowski spacetime. As an application of the obtained results, the Huygens' principle for the generalized Dirac operator in the de Sitter spacetime is dicussed. We present sufficient conditions for the Huygens' principle for the generalized Dirac operator and necessary conditions for the Dirac operator in the de Sitter spacetime. In particular, we give different proof of the result by Volkmar Wunsch'85 for the Dirac equation in the de Sitter spacetime. For the particle field in the de Sitter spacetime there are three values $m = 0$ and $m = \pm iH$ of mass for the equation that obeys Huygens principle. The last two appear due to the curvature of the spacetime $R = -12H^2 \neq 0$, where H is the Hubble constant. It is now known that there are three discrete neutrino masses with different tiny values. It is very tempting to think that these three masses $m = 0, \pm iH$ belong to three neutrinos.

Asymptotic decay for defocusing semilinear wave equations

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Joint work with Dongyi Wei

Consider the energy subcritical defocusing semilinear wave equation with pure power nonlinearity. By introducing new weighted vector fields as multipliers, we obtain improved time decay for the potential energy, which leads to the improved scattering results and pointwise decay estimates for the solutions. These are based on joint works with Dongyi Wei.

Session 13: Pseudo Differential Operators

Organizer: Man Wah Wong

Pseudo-differential calculus in anisotropic Gelfand-Shilov setting

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Joint work with Marco Cappiello and Joachim Toft

We study some classes of pseudo-differential operators with symbols a admitting anisotropic exponential growth at infinity and we prove mapping properties for these operators on Gelfand-Shilov spaces. Moreover, we deduce algebraic and certain invariance properties of these classes.

Weyl operator by direct substitution

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Weyl developed a correspondence rule between classical expressions and operators. The rule involves the inversion of the Fourier transform of the classical quantity, where the noncommuting position and momentum operators appear in the inversion, instead of the classical position and momentum. We present a simple direct substitution method that obtains the same operators as the Weyl rule. In addition, we show a direct substitution that does the reverse, that is, obtains the classical quantities from the Weyl operator. We also show how to obtain infinitely many operators that are the same as their classical counterparts.

Localization operators for the windowed linear canonical transform

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The main aim of this work is to introduce and study a class of bounded linear operators $L_{\sigma,\varphi,\psi}$ from suitable Lebesgue spaces $L^p(\mathbf{R})$ to themselves, $1 \leq p \leq \infty$, which are related to the windowed linear canonical transform (WLCT) G^A_φ, where $\sigma \in L^1(\mathbf{R}^2) \cup L^\infty(\mathbf{R}^2)$ in the beginning and afterwards $\sigma \in L^r(\mathbf{R}^2)$, $1 \leq r \leq \infty$, $\varphi, \psi \in L^q(\mathbf{R})$, $1 \leq q \leq \infty$ and A is a 2×2 real parameter matrix such that $\det(A) = 1$.
Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be such a unimodular real matrix (i.e. $a, b, c, d \in \mathbf{R}$ and $\det(A) = ad - bc = 1$). Then the linear canonical transform (LCT) of a signal $f \in L^2(\mathbf{R})$ with respect to parameter matrix A is defined by $L_A(f)(\omega) = \begin{cases} \int_{\mathbf{R}} K_A(x,\omega) f(x) dx, \ b \neq 0 \\ \sqrt{d}\, e^{i\frac{cd}{2}\omega^2} f(d\omega), \ b = 0, \end{cases}$ where $K_A : \mathbf{R}^2 \to \mathbf{C}$ is called the kernel of the LCT and is defined by

$$K_A(x,\omega) = \frac{1}{\sqrt{2\pi i b}} e^{\frac{i}{2b}(ax^2 - 2x\omega + d\omega^2)}, \ b \neq 0,$$

for all $(x,\omega) \in \mathbf{R}^2$. The windowed linear canonical transform (WLCT) of a function $f \in L^1(\mathbf{R})$ (or $L^2(\mathbf{R})$) with respect to the window $\varphi \in L^\infty(\mathbf{R})$ (or $L^2(\mathbf{R})$) is defined by

$$G^A_\varphi(f)(\omega,u) = \int_{\mathbf{R}} f(x)\overline{\varphi^A_{\omega,u}(x)} dx, \ (\omega,u) \in \mathbf{R}^2,$$

where $\varphi^A_{\omega,u}(x) = K_A(x,\omega)\varphi(x-u), \ x \in \mathbf{R}, \ (\omega,u) \in \mathbf{R}^2$. Now, let $\varphi, \psi \in L^2(\mathbf{R})$ be such that $(\varphi,\psi) \neq 0$, where (,) denotes the inner product in $L^2(\mathbf{R})$.
Then the two window localization operator associated to the WLCT is defined on $L^2(\mathbf{R})$ by

$$L_{\sigma,\varphi,\psi}(f)(x) = \frac{1}{(\psi,\varphi)} \int_{\mathbf{R}} \int_{\mathbf{R}} \sigma(\omega,u) G^A_\varphi(f)(\omega,u) \psi^A_{\omega,u}(x) d\omega du,$$

for all f in $L^2(\mathbf{R})$ and $x \in \mathbf{R}$, where $\sigma \in L^1(\mathbf{R}^2) \cup L^\infty(\mathbf{R}^2)$ or equivalently

$$(L_{\sigma,\varphi,\psi}(f), g) = \frac{1}{(\psi, \varphi)} \int_{\mathbf{R}} \int_{\mathbf{R}} \sigma(\omega, u) G^A_\varphi(f)(\omega, u) \overline{G^A_\psi(g)(\omega, u)} d\omega du,$$

for all f and g in $L^2(\mathbf{R})$.
Our main goal in this work is to study the boundedness, compactness and Schatten-von Neumann properties of two window localization operators associated to the WLCT.

Time dependent Weyl transform

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The Weyl operator or transform associates a function of two ordinary variables, say q and p, with a function of two operators, say Q and P. But suppose the variables q and p or the operators Q and P are time-dependent, how then does the Weyl operator change with time? We address this question for cases where the time dependence is governed by some of the standard evolution equations, such as the Heisenberg equation of motion for operators. We discuss the question in general and give some explicit results for special cases, for example, when the operators are evolved by a unitary transformation.

Microlocal propagation of singularities for semilinear partial differential equations in weighted Fourier Lebesgue spaces

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Joint work with A. Morando

Some results of microlocal continuity for pseudodifferential operators whose non regular symbols belong to weighted Fourier Lebesgue spaces are given.
Inhomogeneous local and microlocal propagation of singularities of Fourier Lebesgue type are then studied, with applications to some classes of semilinear equations.

Basic calculus of pseudodifferential operators with nonsmooth symbols

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Joint work with Ivana Vojnovic

We show that basic calculus of pseudodifferential operators including asymptotic formulae and oscillatory integral representations for the symbol of the composition and the formal adjoint can partially be saved even when we deal with symbols that are not necessarily of class C^∞, but only of class C^{2N} for some $N \in \mathbb{N}_0$.

Pseudo-differential operators and localization operators on $\mathcal{S}^\mu_\nu(\mathbb{R})$ space associated with linear canonical transform

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Joint work with Tusharakanta Pradhan and Michael Alphonse

Gelfand-Shilov's space, namely $\mathcal{S}_\nu^\mu(\mathbb{R})$, (where $\mu, \nu \in \mathbb{R}$, such that $\mu \geq 0$, and $\nu \geq 0$) played an important role in developing the general theory of partial differential equations. This work introduces the definition and some essential properties related to pseudo-differential operators associated with a symbol $\sigma \in \Gamma_{\nu,\mu}^m(\mathbb{R}^2)$, (where $m = (m_1, m_2) \in \mathbb{R}^2$; $\mu, \nu \in \mathbb{R}$, such that $\mu > 1$, and $\nu > 1$) on $\mathcal{S}_\nu^\mu(\mathbb{R})$ using linear canonical transform. Commutator identities for differential operators on Schwartz space $\mathcal{S}(\mathbb{R})$ are obtained using the same transformation. We define general ultra distribution spaces $(\mathcal{S}_\nu^\mu)^{'}(\mathbb{R})$ that contains the Schwartz dual $\mathcal{S}^{'}(\mathbb{R})$. Further, we have also defined the properties of localization operators and Weyl operators on $\mathcal{S}_\mu^\mu(\mathbb{R})$ and it's dual by using linear canonical transformation. Furthermore, we have defined and solved the generalized partial differential equation by exploiting the linear canonical transform technique theory.

L^p-L^q boundedness of pseudo-differential operators on smooth manifolds

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Joint work with Duván Cardona, Marianna Chatzakou, Michael Ruzhanski and Niyaz Tokmagambetov

In this talk, we will discuss the L^p-L^q boundedness of global pseudo-differential operators and Fourier multipliers on smooth manifolds for the range $1 < p \leq 2 \leq q < \infty$ using the nonharmonic Fourier analysis developed by Ruzhansky, Tokmagambetov and Delgado. As an application, we obtain the boundedness of spectral multipliers, embedding theorems and time asymptotic the heat kernels for the anharmonic oscillator.

Boundedness of trilinear pseudo-differential operators with flag type symbols on local Hardy spaces

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In this talk, we will describe some work on the endpoint estimate ($0 < p \leq 1$) for a trilinear pseudo-differential operator, where the symbol involved is given by the product of two standard symbols from the bilinear Hörmander class $BS_{1,0}^0$. More precisely, we will consider the trilinear pseudo-differential operator with flag symbols defined as follows:

$$T_m(f,g,h)(x) := \int_{\mathbb{R}^{3n}} a(x,\xi,\eta,\zeta) b(x,\eta,\zeta) \hat{f}(\xi) \hat{g}(\eta) \hat{h}(\zeta) e^{2\pi i x \cdot (\xi+\eta+\zeta)} d\xi d\eta d\zeta,$$

where $a(x,\xi,\eta,\zeta) \in TS_{1,0}^0(\mathbb{R}^{4n})$, $b(x,\eta,\zeta) \in BS_{1,0}^0(\mathbb{R}^{3n})$. We will show that T_m is bounded from $h^{p_1} \times h^{p_2} \times h^{p_3}$ to L^s for $0 < p_1, p_2, p_3 < \infty$ with $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{s}$ with $0 < s < \infty$.

Microlocal regularity of nonlinear PDE in quasi-homogeneous Fourier-Lebesgue spaces

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Joint work with G. Garello

We study the continuity in weighted Fourier-Lebesgue spaces for a class of pseudodifferential operators, whose symbol has finite Fourier-Lebesgue regularity with respect to x and satisfies a quasi-homogeneous decay of derivatives with respect to the ξ variable. Applications to Fourier-Lebesgue microlocal regularity of fully nonlinear partial differential equations are given.

Wigner analysis of operators

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Joint work with E.Cordero

We apply the Wigner distribution to the time-frequency analysis of the linear operators. In particular, we introduce a generalized definition of Wigner transform and provide applications to pseudo-differential operators.

Analytic pseudo-differential calculus via the Bargmann transform

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Joint work with Nenad Teofanov, Patrik Wahlberg

The Bargmann transform maps Fourier-invariant function spaces and their duals to spaces of formal power series expansions, which sometimes are convenient classes of analytic functions.
In the 70th, Berezin used the Bargmann transform to translate problems in operator theory into an analytic pseudo-differential calculus, the so-called Wick calculus, where the involved symbols are analytic functions, and the corresponding operators map suitable classes of entire functions into other classes of entire functions. In the same manner, the Toeplitz operators correspond to so-called anti-Wick operators on the Bargmann transformed side.
Recently, the author performed some investigations on the so-called Pilipović spaces, defined by imposing suitable boundaries on the Hermite coefficients of the involved functions or distributions. The family of Pilipović spaces contains all Fourier invariant classical Gelfand-Shilov spaces and other subspaces of such Gelfand-Shilov spaces. In the same way, the family of Pilipović distribution spaces contains spaces which are strictly larger than any Fourier invariant Gelfand-Shilov distribution space.
In the talk we find convenient characterisations of Wick and anti-Wick operators acting on the Bargmann images of Pilipovicć spaces. We also discuss some links between global ellipticity in the real pseudo-differential calculus and the Wick calculus, as well as links between Wick and anti-Wick operators.

H-distributions on Hörmander spaces

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Joint work with Ivan Ivec

We define an important microlocal tool – H-distributions (also called defect distributions), a generalization of H-measures. Using pseudo-differential calculus we construct defect distributions associated with weakly convergent sequences in Hörmander $B_{p,k}$ spaces. Results are applied to linear partial differential equations with non-smooth coefficients and to some semilinear equations.
Let (u_n) be a weakly convergent sequence of approximate solutions of the given semilinear pseudo-differential equation. We obtain a relation between appropriate defect distribution and (locally) strong convergence of the sequence (u_n^2).

Extension of Pseudo differential operators through antiwick calculus with super-exponential kernels

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Joint work with Stevan Pilipovic, Bogan Prangoski

Our aim is to show that the Weyl quantization usually considered in the framework of $\mathcal{S}'^{\{M_p\}}(\mathbb{R}^{2d})$ can be extended to a wider class of ultradistributions related to the growth order.
This talk is based on the joint work with Stevan Pilipović and Bojan Prangoski

The Bargmann transform for Shubin pseudodifferential operators

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Joint work with N. Teofanov and J. Toft

We study Wick operators obtained from Weyl pseudodifferential operators by means of conjugation with the Bargmann transform. Formulas expressing the Wick symbol from the Weyl symbol and vice versa are deduced. These formulas admit characterization of Wick symbols corresponding to Shubin pseudodifferential operators. As a by-product we obtain a short proof of the result on composition for the Shubin calculus.

Session 14: Quaternionic and Clifford Analysis

Organizers: Swanhild Bernstein, Uwe Kähler, Irene Sabadini and Franciscus Sommen

Sets of uniqueness for inframonogenic functions

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Joint work with A. Moreno García, T. Moreno García

As a consequence of the maximum principle, it is obvious that one sphere is a set of uniqueness for harmonic functions. This means that any harmonic function in a domain $\Omega \subset \mathbb{R}^m$, which vanishes on a sphere contained together with its interior in Ω, is identical to zero there.
Inframonogenic functions are the solutions of the equation $\partial f \partial = 0$ and recently it became clear that they have interesting connections with some topics of linear elasticity theory.
The aim of this talk is to show how, even in absence of the maximum principle, a sphere is a set of uniqueness for inframonogenic functions in euclidean spaces of odd dimension. In even dimension we provide examples of non-zero inframonogenic functions which vanish on a sphere.

On a one-dimensional Continuous Octonion Fourier Transform

Eusebio Ariza Garcia

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Joint work with Claudia Jimenez Heredia and Carmen Judith Vanegas

The Fourier transform is a very useful tool in many areas. In this talk, we introduce a one-dimensional octonion Fourier transform, exhibit some of its properties, including Plancherel's and Parseval's theorems and an inversion formula, and apply these properties to prove an important inversion formula for a three-dimensional octonion Fourier Transform.

HIGHER DIMENSIONAL SPHERICAL MONOGENICS, REPRODUCING KERNELS AND GRADIENT DESCENT

Hamed Baghal Ghaffari

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In this talk, we build higher dimension spherical monogenics through the use of appropriate reproducing kernels and an application of a gradient descent-based optimization. These functions are an essential component in the development of higher dimensional Clifford-valued prolate spheroidal wave functions (CPSWFs). We will show that in dimension at least 2, the CPSWFs are eigenfunctions of the restriction of the Fourier transform to the unit ball.

Time-frequency analysis for the fractional Dirac equation

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Joint work with N. Faustino

We investigate several issues concerning the fractional Dirac equation from a time-frequency analysis perspective. Recall that the fractional Hilbert operators $\mathcal{H}^\theta$ and H^θ are interrelated by

$$H^\theta = e^{-i\frac{\pi\theta}{2}}\mathcal{H}^\theta$$

are associated to the Fourier symbols $h_\theta(\xi)$ and $e^{i\frac{\pi\theta}{2}}h_\theta(\xi)$, with

$$h_\theta(\xi) = e^{-i\frac{\pi\theta}{2}}\left(\cos\left(\alpha\frac{\pi}{2}\right) + \frac{\xi}{|\xi|}\sin\left(\theta\frac{\pi}{2}\right)\right).$$

Then the fractional Dirac operator is defined by

$$D^{\alpha,\theta} = |D|^\alpha H^\theta, \quad 0 \le \alpha, \theta \le 1$$

and it's Fourier symbol is given by

$$|\xi|^\alpha \exp(-i\tfrac{\pi}{2}\theta)\exp(\tfrac{\pi}{2}\theta\frac{\xi}{|\xi|})$$

We will consider the solution to he Cauchy problem

$$\begin{cases} i\frac{\partial u}{\partial t}(x,t) &= D_x^{\alpha,\theta}u(x,t) \\ u(x,0) &= f(x), \quad x \in \mathbb{R}^d, t \ge 0. \end{cases}$$

It turns out the the appropriate spaces are modulation spaces with weights which admit sub-exponetial growth at infinity.

On a quaternionic Picard Theorem

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Joint work with Joerg Winkelmann

The classical theorem of Picard states that a non-constant holomorphic function $f : \mathbb{C} \to \mathbb{C}$ can avoid at most one value. We investigate how many values a non-constant slice regular function f of a quaternionic or octonionic variable may avoid.

Meridional Electrostatic Fields in Inhomogeneous Media and the Reduced Quaternion-valued Functions Associated with Classical Holomorphic

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Class of meridional solutions of the static Maxwell system in some special cylindrically layered media in $\mathbb{R}^3 = \{(x_0, x_1, x_2)\}$

$$\begin{cases} \text{div } (\rho^{-1}\vec{E}) = 0, \\ \text{curl } \vec{E} = 0, \end{cases}$$

where $\vec{E} = (E_0, E_1, E_2)$, $\rho = \sqrt{x_1^2 + x_2^2} > 0$, is equivalently represented as class of the reduced quaternion-valued functions associated with classical holomorphic into the framework of Fueter's construction in $\mathbb{R}^3$

$$F = F(x) = u_0 + iu_1 + ju_2 = u_0(x_0, \rho) + Iu_\rho(x_0, \rho),$$

$$x = x_0 + I\rho, \quad I = \frac{ix_1 + jx_2}{\rho}, \quad I^2 = i^2 = j^2 = -1,$$

taking into account that $(u_0, u_1, u_2) := (E_0, -E_1, -E_2)$.
Roots of the characteristic equation of the electric field gradient tensor $\mathbf{J_{lm}}(\vec{E}) = \frac{\partial E_l}{\partial x_m}$ $(l, m = 0, 1, 2)$ into the framework of Fueter's construction in $\mathbb{R}^3$ are given by formulas $\lambda_0 = -\frac{u_\rho}{\rho}$, $\quad \lambda_{1,2} = \pm\sqrt{\left(\frac{\partial u_\rho}{\partial x_0}\right)^2 + \left(\frac{\partial u_\rho}{\partial \rho}\right)^2}$.
New concept of the reduced quaternion-valued Fourier-Fueter cosine and sine transforms of real-valued originals $\tilde{\eta}(\tau)$ into the framework of Fueter's construction in $\mathbb{R}^3$ allows us to demonstrate explicitly properties of surprising analytic models of meridional electrostatic fields in cylindrically layered media described by the dielectric permittivity $\varepsilon(\rho) = \rho^{-1}$.

Triangular representations of quaternionic operators

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Joint work with F. Colombo, U. Kähler, I. Sabadini

The development of the quaternionic operator theory started in the 1930s, mainly motivated by a need for a mathematical foundation of quaternionic quantum mechanics. However, the non-existence of a precise definition of the spectrum of a quaternionic linear operator (due to the lack of symmetry) made it difficult to construct a spectral theory for such operators. Indeed, for quaternionic operators the notion of a left spectrum has little practical use while the notion of a right spectrum is based on a nonlinear eigenvalue problem. In this talk we recall the notion of S-spectrum as a natural concept of spectrum in a noncommutative setting. We will use it to discuss quaternionic Volterra operators and triangular representations of quaternionic operators similar to the classic approaches by Gohberg, Krein, Livsic, Brodskii, and de Branges. To this end we construct spectral representations of quaternionic operators via integration with respect to quaternionic eigenchains and discuss the concept of P-triangular operators in the quaternionic setting.

Superoscillations, infinite order differential operators and hyperholomorphic functions

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Infinite order differential operators appear in different fields of mathematics and physics and in the past decade they turned out to be of fundamental importance in the study of the evolution of superoscillations as initial datum for Schrödinger equation. Inspired by the operators arising in quantum mechanics, we investigate the continuity of a class of infinite order differential operators acting on spaces of entire hyperholomorphic functions. We will consider two classes of hyperholomorphic functions, both being natural extensions of holomorphic functions of one complex variable. We show that, even though these two notions of hyperholomorphic functions are quite different from each other, in both cases, entire hyperholomorphic functions with exponential bounds play a crucial role in the continuity of infinite order differential operators acting on these two classes of functions.

$*$-exponential and $*$-logarithm of slice regular functions of a quaternionic variable

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Joint work with A. Altavilla

Given a slice regular function f, Colombo, Sabadini and Struppa introduced its $*$-exponential $\exp_*(f)$, which was later studied by Altavilla and de Fabritiis, showing in particular that $\exp_*(f)$ is a never vanishing function.
In this talk, given a never vanishing slice regular function g defined on a circular domain contained in the algebra of quaternions $\mathbb{H}$, we will present some results on its $*$-logarithm, that is any slice regular function f such that $\exp_*(f) = g$.
In particular, we will give topological conditions on the domain of definition of g and on the characteristics of g itself in order to obtain existence results for a $*$-logarithm.
Quite unexpectedly, we are able to state conditions on the behaviour of the function g that ensure the uniqueness of the solution of the equation $\exp_*(f) = g$, in sharp contrast with the case of holomorphic functions defined on a domain of the complex line $\mathbb{C}$.

The Poisson kernel and the Fourier transform of the slice monogenic Cauchy kernels

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In this talk we show that the relation $\Delta_{n+1}^{(n-1)/2} S_L^{-1} = \mathcal{F}_n^L$ between the slice monogenic Cauchy kernel S_L^{-1} and the F-kernel $\mathcal{F}_n^L$, that appear in the integral form of the Fueter-Sce-Qian-theorem for n odd, holds also in the case we consider the fractional powers of the Laplace operator Δ_{n+1} in dimension $n+1$, i.e., for n even. Moreover, this relation is proven computing explicitly Fourier transform of the kernels S_L^{-1} and $\mathcal{F}_n^L$ as functions of the Poisson kernel. Similar results hold for the right kernels S_R^{-1} and of $\mathcal{F}_n^R$.

Weighted Rarita-Schwinger Type Operators

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Joint work with C.J. Vanegas

The Rarita-Schwinger type operators are generalizations of the Dirac operator which in turn is a natural generalization of the Cauchy-Riemann operator. They have been studied in the context of the usual Clifford algebras and with the obtaining of their fundamental solutions, integral formulas have been proposed for solving problems with boundary values. In this talk we will show Rarita-Schwingertype operators involving weighted Dirac operators and also construct fundamental solutions for these operators. We will call these operators weighted Rarita-Schwinger type operators. Finally some physical and mathematical applications are proposed.
Keywords: Rarita-Schwinger type operators, weighted Dirac operators, fundamental solutions.

A new family of QRKHS in the Clifford-Appell case

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Joint work with D. Alpay, F. Colombo, S. Krausshar, I. Sabadini

We will use the classical Fueter mapping theorem in order to construct a special system of Clifford-Appell polynomials in the quaternionic setting. We will discuss various properties of such polynomials and study their algebraic behaviour with respect to the well-known Cauchy-Kowalevski product. Then, based on this system we can introduce a new family of quaternionic reproducing kernel Hilbert spaces (QRKHS) in this framework. As particular examples we will treat further results on the Hardy and Fock spaces and their related operators such as creation, annihilation, shift and backward shift operators. Finally, if time allows we will discuss some connections and applications of this new approach to the recent theory of quaternionic slice polyanalytic functions and Schur analysis. This talk is based on recent works that are in collaboration with Daniel Alpay, Fabrizio Colombo, Soeren Krausshar and Irene Sabadini.

Fractional Multicomplex Polynomials in The Riemann-Liouville Sense

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Joint work with Francisco Ponce, Nicolas Coloma and Diego Ochoa

In this talk, we will present a development of the fractional Multicomplex calculus in the Riemann-Liouville sense based on the modification of the Cauchy-Riemann operator using the one dimensional fractional derivative on each direction of the Multicomplex space. With this operator, we will introduce the analytic polynomials built recursively from monomials from the real numbers, through the complex, bicomplex, until we reach $\mathbb{BC}_n$ (the space generated over the reals by n commuting imaginary units)

A feasibility approach to quaternion-valued wavelet construction

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Joint work with J.A. Hogan

The construction of non-separable, smooth, compactly supported and complex-valued multidimensional wavelets has been recently formulated as feasibility problems subsequently solved by employing projection algorithms. In another timeline, the development of Clifford–Fourier transforms has laid down the foundation for generalizing the classical Fourier and wavelet analyses to provide the basic theory required for the construction of quaternion-valued wavelets with compact support, prescribed regularity, and multiresolution structure. In this talk, we present a feasibility problem formulation for wavelet construction yielding novel examples of quaternion-valued wavelets on the plane.

The Pizzetti formula on Gr$(m, 2)$

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Joint work with Yasushi Homma

In this talk we focus on the role played by the Higgs algebra H_3 in the generalisation of classical harmonic analysis from the sphere S^{m-1} to the (oriented) Grassmann manifold Gr$_o(m, 2)$ of 2-planes. This algebra is identified as the dual partner (in the sense of Howe-duality) of the orthogonal group SO(m) acting on functions on the Grassmannian. This is then used to obtain a Pizzetti formula for integration over this manifold. The resulting formulas are finally compared to formulas obtained earlier for the Pizzetti integration over Stiefel manifolds, using an argument involving symmetry reduction.

Quaternion Fourier Transform and Generalized Lipschitz classes.

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Joint work with El Mehdi Loualid and Radouan Daher

For functions $f \in L^1(\mathbb{R}^2, \mathbb{H})$ with the Quaternion Fourier Transform (QFT) $\widehat{f}$ we give necessary and sufficient conditions in the termes of $\widehat{f}$ to ensure that f belongs to one of the Generalized Lipschitz classes $H^m_{\alpha_1,\alpha_2}$ and $h^m_{\alpha_1,\alpha_2}$ for $0 < \alpha_1, \alpha_2 < m$.

Integral theorems for paravector-valued α-hypermonogenic functions

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We research hyperbolic function theory connected to α-hyperbolic harmonic functions that are harmonic with respect to the Laplace-Beltrami operator of the Riemannian metric $ds^2 = x_n^{-\frac{2\alpha}{n-1}} \left(\sum_{i=0}^n dx_i^2\right)$ on the upper half space $\mathbb{R}^{n+1}_+ = \{(x_0, ..., , x_n,) \in \mathbb{R}^{n+1} : x_n > 0\}$. The term hyperbolic is used to emphasize the connections to the Poincaré upper half space model $\alpha = n - 1$. In this case hyperbolic harmonic functions are solutions of the hyperbolic Laplace operator $\Delta_h f = x_n^2 \Delta f - (n-1)\, x_n \frac{\partial f}{\partial x_n}$ and generally α-hyperbolic harmonic functions can be transferred to the eigen functions of this operator.

We call generalized holomorphic functions $\alpha-$hypermonogenic functions. In this talk we are particularly interested in paravector valued α-hypermonogenic functions, called H_α-solutions. They are mapping from a subset of $\mathbb{R}^{n+1}$ into $\mathbb{R}^{n+1}$. Heinz Leutwiler started to study these type of function in case $\alpha = n-1$ around 1990 and noticed that the power function x^m $(m \in \mathbb{Z})$, is a conjugate gradient of a $n-1$-hyperbolic harmonic function. An important example of α-hypermonogenic functions is the function $|x|^{\alpha-n+1}x^{-1}$. The Cauchy type integral formulas for α-hypermonogenic functions where the kernels are calculated using the hyperbolic distance of the Poincaré upper half space model is verified by Vesa Vuojamo, Heikki Orelma and the author. We present the Cauchy integral formula for H_α-solutions and study related integral operators. We consider also its probabilistic interpretations.

On fractional semidiscrete Dirac operators of Lévy-Leblond type

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In this paper we introduce a wide class of space-fractional and time-fractional fractional Dirac operators of Lévy-Leblond type on the semidiscrete lattice $h\mathbb{Z}^n \times [0,\infty)$, corresponding to fractional discrete counterparts of the so-called parabolic Dirac operators. The cornerstone this approach is centered around the study of the null solutions of the underlying space-fractional resp. time-fractional fractional Dirac operators, and on the study of representation of the null solutions for both operators with the aid of the analytic fractional semidiscrete semigroup $\{\exp(-te^{i\theta}(-\Delta_h)^\alpha)\}_{t\geq 0}$ carrying the parameter constraints $0<\alpha\leq 1$ and $|\theta| \leq \frac{\alpha\pi}{2}$. The results obtained involve the study of Cauchy problems on $h\mathbb{Z}^n \times [0,\infty)$.

Spin(4) representations, Fourier transform, and symbol calculus

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Joint work with P. Cerejeiras, U. Kähler, J. Wirth

In this talk, we consider the Spin(4) representations in the spaces of simplicial harmonic polynomials and simplicial spinor-valued monogenic polynomials and decompose them as the tensor product of Spin(3) representations. Using the Kronecker product and the properties of Spin(3) representations we study recurrence relations for the matrix coefficients of Spin(4) representations and establish a differential and symbol calculus for some left/right invariant differential operators. With the Fourier transform on Spin(4) in hand and a family of admissible first-order difference operators chosen we study pseudo-differential operators on the group Spin(4). We obtain results concerning the ellipticity and the global hypoellipticity of pseudo-differential operators in Spin(4), in terms of their matrix-valued full symbols.

Slice conformality: Riemann manifolds and logarithm on quaternions and octonions

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Joint work with Jasna Prezelj, Fabio Vlacci

This talk will present the quaternionic and octonionic analogs of the classical Riemann surfaces of the complex logarithm and n-th root function, and give a unifying definition of such functions in the quaternionic and octonionic settings. The construction of these manifolds has nice peculiarities and an approach mainly based on conformality leads to the definition of slice conformal or slice isothermal parameterization of a Riemann 4-manifold and 8-manifold. These new classes of manifolds include slice regular quaternionic and octonionic curves and graphs of slice regular functions.

Quaternionic approach in the transversely anisotropic elasticity

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Joint work with A. Yakovlev

The quaternionic functions method is an analytical tool used in elasticity theory. For isotropic elasticity, there are known a few variants of three-dimensional analogues of the Kolosov-Muskhelishvili formulae. In this case, a general solution of the Lame equation for the spatial theory of elasticity is expressed in terms of two regular quaternionic or monogenic Clifford functions. For the anisotropic elasticity, a close approach exists when equations of an equilibrium are factorized by means of matrix algebra. In this report, we will discuss a quaternionic function method in transversely anisotropic theory of elasticity. The model of an elastic media with such symmetry is described by five elastic constants, and for example, can be used in the mechanics of rocks in permafrost conditions.

The Radon transform in superspace

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Joint work with Irene Sabadini, Frank Sommen

In this talk, we study inversion formulas for the Radon transform in superspace following two different approaches. The first one relies on the decomposition into plane waves of the Dirac Delta distribution in superspace, provided that the superdimension is not odd and negative. Such a decomposition is obtained by adopting the point of view of hyperfunctions, namely by using the fact that the Dirac delta is a suitable boundary value of the super Cauchy kernel. In the cases of negative and even superdimension, the obtained formulas no longer resemble the structure of the classical plane wave decompositions in $\mathbb{R}^m$. In turn, the explicit inversion formulas obtained for the super Radon transform in these cases show important differences with the classical case.
On the other hand, we show how to invert the super Radon transform using the classical approach, i.e. by composing the dual Radon transform with a certain power of the super Laplace operator. This approach yields a unified inversion formula that is valid for all possible integer values of the superdimension. The proof of this result comes along with the study of fractional powers of the super Laplacian, their fundamental solutions, and the plane wave decompositions of super Riesz kernels.

Generalization of special affine Fourier transform to spacetime algebra $Cl(3,1)$

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In this presentation we explain how the Spacetime Fourier Transform of [E. Hitzer, Special relativistic Fourier transformation and convolutions, Mathematical Methods in the Applied Sciences, First published: 04 Mar. 2019, Vol. 42, Issue 7, pp. 2244–2255, 2019, DOI: 10.1002/mma.5502, URL: Preprint: vixra 1601.0283v3] can be generalized to a Special Affine Spacetime Fourier Transform (SASFT) operating on spacetime functions in $L^1(\mathbb{R}^{3,1}; Cl(3,1)$, and we establish some properties of this new SASFT.

Clifford translations, wavelets and splines

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Joint work with Joseph Lakey, Peter Massopust

Modern constructions of multi-dimensional wavelets and splines rely on translations along uniform grids that run parallel to the coordinate axes. In the wavelet case this leads to a bias towards separability, while in the spline case it causes rotational non-covariance. In this talk we introduce a one-parameter family of Clifford-analytic translations and modulation operators that act isotropically on Clifford-valued functions on n-dimensional Euclidean space. Applications to the construction of higher-dimensional wavelets, splines and bandpass prolates will be investigated.

Szegö-Radon transform for hypermonogenic functions

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Joint work with Ali Guzman Adan, Tim Raeymaekers, Franciscus Sommen

In our work, we study a refinement of the Szegö-Radon transform in the hypermonogenic setting. Hypermonogenic functions form a subclass of monogenic functions arising in the study of a modified Dirac operator, which allows for weaker symmetries and also has a strong connection to the hyperbolic metric. In particular, we construct a projection operator from a module of hypermonogenic functions in $\mathbb{R}^{p+q}$ onto a suitable submodule of plane waves parameterized by a vector on the unit sphere of $\mathbb{R}^q$. Moreover, we study the interaction of this Szegö-Radon transform with the generalized Cauchy-Kovalevskaya extension operator. Finally, we develop a reconstruction (inversion) method for this transform.

The spectral theorem for a normal operator on a Clifford module

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Joint work with F. Colombo

In this talk we will consider the problem of obtaining a spectral resolution for a densely defined closed normal operator on a Clifford module $\mathcal{H}_n := \mathcal{H} \otimes \mathbb{R}_n$, where $\mathcal{H}$ is a real Hilbert space and $\mathbb{R}_n := \mathbb{R}_{0,n}$ is the Clifford algebra generated by the units $e_1, \dots, e_n$ with $e_ie_j = -e_je_i$ for $i \neq j$ and $e_j^2 = -1$ for $j = 1, \dots, n$. We shall see that any densely defined closed normal operator T on a Clifford module admits an integral representation which is analogous to the integral representation for a densely defined closed normal operator on a quaternionic Hilbert space (which one may think of as a Clifford module $\mathcal{H}_2$) discovered by Daniel Alpay, Fabrizio Colombo and the speaker in 2014. However, the Clifford module setting sketched above with $n > 2$ presents a number of technical difficulties which are not present in the quaternionic Hilbert space case.
In order to prove this result, one needs to utilise spectra of operators which are not necessarily paravector operators, i.e., operators of the form $T = T_0 + \sum_{j=1}^n T_je_j$. This observation has implications on a generalisation of the S-functional calculus and some related function theory which we shall briefly highlight.
The main thrust of this talk is based on joint work with Fabrizio Colombo. The work on the S-functional calculus is joint work with Fabrizio Colombo, Jonathan Gantner and Irene Sabadini. The work on the related function theory is joint work with Fabrizio Colombo, Irene Sabadini and Stefano Pinton.

Reproducing kernel spaces of octonionic monogenic functions

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Joint work with D. Constales

In this talk we present some essential differences between the theories of reproducing kernels in the Clifford analysis setting and the setting of octonionic monogenic functions. The lack of associativity has an essential influence on the treatment of RHKS since generalizations of the Cauchy Schwarz inequality and the Fischer Riesz representation theorem as well as the existence of an adjoint operator are not for granted at all. Therefore, a fundamental issue consists in defining an inner product in an appropriate way to guarantee the existence of a reproducing kernel function and to apply analogues of these fundamental theorems. In the main focus of the talk we look at appropriate generalizations of the Szegö projection of octonionic monogenic functions and on generalizations of a dual Cauchy transform. For some special cases we present explicit representation formulas for the reproducing kernels. We also present a generalization of a compact Kerzman-Stein operator. As an application we establish a relation to the Hilbert-Riesz transform on eight-dimensional upper half-space.

Fischer decomposition for massless fields of spin $3/2$

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Joint work with F. Brackx, H. De Schepper, V. Soucek, W. Wang

As an analogue of massless field equations in Euclidean spaces, we consider the so-called generalized Cauchy-Riemann equations introduced by E. Stein and G. Weiss. For spin 1/2, this is the Dirac equation for spin 1/2 fields studied intensively in Clifford analysis. For a general spin in dimension 4, it turns out that homogenous solutions form irreducible Spin modules. The next step is to describe the corresponding Fischer decomposition, that is, an irreducible decomposition of the space of spinor fields. We review the well-known Fischer decompositions for spinor fields of spin $1/2$ and spin 1. The main aim of the talk is to describe the Fischer decomposition in dimension 4 for spin $3/2$.

Finite element method based on script geometry

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Joint work with P. Cerejeiras, U. Kähler

Additionally to the classical numerical methods, which are based on the ideas of approximation the continuous problem under consideration, the methods working directly with discrete structures, such as discrete potential theory, discrete function theory, and finite element exterior calculus, have gained popularity nowadays. The finite element exterior calculus utilises tools of algebraic topology, such as de Rham cohomology and Hodge theory, to address the stability of the continuous problem. By its construction, the finite element exterior calculus is limited to triangulation based on simplicial complexes. However, practical applications often require triangulations containing elements of more general shapes. Therefore, it is necessary to extend the finite element exterior calculus to overcome the restriction to simplicial complexes. In this talk, we discuss a generalisation of the finite element exterior calculus by help of the script geometry, a recently introduced new kind of discrete geometry and calculus. We discuss basic ideas about scripts and their properties, as well as presenting how finite element method can be established based on the script geometry.

Algebra of Quaternionic Functions

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Joint work with Igor Frenkel (Yale University)

I will talk about a new algebra structure on quaternionic functions that intertwines the conformal group action. This algebra structure is expected to have many interesting properties and have a close relation with quantum electrodynamics.
This is a joint work with Igor Frenkel from Yale University and based on the paper "Quaternionic Analysis, Representation Theory and Physics II" arXiv:1907.01594 accepted for publication in the Advances in Theoretical and Mathematical Physics.

Hyperbolic curves and integration of bicomplex functions

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The notions of hyperbolic straight lines and hyperbolic curves in the set $\mathbb{BC}$ of bicomplex numbers, or in the n dimensional hyperbolic module $\mathbb{D}^n$ have been helpful for a better understanding of some analytical facts in both theories. In this talk we will analyze some of these facts. For example, how the bicomplex stereographic projection can be efficiently described or, as the title of the talk says, how the hyperbolic curves help to establish some of the integral theorems in the bicomplex context.

Quaternionic Iterated Function Systems

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We introduce the novel concept of a quaternionic iterated function system (IFS) on the complete metric space $(\mathbb{H}^k, d)$ and define its quaternionic attractor. Systems of quaternionic function systems arising from quaternionic IFSs and their backward trajectories are also introduced and it is shown that the attractors of such backward trajectories possess different local (fractal) shapes.

Solutions to the parabolic Dirac equation and its generalizations

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Joint work with S. Bao, H. De Bie, D. Constales

In classic Clifford analysis one uses Clifford algebra to factorize the Laplacian $\Delta_{\underline{x}}$ as $-\partial_{\underline{x}}^2$ where $\partial_{\underline{x}}$ is the Dirac operator. In this talk I will discuss how one can use the same techniques to factorize the heat operator $-\Delta_{\underline{x}} + \partial_t$ as $D_{\underline{x},t}^2$, with $D_{\underline{x},t}$ the so-called parabolic Dirac operator.
Furthermore, I will discuss a special class of null-solutions of $D_{\underline{x},t}$ generated as a series expansion.
Finally I will show a way to generalize the parabolic Dirac operator and show that one can find the same type of solutions as for the parabolic Dirac operator.

Contragenic Spheroidal Functions of a Quaternionic Variable

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A contragenic function in a domain $\Omega \subseteq \mathbb{R}^3$ is a reduced-quaternion-valued (i.e., the last coordinate function is zero) harmonic function, which is orthogonal in $L_2(\Omega)$ to all monogenic functions and their conjugates. Contragenicity is not a local property. We investigate the relationships between standard orthogonal bases of harmonic and contragenic functions for one domain to another via computational formulas for spheroids of different eccentricities, showing that there are common contragenic functions to all spheroids of all eccentricities.

Complete systems of time-harmonic Maxwell equations for chiral media

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Joint work with Vladislav Kravchenko, Michael Porter

Maxwell's equations for time-harmonic electromagnetic fields in a chiral medium have the form

$$\operatorname{div} \widetilde{E}(x) = \operatorname{div} \widetilde{H}(x) = 0, \tag{1}$$
$$\operatorname{curl} \widetilde{E}(x) = \omega \widetilde{B}(x), \tag{2}$$
$$\operatorname{curl} \widetilde{H}(x) = -\omega \widetilde{D}(x), \tag{3}$$

These equations can be transformed into a equivalent system of the form:

$$\left(D + \frac{\alpha}{1+\alpha\beta}\right) \vec{\zeta}(x) = 0,$$
$$\left(D - \frac{\alpha}{1-\alpha\beta}\right) \vec{\eta}(x) = 0.$$

We construct an *invertible operators* $\mathbf{T}, \widetilde{\mathbf{T}}$ that acts on complex quaternion functions such that related the well know monogenic functions (i,e functions who satisfies $Du = (\mathbf{e}_1\partial_1 + \mathbf{e}_2\partial_2 + \mathbf{e}_3\partial_3)u = 0$) into solutions of $D + M^{Df(x_3)/f(x_3)}$ (M is the operator of multiplication on the right) as follows:
For $v \in C^1(\Omega, \mathbb{H}(\mathbb{C}))$,

$$\left(D + M^{Df(x_3)/f(x_3)}\right) \mathbf{T}[v] = \widetilde{\mathbf{T}}[Dv]. \tag{4}$$

Using the operators $\mathbf{T}, \widetilde{\mathbf{T}}$ to find a complete system of solutions to Maxwell's equations (1)-(3).

Clifford Siegel Spaces

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The Siegel upper half space consists of complex combinations $X + iY$ where X and Y are symmetric n by n real matrices with Y positive definite. This generalizes the complex upper half plane. Symplectic groups act on these spaces which generalize $PSL(2, \mathbb{R})$. We extend the Siegel upper half space to paravectors in a Clifford algebra. We also consider replacing the n by n matrices with bounded linear operators on a real Hilbert space.

Higher order gradients of monogenic functions

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Joint work with L. Baracco

It was proved by Stein and Weiss (1960) that for an harmonic function f in $\mathbb{R}^n$ the power of the gradient

$$|\nabla f|^\alpha := \left(\sum_{i=1}^{n} (\partial_{x_i} f)^2\right)^{\frac{\alpha}{2}}$$

is subharmonic for $\alpha \geq \frac{n-2}{n-1}$ (when $n = 2$ the previous result means that for any $\alpha > 0$ the function $|\nabla f|^\alpha$ is subharmonic and also $\log(|\nabla f|)$ is a subharmonic function). Calderón and Zygmund (1964) extended this result to higher order gradients proving that

$$|\nabla^m f|^\alpha := \left(\sum_{|\beta|=m} |\partial^\beta f|^2\right)^{\frac{\alpha}{2}}$$

($\partial^\beta f = \partial_{x_{\beta_1}} \cdots \partial_{x_{\beta_m}} f$, $\beta \in \{1, \ldots, n\}^m$ and $\beta_1 \leq \cdots \leq \beta_m$) is subharmonic for $\alpha \geq \frac{n-2}{n+m-2}$ and that such lower bound is optimal. Since holomorphic functions $f : \Omega \subset \mathbb{C} \to \mathbb{C}$ are the gradient of an harmonic

function, the previous results imply that if $\gamma > 0$ the functions $|\nabla^m f|^\gamma$ are subharmonic for any integers $m \geq 0$ where ∇^0 is meant to be the identity operator. Moreover in these cases also $\log(|\nabla^m f|)$ is subharmonic. In the context of quaternionic, Clifford algebraic or octonionic analysis it is no more true that regular functions are the gradient of an harmonic function. Nevertheless Stein and Weiss (1968) proved that if f is a monogenic quaternionic function then $|f|^\alpha$ is subharmonic for $\alpha \geq \frac{2}{3}$. Kheyfits and Tepper (2006) proved that the same result holds in the octonions for $\alpha \geq \frac{6}{7}$.
In this talk I will explain how to extend the technique of Calderón and Zygmund to higher order gradients of monogenic functions on quaternions, Clifford algebras and octonions.

Slice analysis of several variables

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Joint work with I. Sabadini, X. Dou, M. Jin X. Wang, Z. Xu, T. Yang

This talk provides a survey of our recent progress in slice analysis of several variables. We highlight some remarkable phenomena. For example, to show the Hartogs phenomena, we have to introduce new regular products induced by commutative complex structures other than non-commutative complex structures. To study the slice regular extension, we need to introduce a new theory called slice Riemann surfaces; the canonical topology in slice analysis turns out to be a non-Euclidean topology; slice analysis in non-axially symmetric domains yields two distinct theories related to strong or week slice regular functions. We also generalize slice analysis to other settings such as for non-alternative algebras and for Dirac operators.

Cells of Harmonicity and the Holomorphic Dirac Operator

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We shall review aspects of the holomorphic Dirac operator over domains in several complex variables and on the complex sphere. Results reviewed will include a Runge approximation theorem.

Processes for generating fractals over the hyperbolic numbers plane

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Joint work with J. Bory Reyes

On this talk we will develop processes to generate fractals over the hyperbolic numbers plane, such as: iterated function system, chaos game algorithm and percolation. Notably, we consider hyperbolic interval, entropy, and partitions. Our framework includes hyperbolic distance between points, bijections with their complement, and quantity of information based in hyperbolic probability theory.

Schur Analysis in the Complex Ternary Algebra

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Joint work with D. Alpay

We develop a Schur analysis in the setting of the complex ternary algebra, and thus this work can be seen as a study of linear systems over a commutative ring. When leaving the realm of complex numbers and going to another set of hypercomplex numbers, a number of questions need to be answered. In particular, one has to define the corresponding notion of holomorphicity and the definition of positive matrices. Previously I defined a notion of holomorphicity in this case with applications to several types of C-K extensions, as well as notions of positivity. We now expand on these notions and build a Schur analysis in the ternary complex algebra.

Fractional Sturm-Liouville problem in higher dimensions

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Joint work with M. Ferreira, M.M. Rodrigues

In this talk, we consider the n-dimensional fractional Sturm-Liouville eigenvalue problem, by using fractional versions of the gradient operator involving left and right Riemann-Liouville fractional derivatives. We study the main properties of the eigenfunctions and the associated eigenvalues of the associated fractional boundary problem. More precisely, we show that the eigenfunctions are orthogonal and the eigenvalues are real and simple. Moreover, using techniques from fractional variational calculus, we prove in the main result that the eigenvalues are separated and form an infinite sequence, where the eigenvalues can be ordered according to increasing magnitude. Finally, a connection with Clifford analysis is established.

The Taylor Expansions of Weighted Regular Functions

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Joint work with Liping Luo, Fen Qiu

In this paper, we will express weighted regular functions as series composed of weighted regular polynomials. Firstly, the definition of p order homogeneous weighted regular polynomials is given. In order to obtain the basis of the set composed of the above polynomials, we introduce the hypercomplex variable. Secondly, we prove the relationship between the analytic as well as weighted regular functions and the p order homogeneous weighted regular polynomials.By using their relationship, the Taylor expansions of the weighted regular functions at a certain point are given. Then, the inner closed uniform convergence of the Taylor expansions of the fundamental solutions $E_\omega(x,\xi)$ of weighted Dirac operator in a certain domain is proved, and the inner closed uniform convergence of Taylor expansions of weighted regular functions in this region is obtained by using the Cauchy integral formula, then the inverse theorem of Taylor expansions is obtained. Finally, the uniqueness theorem is obtained from the Taylor expansions and the connectivity of Ω.

The resolution of 6-dimensional Euclidean massless field operators of higher spins and the L^2 method

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Joint work with Q.-Q. Kang and Y. -C. Zhang

The resolution of 4-dimensional massless field operators of higher spins was constructed by Eastwood-Penrose-Wells by using the twistor method. Recently physicists are interested in 6-dimensional physics including the massless field operators of higher spins on Lorentzian space $\mathbb{R}^{5,1}$. I will talk about the Euclidean version $\mathscr{D}_0$ and the construction of an exact sequence of Hilbert spaces as weighted L^2 spaces resolving $\mathscr{D}_0$:

$$L^2_\varphi(\mathbb{R}^6,\mathscr{V}_0) \xrightarrow{\mathscr{D}_0} L^2_\varphi(\mathbb{R}^6,\mathscr{V}_1) \xrightarrow{\mathscr{D}_1} L^2_\varphi(\mathbb{R}^6,\mathscr{V}_2) \xrightarrow{\mathscr{D}_2} L^2_\varphi(\mathbb{R}^6,\mathscr{V}_3) \longrightarrow 0,$$

with suitable operators $\mathscr{D}_l$ and vector spaces $\mathscr{V}_l$. Namely, we can solve $\mathscr{D}_l u = f$ in $L^2_\varphi(\mathbb{R}^6,\mathscr{V}_l)$ when $\mathscr{D}_{l+1} f = 0$ for $f \in L^2_\varphi(\mathbb{R}^6,\mathscr{V}_{l+1})$. This is proved by using the L^2 method in the theory of several complex

variables, which is a general framework to solve overdetermined PDEs under the compatibility condition. To apply this method here, it is necessary to consider weighted L^2 spaces, an advantage of which is that any polynomial is L^2_φ integrable. As a corollary, we prove that

$$P(\mathbb{R}^6, \mathscr{V}_0) \xrightarrow{\mathscr{D}_0} P(\mathbb{R}^6, \mathscr{V}_1) \xrightarrow{\mathscr{D}_1} P(\mathbb{R}^6, \mathscr{V}_2) \xrightarrow{\mathscr{D}_2} P(\mathbb{R}^6, \mathscr{V}_3) \longrightarrow 0$$

is a resolution, where $P(\mathbb{R}^6, \mathscr{V}_l)$ is the space of all $\mathscr{V}_l$-valued polynomials. This provides an analytic way to construct a resolution of a differential operator acting on vector valued polynomials.

Interior estimates in the sup-norm for meta-monogenic functions and their application to initial value problems

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Initial value problems of type

$$\partial_t u = \mathcal{F}(t, x, u, \partial_{x_i} u), \quad u(0, x) = \varphi(x) \tag{1}$$

can be solved by the contraction-mapping principle in case the initial function φ belongs to an associated space whose elements satisfy an interior estimate. The present article proves such an interior estimate in the sup-norm for meta-monogenic functions in Clifford analysis. The proof is based on a representation of meta-monogenic functions by harmonic functions. The constants in the interior estimate are represented explicitly.

As an application of the interior estimate, the unique solvability of initial value problems of type (1) is proven, provided $\mathcal{F}$ satisfies a Lipschitz condition.

Session 15: Recent Progress in Evolution Equations

Organizers: Marcello D'Abbicco and Marcelo Rempel Ebert

The Cauchy Problem for $3-$evolution equations in Gevrey type spaces

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Joint work with A. Ascanelli, M. Cappiello

We consider the Cauchy Problem for evolution operators of the form

$$P(t,x,D_t,D_x)=D_t+a_p(t)D_x^p+\sum_{i=0}^{p-1}a_i(t,x)D_x^i,\quad t\in[0,T],\, x\in\mathbb{R},$$

where $T>0$, $D=-i\partial$, $a_p(t)\in C([0,T];\mathbb{R})$, $a_p(t)\neq 0$ for every t and $a_i(t,x)\in C([0,T];C^\infty(\mathbb{R};\mathbb{C}))$, $i=0,1,\dots,p-1$. The operator P is known in the literature as $p-$evolution operator.
In the situation that each coefficient $a_i(t,x)$, $i=0,1,\dots,p-1$, is purely real, it is well known that that the Cauchy Problem is well posed in Sobolev spaces $H^m(\mathbb{R})$, $m\in\mathbb{R}$. When there are complex valued coefficients, some decay conditions at infinity are required in order to achieve well-posedness in $H^m(\mathbb{R})$ or $H^\infty(\mathbb{R})=\cap_m H^m(\mathbb{R})$.
Concerning well-posedness results in Gevrey type spaces, as far as the authors know, there are only results for $p=1$ (strictly hyperbolic operators) and $p=2$ (Schrödinger type operators). In this lecture we focus in the case $p=3$ and we prove necessary conditions on the coefficients $a_i(t,x)$, $i=0,1,2$, in order to obtain well-posedness of the Cauchy Problem associated with P, with initial data in Gevrey type spaces.

Solution theory to semilinear parabolic stochastic pdes with polynomially bounded coefficients

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Joint work with Sandro Coriasco, André Süss

We consider stochastic partial differential equations of the form

$$Lu(t,x)=\gamma(t,x,u(t,x))+\sigma(t,x,u(t,x))\dot{\Xi}(t,x),\quad (t,x)\in[0,T]\times\mathbb{R}^d,$$

where L is a parabolic operator with (t,x)-depending coefficients possibly admitting a polinomial growth as $|x|\to\infty$, and Ξ is an $\mathcal{S}'(\mathbb{R}^d)$-valued Gaussian process, white in time and coloured in space. We provide conditions on the initial data and on the noise (namely, on the spectral measure associated to the noise) such that a unique function-valued mild solution u exists, valued in weighted Sobolev spaces where both regularity and behavior at spatial infinity can be described.

Time -periodic Gelfand-Shilov spaces and global hypoellipticity on $\mathbb{T}\times\mathbb{R}^n$

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Joint work with Fernando de Ávila Silva (Federal University of Parana, Curitiba, Brazil)

We introduce a class of time-periodic Gelfand-Shilov spaces of functions on $\mathbb{T}\times\mathbb{R}^n$, where $\mathbb{T}\sim\mathbb{R}\setminus\mathbb{Z}$ is the one-dimensional torus. We develop a Fourier analysis inspired by the characterization of the Gelfand-Shilov spaces in view of the eigenfunction expansions given by a fixed self-adjoint, globally elliptic differential operator on $\mathbb{R}^n$. In this setting, as an application, we characterize the global hypoellipticity for a class of linear differential evolution operators on $\mathbb{T} \times \mathbb{R}^n$.

The Cauchy problem for acoustic waves: global wellposedness, asymptotic profiles and singular limits

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Joint work with R. Ikehata, A. Palmieri

In this talk, we will introduce the Cauchy problem for linear and nonlinear Moore-Gibson-Thompson(MGT) equation in acoustic waves. First of all, we consider the linear MGT equation in $\mathbb{R}^n$, namely,

$$\mathcal{L}_{\mathrm{MGT}}(\psi) := \tau\psi_{ttt} + \psi_{tt} - \Delta\psi - (\tau+\delta)\Delta\psi_t = 0,$$

where $\psi = \psi(t,x) \in \mathbb{R}$ is an acoustic velocity potential, $\tau > 0$ denotes the thermal relaxation (in the Cattaneo's law heat conduction) and $\delta > 0$ stands for the diffusivity of the sound. Later, some qualitative properties will be shown, including sharp decay estimates, asymptotic profiles, large-time approximations and singular limits with respect to $\tau \downarrow 0$. Then, two different nonlinear MGT equations in $\mathbb{R}^n$ will be studied as follows:

$$\mathcal{L}_{\mathrm{MGT}}(\psi) = \begin{cases} \partial_t\left(\frac{B}{2A}(\psi_t)^2 + |\nabla\psi|^2\right), & \text{Jordan-MGT equation,} \\ |\psi|^p, & \text{semilinear MGT equation,} \end{cases}$$

where the positive constants A, B describe behaviors of nonlinearity in high-intensity ultrasound, and the power $p > 1$. We completed some results for global (in time) existence of small data solution and blow-up of solutions. Particularly, the blow-up results for the semilinear MGT equation in the conservative case ($\delta = 0$) and the dissipative case ($\delta > 0$) are quite different. Finally, asymptotic profiles and singular limits for the corresponding models in hereditary fluids will be analyzed. This talk is based on a joint work with R. Ikehata and A. Palmieri.

A dissipative logarithmic-Laplacian type of plate equation

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Joint work with Alessandra Piske, Ryo Ikehata

We consider a new model of a dissipative logarithmic-Laplacian type of plate equation based on an operator L, that combine the composition of logarithm function with the Laplacian operator, as follows

$$\begin{aligned} &u_{tt} + Lu + (I+L)^{-1}u_t = 0, \quad (t,x) \in (0,\infty) \times \mathbf{R}^n, \\ &u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \mathbf{R}^n, \end{aligned}$$

where $L : D(L) \subset L^2(\mathbf{R}^n) \to L^2(\mathbf{R}^n)$, with domain
$D(L) := \{f \in L^2(\mathbf{R}^n) \;\big|\; \int_{\mathbf{R}^n} (\log(1+|\xi|^2))^2 |\hat{f}(\xi)|^2 d\xi < +\infty\}$,
is the operator with symbol $\log(1+|\xi|^2)$. Symbolically writing, one can see that $L = \log(I - \Delta)$, where Δ is the usual Laplace operator defined on $H^2(\mathbf{R}^n)$. The operator L was recently introduced Charão-Ikehata [ZAMP, 2020] as a new type of damping to the wave equation.
To the above model we study the asymptotic profile and optimal decay rates of solutions as $t \to \infty$ in L^2-sense. We also discuss asymptotic properties of the solution as time goes to infinity to our Cauchy problem depending on the regularity of the initial data and, in particular, we classify the structure of the solution into three types from the viewpoint of regularity of the initial data and the dimension n, that is, diffusion-like, wave-like, and both of them. Important part of the results is obtained by using results from the hypergeometric functions that appear in the works by Charao-Ikehata [ZAMP 2020] and Charao-D'Abbicco-Ikehata [arXiv 2020].

Sharp lifespan estimates for the weakly coupled system of semi-linear damped wave equations in the critical case

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Joint work with Wenhui Chen

In this talk, we mainly investigate the sharp lifespan estimates for solutions to the Cauchy problem for the weakly coupled system of semi-linear damped wave equations in the critical case to answer the open question, which seems to be also the final part in terms of studying such kind of system. By using a suitable test function method associated with nonlinear differential inequalities, we catch upper bound estimates for the lifespan. Moreover, we establish polynomial-logarithmic type time-weighted Sobolev spaces to obtain lower bound estimates for the lifespan in low spatial dimensions. Then, together with the derived lifespan estimates, new and sharp results on estimates for the lifespan in the critical case are claimed. Finally, we give an application of our results to the semi-linear reaction-diffusion system in the critical case.

On odd-order quasilinear evolution equations on bounded intervals

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On an interval $(0,R)$ for an arbitrary $R>0$ consider an initial-boundary value problem for an equation

$$u_t-(-1)^l(\partial_x^{2l+1}u+a_{2l}\partial_x^{2l}u)-\sum_{j=0}^{l-1}(-1)^j\partial_x^j\big[a_{2j+1}(t,x)\partial_x^{j+1}u+a_{2j}(t,x)\partial_x^j u\big]$$

$$+\sum_{k=0}^{l}(-1)^k\partial_x^k\big[g_k(t,x,u,\dots,\partial_x^{l-1}u)\big]=f(t,x),\quad l\in\mathbb{N},$$

with an initial condition $u(0,x)=u_0(x)$ and boundary conditions $\partial_x^j u(t,0)=\partial_x^j u(t,R)=0,\ j=0,\dots,l-1$, $\partial_x^l u(t,R)=\nu(t)$.
Equations of such a type are the class of quasilinear evolution dispersive equations describing wave processes in various media.
It is assumed that $a_{2l}\le 0$, the coefficients a_j for $j\le 2l-1$ are small in some sense or have an appropriate sign, the functions $g_k(t,x,y_0,\dots,y_{l-1})$ satisfy certain growth restrictions with respect to y_j. Then for small functions $u_0\in L_2(0,R)$, $\nu\in L_2(0,+\infty)$ and $f\in L_2((0,+\infty)\times(0,R))$ there exists a unique weak solution to the considered problem $u(t,x)$ such that $u\in C([0,T];L_2(0,R))$, $\partial_x^l u\in L_2((0,T)\times(0,R))\ \forall T>0$. Moreover, if ν and f decay exponentially when $t\to+\infty$ then the solution also decays exponentially in $L_2(0,R)$.
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Global existence and blow up for periodic non-gauge invariant NLS

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Joint work with Kazumasa Fujiwara

We study NLS with non-gauge invariant quadratic nonlinearity on the torus. The Cauchy problem admits trivial global solutions which are constant with respect to space. The non-existence of global solutions also has been studied only by focusing on the behavior of the Fourier 0 mode of solutions. However, the earlier works are not sufficient to obtain the precise criteria for the global existence for the Cauchy problem in 1D case. The exact criteria for the global existence of L^2 solutions is shown by studying the interaction between the Fourier 0 mode and oscillation of solutions. Namely, L^2 solutions are shown a priori not to exist globally if they are different from the trivial ones.

Asymptotic profile for a time-fractional telegraph equation

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Joint work with Prof. Marcello D'Abbicco

In the present paper we consider the Cauchy-type problem associated to a telegraph time-fractional differential equation:

$$\begin{cases} \partial_t u + \partial_t^{\frac{1}{2}} u - \Delta u = g(t,x), \quad t>0,\ x\in\mathbb{R}^n \\ u(0,x) = u_0(x), \end{cases}$$

where the fractional derivative $\partial_t^{\frac{1}{2}}$ is in Caputo sense. We provide a sufficient condition on the right-hand term $g(t,x)$ to obtain a solution in $\mathcal{C}_b([0,\infty),H^s)$. In low space dimension, we also describe the asymptotic profile of the solution.

Finite vs infinite derivative loss for abstract wave equations with singular time-dependent propagation speed

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Joint work with M. Ghisi

We consider an abstract wave equation with a propagation speed that depends only on time. We investigate well-posedness results with finite derivative loss in the case where the propagation speed is smooth for positive times, but potentially singular at the initial time. We prove that solutions exhibit a finite derivative loss under a family of conditions that involve the blow up rate of the first and second derivative of the propagation speed, in the spirit that the weaker is the requirement on the first derivative, the stronger is the requirement on the second derivative. Our family of conditions interpolates between the two limit cases that were already known in the literature. We also provide the counterexamples that show that, as soon as our conditions fail, solutions can exhibit an infinite derivative loss. The existence of such pathologies was an open problem even in the two extreme cases.

On the energy estimates of semi-discrete wave equations with time dependent propagation speed

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We consider the energy estimate for the initial value problem of d-dimensional semi-discrete wave equation with time dependent propagation speed:

$$\begin{cases} u''(t)[k] - a(t)^2 \sum_{j=1}^d D_j^+ D_j^- u(t)[k] = 0, & (t,k)\in\mathbb{R}_+\times\mathbb{Z}^d, \\ u(0)[k] = u_0[k], \quad u'(0)[k] = u_1[k], & k\in\mathbb{Z}^d, \end{cases} \tag{1}$$

where D_j^+ and D_j^- are the forward and the backward difference operators for $f = \{f[k]\}_{k\in\mathbb{Z}^d}$ defined by $D_j^+ f = \{f[k+e_j] - f[k]\}_{k\in\mathbb{Z}^d}$ and $D_j^- f = \{f[k] - f[k-e_j]\}_{k\in\mathbb{Z}^d}$, respectively with the standarad basis $\{e_1,\dots,e_d\}$ of $\mathbb{R}^d$. Here the totanl enery of the solution is defined by

$$E(t) = \left\|u'(t)\right\|_{l^2(\mathbb{Z}^d)}^2 + a(t)^2 \sum_{j=1}^d \left\|D_j^+ u(t)\right\|_{l^2(\mathbb{Z}^d)}^2. \tag{2}$$

In this talk, we consider the difference between the continuous model and the semi-discrete model for the effect of the time-dependent propagation speed $a(t)$ on the energy estimate of the solutions.

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A Kolmogorov ε-entropy estimate for the attractor related to an extensible beam with intrinsic polynomial behavior

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Joint work with V. Narciso, E.H. Gomes Tavares

The main goal of this talk is to present a model in hyperbolic evolution equations related to extensible beams with a nonlocal possibly degenerate damping coefficient of Balakrishnan-Taylor's type. The damping structure prevents any kind of exponential behavior in the topology of the weak phase space along the time, but it allows an intrinsic polynomial range of stability for the corresponding Lyapunov functional. As a consequence, we provide a first study on the long-time dynamics of the associated dynamical system by analyzing the Kolmogorov ε-entropy of the compact attractors.

Non-linear evolution equation with non-local coefficients

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We investigate the global existence in time and the asymptotic behaviour of solutions of non-linear evolution equations with strong dissipation and non-local coefficients. We consider the initial boundary value problem for the equations and show the existence theorem. Applying obtained results we study a non-local chemotaxis model arising in mathematical biology.

Small data blow-up for the weakly coupled system of the generalized Tricomi equations with multiple propagation speeds

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Joint work with M. Ikeda and Z. Tu

In this talk, we consider the Cauchy problem for the weakly coupled system of the generalized Tricomi equations with multiple propagation speeds. Our aim is to prove a small data blow-up result and an upper estimate of lifespan of the problem for a suitable compactly supported initial data in the subcritical and critical cases of the Strauss type. The proof is based on the framework of the test function argument. One of our new contributions is to construct two families of special solutions to the free generalized Tricomi equation as the test functions and prove their several properties. We emphasize that the system with two different propagation speeds is treated in this paper and the assumption on the initial data is improved from the point-wise positivity to the integral positivity.

The influence of slow decaying initial data on the critical exponent for nonlinear scale invariant damped wave equations.

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Joint work with F. Chiarello, G. Girardi

In the present talk, we will consider scale invariant damped nonlinear wave equations. In the last decade many papers concern blow up and global small data solution existence with different nonlinear terms. According to the growth of the nonlinear term, we can have parabolic threshold (Fujita-type critical exponents) or hyperbolic threshold (Strauss-type critical exponent). Many others parameters describes this threshold, for example the space dimension. Moreover considering the linear part of the equation one can find in the critical exponent the influence of the speed, damping and mass coefficients. On the other hand, also the class of solutions can influence the threshold level. For compactly initial data and energy or L^q solutions, the order of the energy and the summability of the solution appear in the threshold, but they do not seem to influence the fact that this threshold is parabolic one or hyperbolic one. In a joint work with Felisia Chiarello and Giovanni Girardi [Journal of Evolution Equations 2021] we consider radial solutions, observing that the blow-up/global-existence threshold passes for hyperbolic one to parabolic according to the rate of the (slow) decaying of initial data at infinity. This result answers to an open question which appeared in D'Abbicco Lucente Reissig [Journal of Differential Equations 2015]. It is based on Takamura's result for semilinear wave equation [Differential and Integral Equations 1995] which can be now interpreted as in the previous discussion.
Finally for blowing-up solutions, we will analyse the dependence of the lifespan by all involved parameters.

Semilinear mixed problems in exterior domains for σ-evolution equations with external damping and coefficients depending on spatial variables

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Joint work with Michael Reissig

In this talk we will present our recent research on the decay estimates for solutions to the Cauchy problem

$$u_{tt} + a_1(x)(-\Delta)^\sigma u + au_t = 0, \;\; u(0,x) = u_0(x), \;\;\; u_t(0,x) = u_1(x) \text{ for } x \in \mathbb{R}^n,$$

and to the corresponding Cauchy-Dirichlet problem in an exterior domain $\Omega \subset \mathbb{R}^n$. The parameter a is a positive constant. The coefficient $a_1 = a_1(x)$ is supposed to be continuous and positive on the closure $\overline{\Omega}$. Finally, we show the global (in time) existence of energy solutions from evolution spaces to the semilinear models

$$u_{tt} + a_1(x)(-\Delta)^\sigma u + au_t = |u_t|^p, \;\; u(0,x) = u_0(x), \;\;\; u_t(0,x) = u_1(x),$$

in domain $(0,\infty) \times \Omega$ with arbitrarily small initial data.

Stein-Weiss inequality in L^1 norm for vector fields

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Joint work with P. De Nápoli (Universidad de Buenos Aires - Argentina)

In this talk, we investigate the limit case $p = 1$ of the Stein–Weiss inequality for the Riesz potential. Our main result is a characterization of this inequality for a special class of vector fields associated to cocanceling operators. As application, we recovered some classical div-curl inequalities in literature. In addition, we also discussed a two-weight inequality with general weights extending the previous result due to Sawyer for the scalar case.

Session 16: Wavelet theory and its Related Topics

Organizers: Keiko Fujita and Akira Morimoto

Meromorphic maps with deficiencies and unicity problems

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In this talk, we will discuss for methods constructing meromorphic maps with deficiencies. In particular, we deal with the derived maps with deficiencies. we also discuss uniqueness problems under conditions on deficiencies.

Density operator formulation in coherent states basis for quantum systems: physical and mathematical aspects

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Joint work with M.N.Hounkonnou, K.Sodoga, E.Baloitcha

In this work, the density operator diagonal representation in the coherent states basis, known as the Glauber-Sudarshan-P representation, is used to study harmonic oscillator quantum systems and models of spinless electrons moving in a two-dimensional noncommutative space, subject to a magnetic field background coupled with a harmonic oscillator. Moroever, quantum models of photon-added are also analyzed and discussed. Relevant statistical properties such as the Q-Husimi distribution, the Wehrl entropy, and other aspects are investigated.

Bernstein theorem related to two-sided quaternion Fourier transform

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Joint work with Mawardi Bahri

The two-sided quaternion Fourier transform is introduced. Essential properties of the two-sided quaternion Fourier transform are given. The Bernstein theorem associated with the classical Fourier transform is presented. The Bernstein theorem associated with the classical Fourier trans- form is generalized in the framework of the two-sided quaternion Fourier transform.

KONTOROVICH-LEBEDEV WAVELET PACKETS

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Joint work with Pr.R.DAHER

Using harmonic analysis associated with the Kontrovich Lebedev transform, we study in this talk some types of wavelets packets and their corresponding wavelet transforms.
We give for this wavelets transforms the related Plancherel and inversion formulas as well as their scale discrete scaling functions.

On some topics related to the Gabor wavelet transform of analytic functionals

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We have studied the Gabor wavelet transformation. Since the Gabor function is an exponential type, we can apply the Gabor wavelet transformation to analytic functionals. In this talk, we will review our previous results, and will consider some characterizations of the Gabor wavelet transform of analytic functional.

Weighted estimates for multilinear Fourier multipliers satisfying Sobolev regularity with mixed norm

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In this talk, we investigate weighted estimates for multilinear Fourier multipliers satisfying Sobolev regularity with mixed norm.

Some results of the time-frequency analysis on the half space

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This talk will give an overview of the theory of time-frequency on the half space, and then focus on our some recent results on a class of dilation-and-modulation frames. Such frames have distinct properties from the usual wavelet and Gabor frames.

Geometric Properties of the Sierpiński Graphs and the extended Sierpiński Graphs

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Joint work with Mai Fujita

In this talk, we investigate the diameters and radii of the Sierpiński Graphs $S(n,k)$ and the extended Sierpiński Graphs $S^+(n,k)$, $S^{++}(n,k)$.

Some Inequalities for Parseval Frames

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Joint work with R. Ashino, A. Morimoto

Let $F = \{f_k\}_{k\in K}$ be a Parseval frame in a Hilbert space H, that is, $\|x\|^2 = \sum_{k\in K} |\langle x, f_k\rangle|^2$ holds for all $x \in H$. It is well-known that if the norm $\|f_{k_0}\| = 1$, then $f_{k_0} \perp f_k$ for all $k \neq k_0$. In general, we can expect that if $\|f_{k_0}\|$ is close to 1, then the angle between other f_k's are close to $\pi/2$. We want to make it clear by some inequalities. In fact, we can get the following inequalities:

$$\begin{aligned}\frac{|\langle f_k, f_l\rangle|}{\|f_k\|\cdot\|f_l\|} &\le \frac{\sqrt{1-\|f_k\|^2}}{\|f_k\|}\cdot\frac{\sqrt{1-\|f_l\|^2}}{\|f_l\|},\\ \|f_k - af_l\|^2 &\ge \|f_k\|^2 + |a|^2\|f_l\|^2 - 2|a|\sqrt{1-\|f_k\|^2}\sqrt{1-\|f_l\|^2}\\ &\ge 1 - \frac{1-\|f_k\|^2}{\|f_l\|^2},\end{aligned}$$

for $k \neq l$ and $a \in \mathbb{C}$. The meaning of the inequalities and some related topics will be presented.

An estimation of scale parameter of images using Fourier-Mellin transform

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The Mellin transform is well known as a scale invariant transform. The Fourier-Mellin transform is not only scale invariant but also time shift invariant. Based on the Fourier-Mellin transform, an algorithm, which checks whether two images $f(x)$, $g(x)$ have a relation $g(x) = \beta f(\alpha x - \gamma)$, where $\alpha > 0$, $\beta > 0$, $\gamma \in \mathbb{R}^2$, is proposed. Another algorithm to estimate α and β is proposed. Several experiments have shown the usefulness of algorithms.

p-adic Time-Frequency Analysis and its Properties

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The $\mathbf{Q}_p$ is the field of p-adic numbers defined by the completion of the field of rational numbers with respect to the p-adic norm. We will give a talk about time-frequency analysis on $\mathbf{Q}_p$. Especially we will construct the p-adic Stockwell transform and see its properties.