An Investigation into Small Weight Code Words of Projective Geometric Codes

Sam Adriaensen

When two computers communicate, there might be some noise on the channel. This means that when sending a message, which consists of multiple symbols, there is a (small) probability for each symbol to not be received the same as it was sent. To arm ourselves against this problem, we can add extra symbols to do some error-correcting. Ofttimes, these symbols are called redundancy. This gives birth to the concept of codes.
An important type of codes are linear codes. These are subspaces of finite vector spaces. One of the advantages is that knowing a base, or more generally a generating set, for this subspace is enough to know the entire code.
Linear codes can be constructed from incidence structures. That is a triple $(\mathcal{P}, \mathcal{B}, I)$, where the elements of $\mathcal{P}$ are called points, elements of $\mathcal{B}$ are called blocks, and $I \subseteq \mathcal{P} \times \mathcal{B}$ is the incidence relation, stating which points are in which blocks. Write $\mathcal{P} = \{P_1, \dots, P_v\}$. The incidence vector of a block $B \in \mathcal{B}$ is $x = (x_1, \dots, x_v)$, with

$$x_i = \begin{cases} 1 & \text{if } P_i I B, \\ 0 & \text{otherwise.} \end{cases}$$

We can interpret these incidence vectors over any finite field $\mathbb{F}_q$. They generate a code in $\mathbb{F}_q^v$.
Choose a prime power $q = p^h$, p prime. Choose numbers $1 \leq j < k < n$. Let $\mathcal{P}$ be the j-dimensional spaces of $\mathbb{F}_q^n$, let $\mathcal{B}$ be the k-dimensional spaces, and let I be containment. The weight of a code word is the number of positions with non-zero symbols. In our research, we focus on these codes. We are interested in code words of small weight, especially giving geometric characterizations of these words.