

# Hocus Pocus Focus-Focus

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In the theory of dynamical systems one can find so-called completely integrable Hamiltonian systems, cases in which there exist as many first integrals as degrees of freedom. This allows for differential-geometric and Lie group-theoretic methods to be used in the analysis of the dynamics. A well understood particular case is that of toric systems, those where all first integrals correspond to circular actions. A complete classification of these systems can be made in terms of a certain class of polynomials.

The next logical step is the study of so-called semi-toric systems, where one action must be circular but the rest might not be so. These systems appear often in theoretical physics and their richness of possible dynamical behaviours is much greater than in the toric case since new types of non-degenerate singularities can arise, such as focus-focus points. We will review the advances made in the last years, where systems have been classified for the particular case of two degrees of freedom and with no hyperbolic singularities. The classification is based on five symplectic invariants but unfortunately some of these are often difficult to calculate.