ON THE NON-TRIVIALITY OF GELFAND-SHILOV SPACES

Let $(M_p)_{p\in\mathbb{N}}$ and $(A_p)_{p\in\mathbb{N}}$ be two sequences of positive real numbers. The space $\mathcal{S}_{\{A_p\}}^{\{M_p\}}$ consists of all $\varphi \in C^\infty(\mathbb{R})$ such that

$$\sup_{p,q\in\mathbb{N}} \sup_{x\in\mathbb{R}} \frac{|x^q D^p \varphi(x)|}{h^{p+q} M_p A_q} < \infty,$$

for some $h > 0$. These spaces were introduced by Gelfand and Shilov in 1958 and since then found numerous applications in different branches of analysis. It is a long standing open problem to find conditions on the weight sequences $(M_p)_{p\in\mathbb{N}}$ and $(A_p)_{p\in\mathbb{N}}$ that characterize the non-triviality of the space $\mathcal{S}_{\{A_p\}}^{\{M_p\}}$. In this talk we will explain how this problem can be tackled and give an overview of several known (partial) solutions.