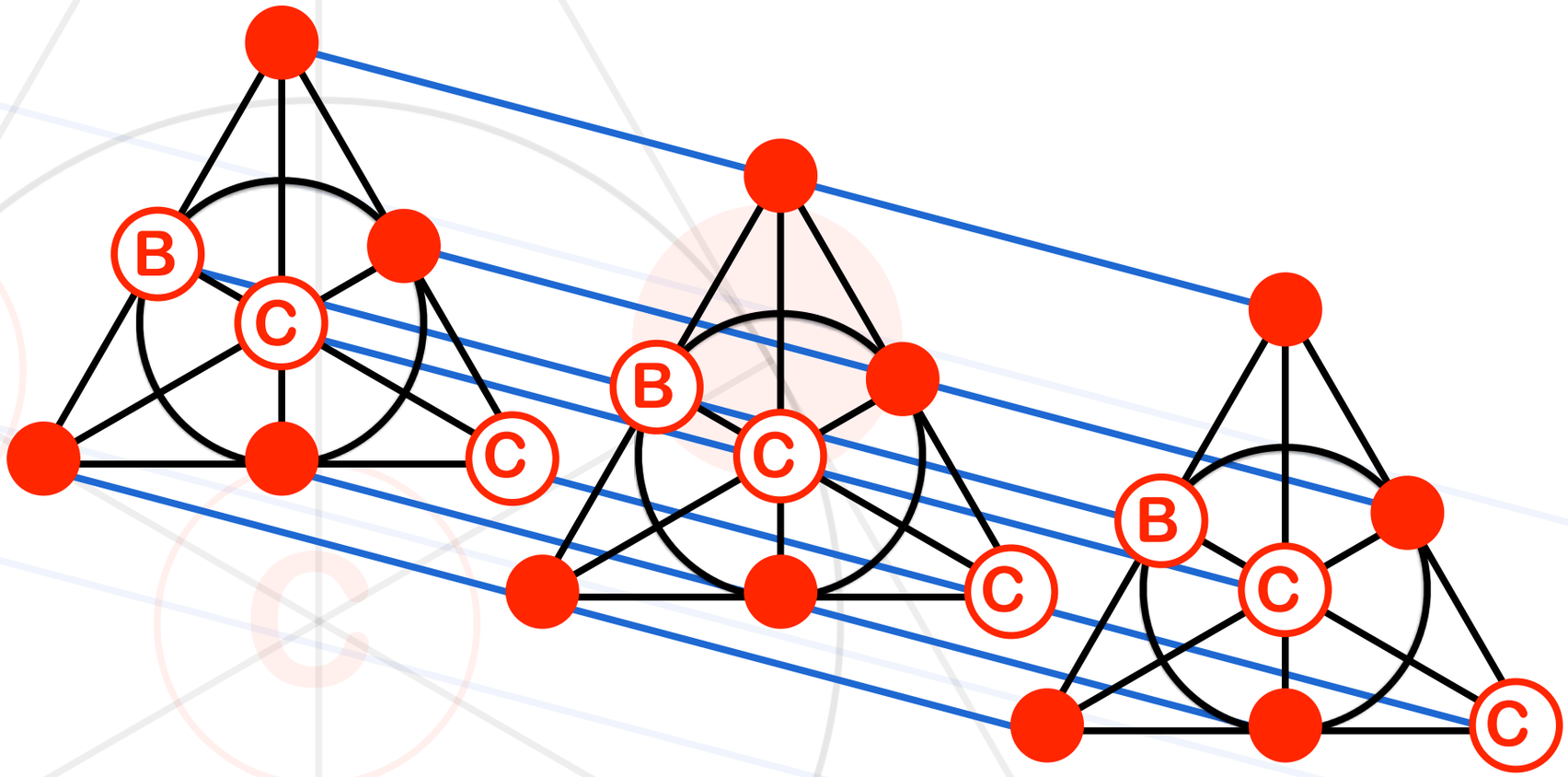
UNIVERSITEIT GENT

Anneleen De Schepper

BCC, July 29, 2019

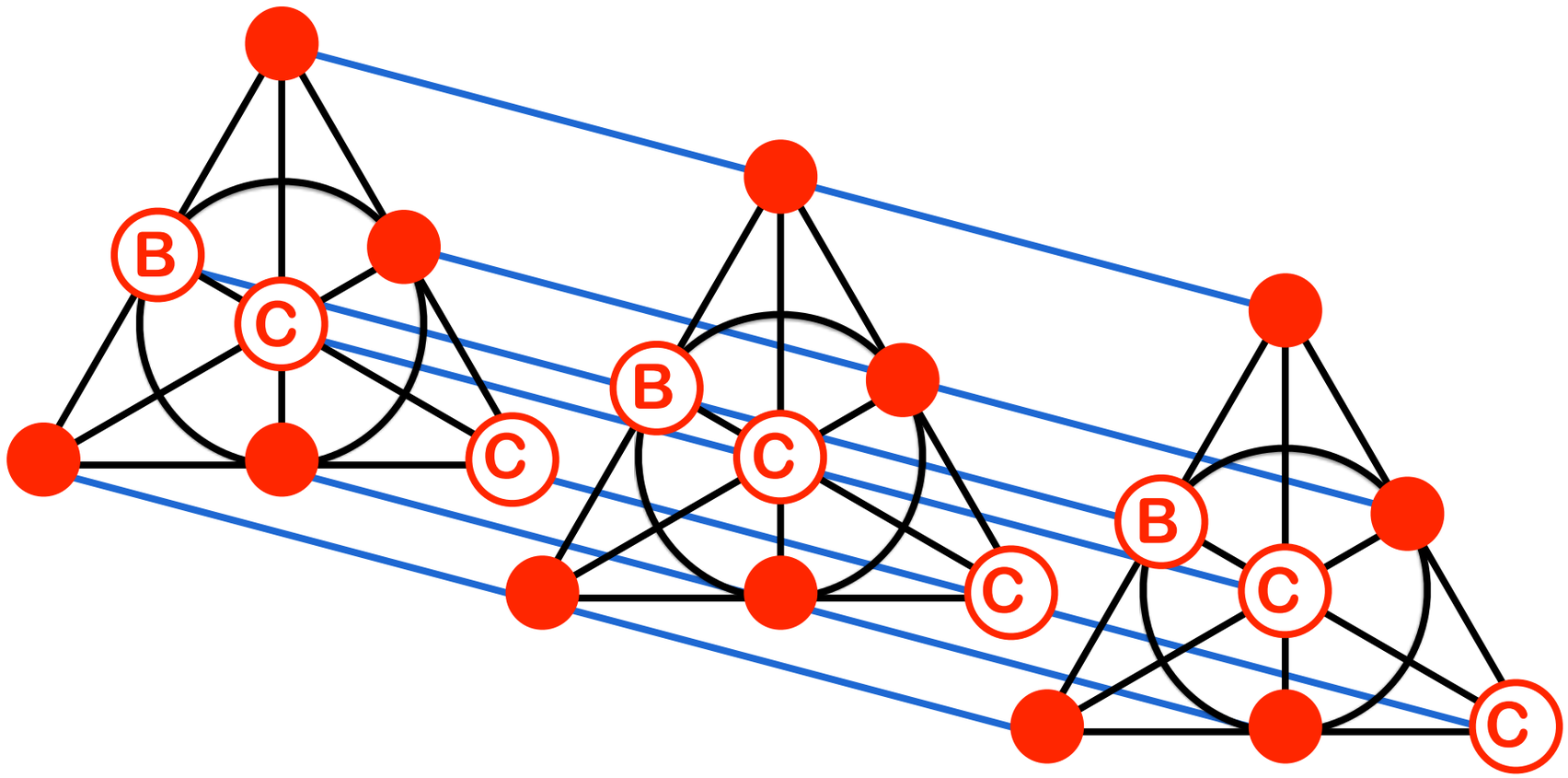


Projective planes with polar spaces

Joint work with:

J. Schillewaert (Auckland), H. Van Maldeghem (Gent) and M. Victoor (Gent)

Projective planes with polar spaces



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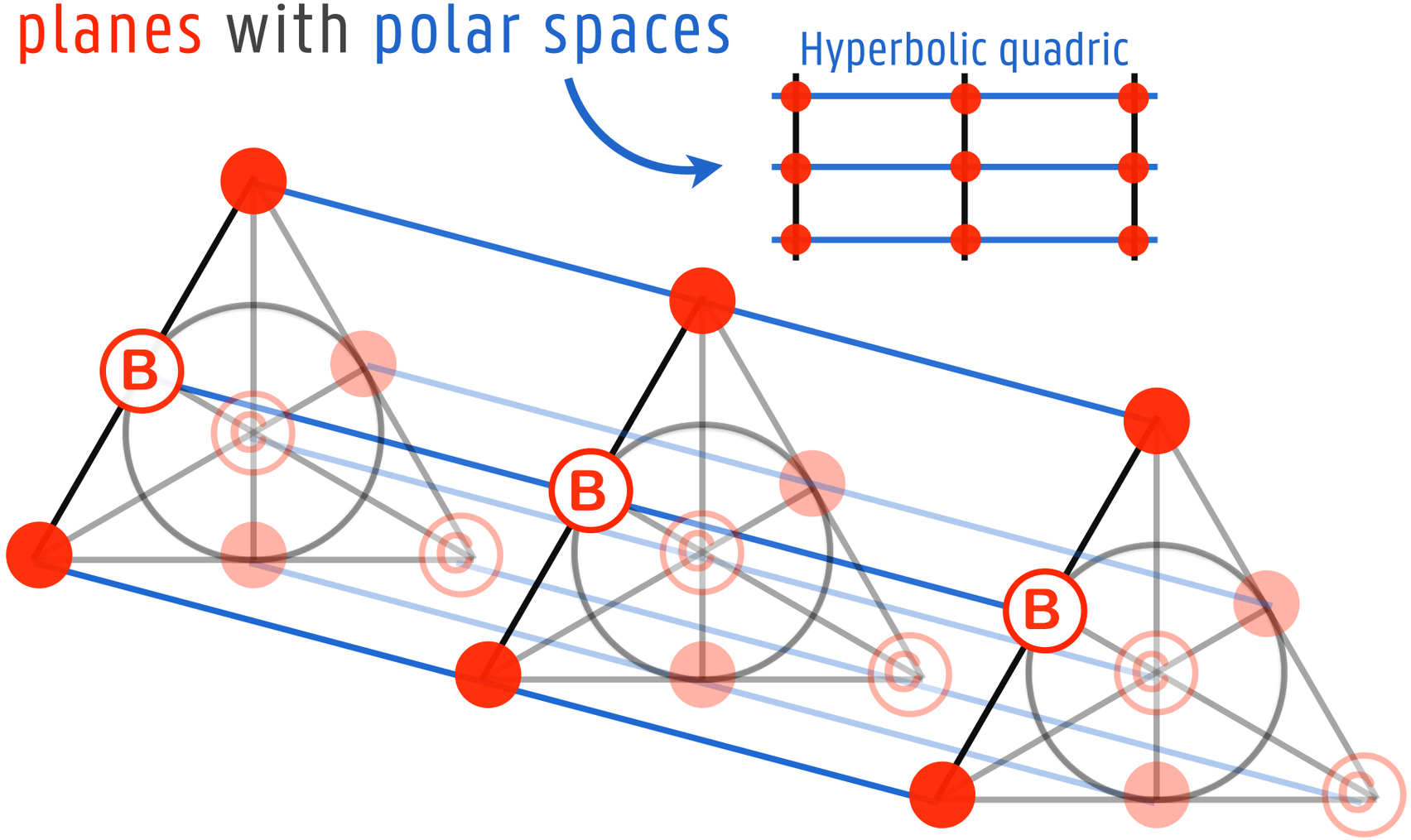

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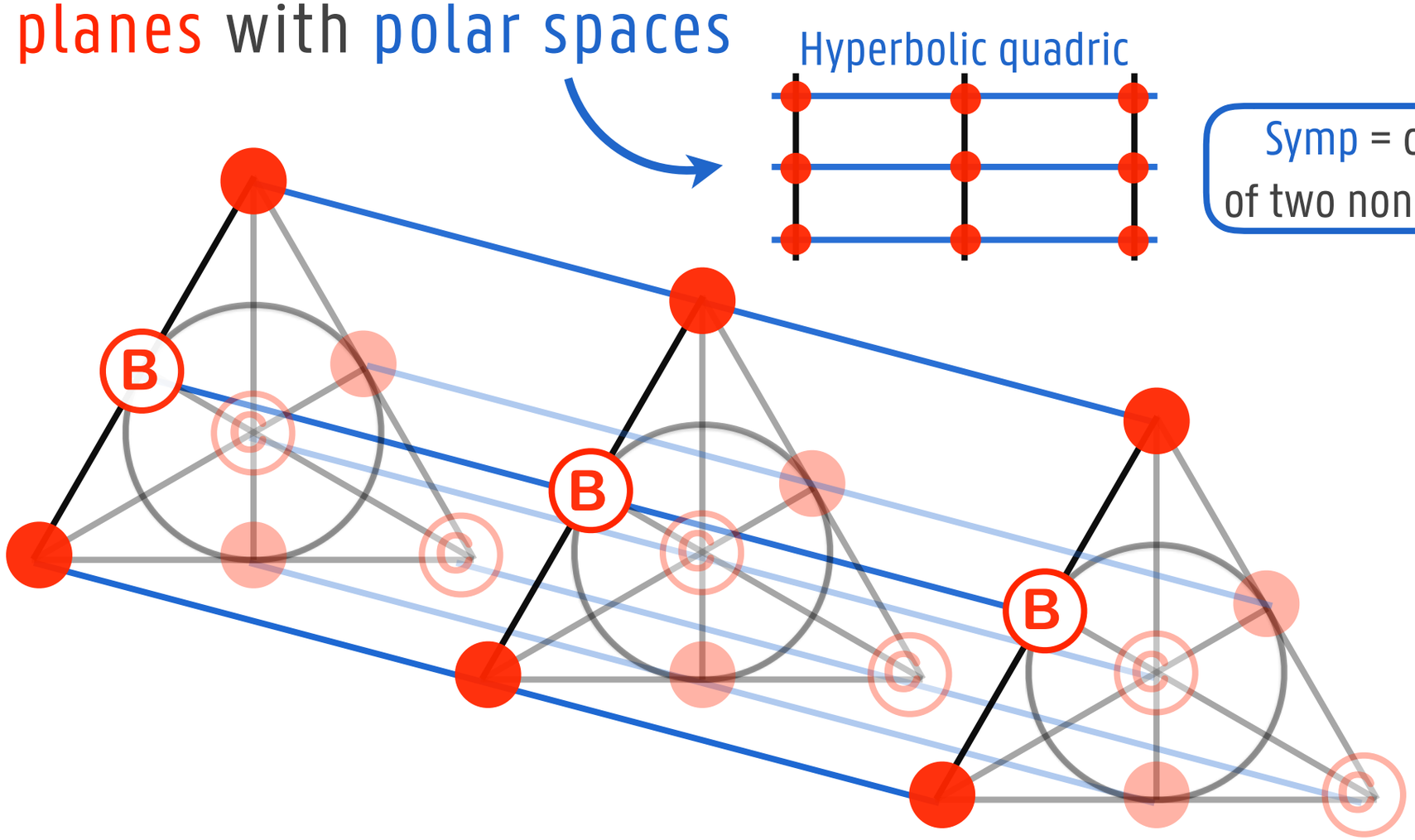
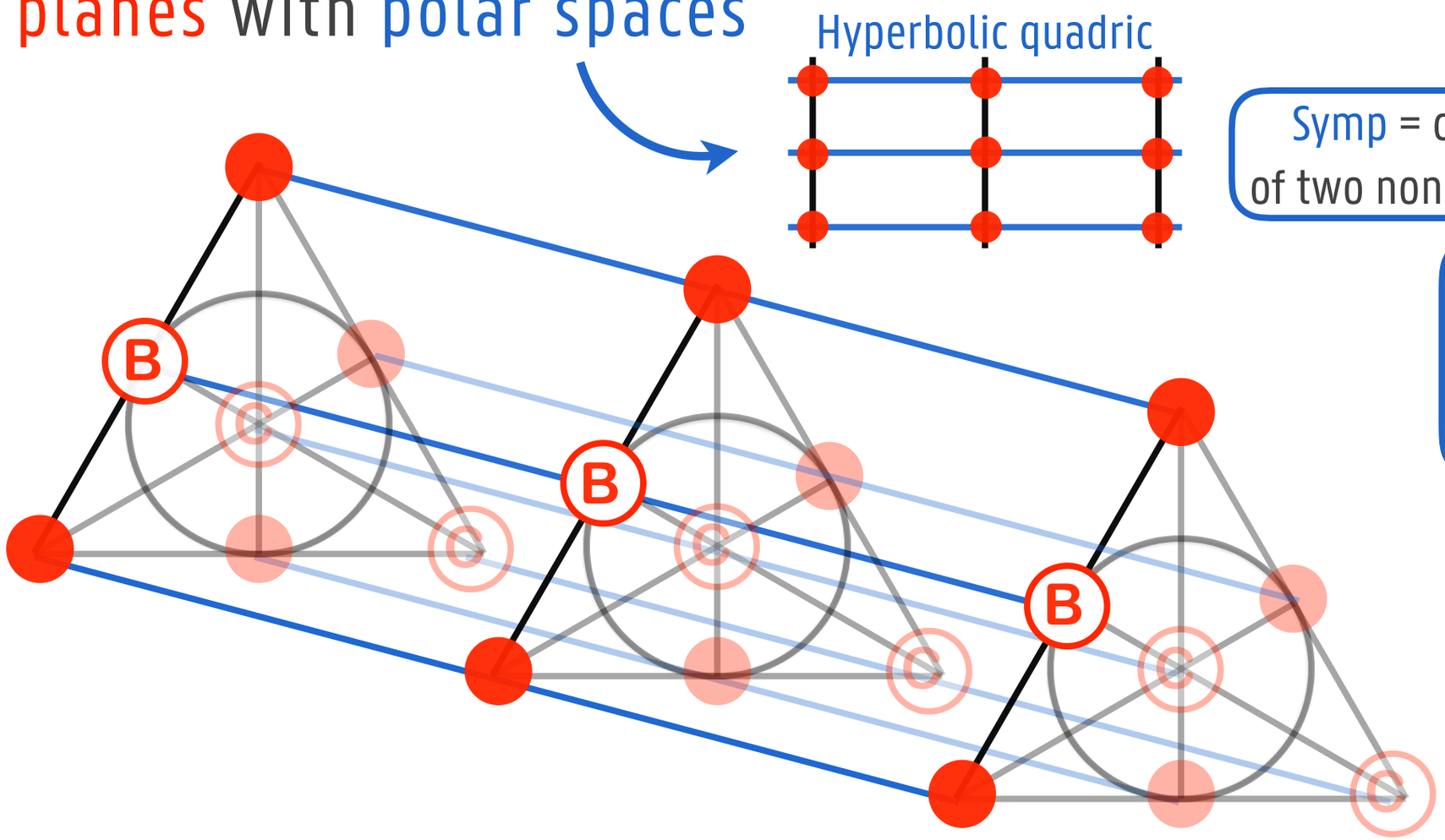


Symp = convex closure of two non-collinear points

Projective planes with polar spaces



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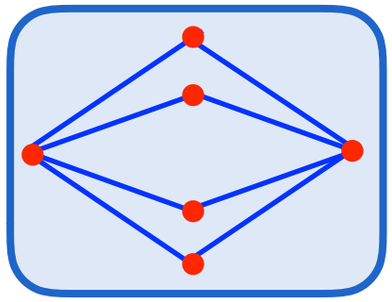

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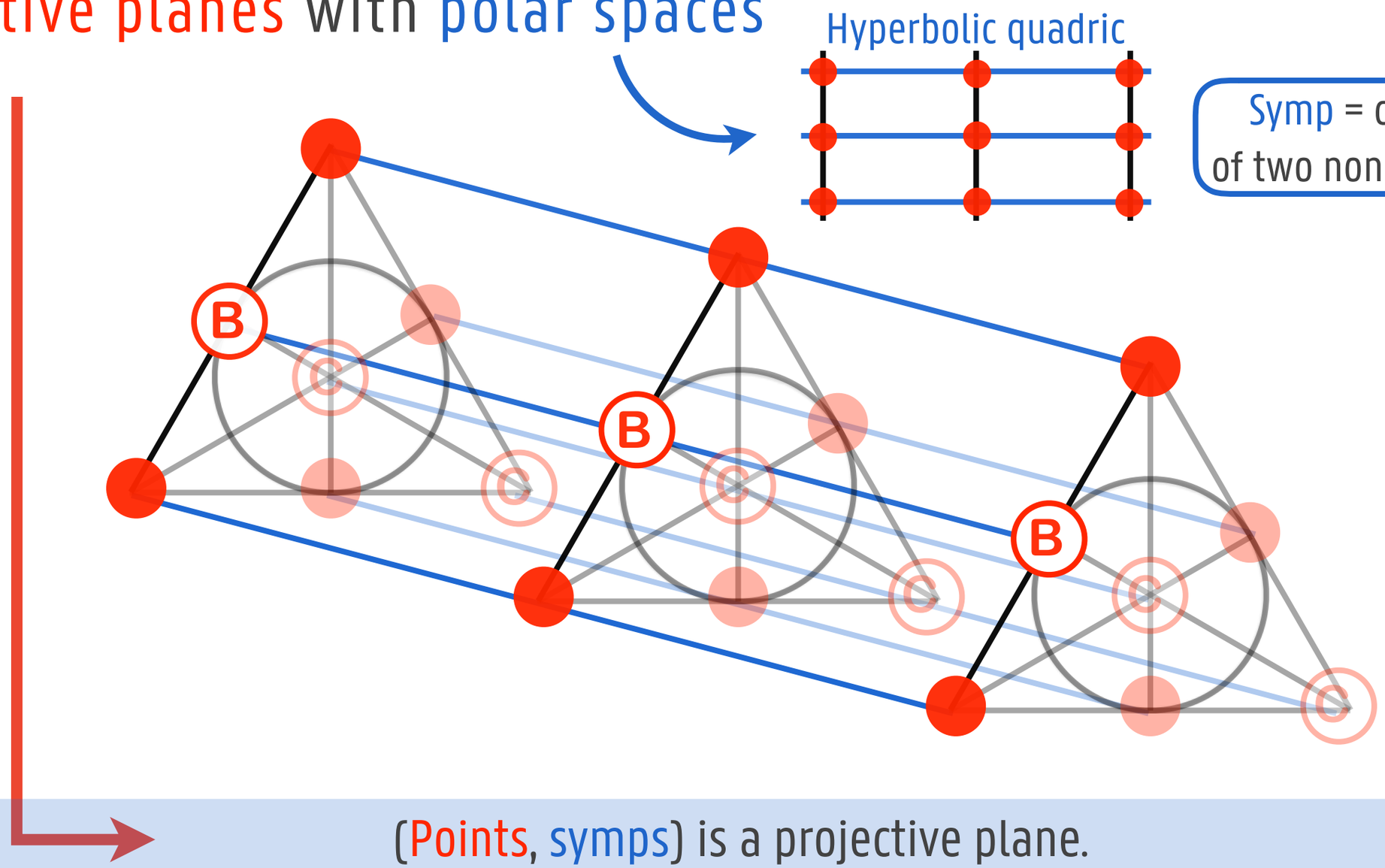
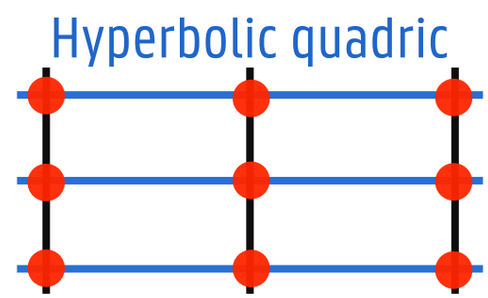


Symp = convex closure of two non-collinear points

(Points, symps) is a projective plane.

Projective planes with polar spaces



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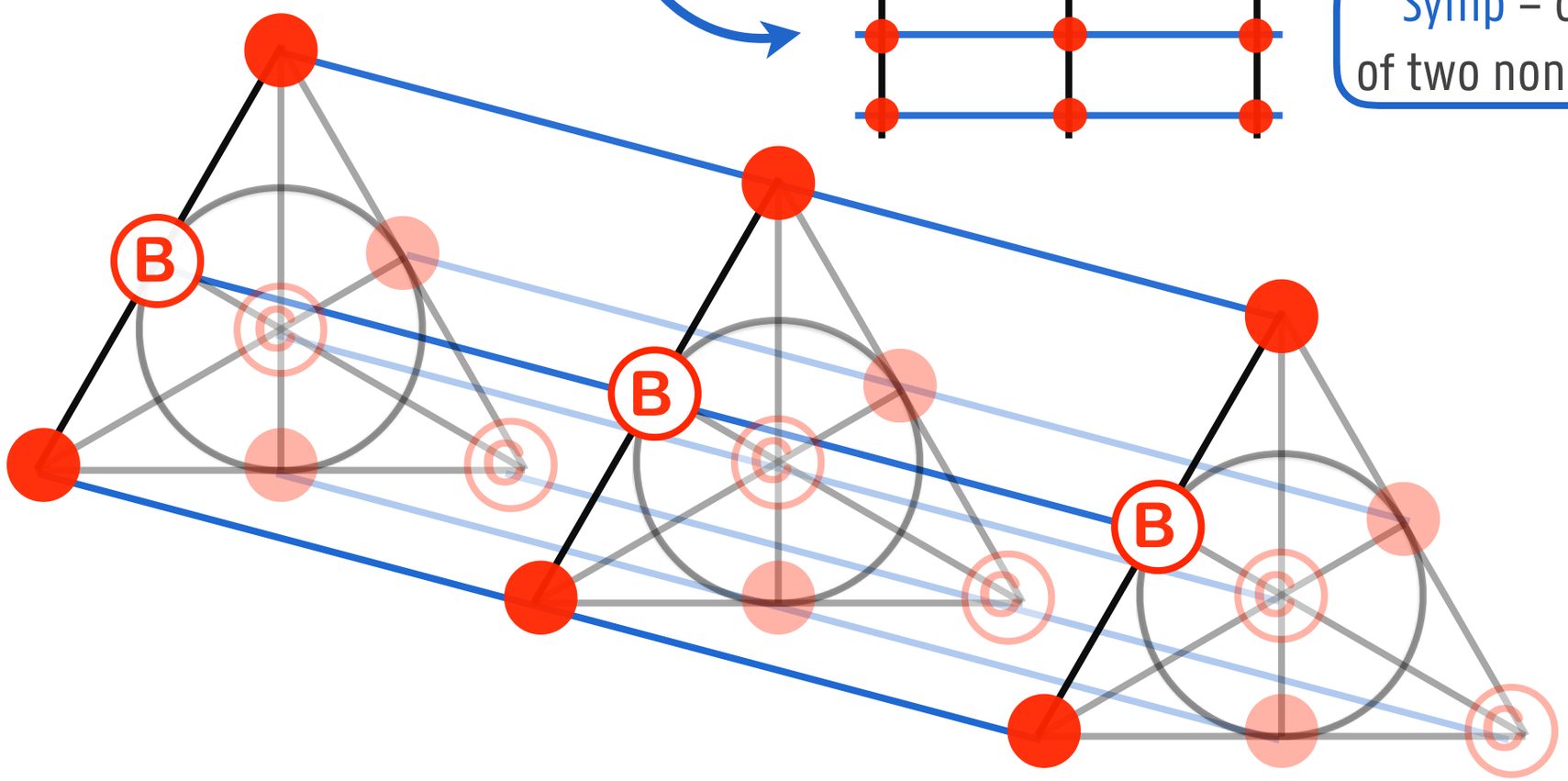
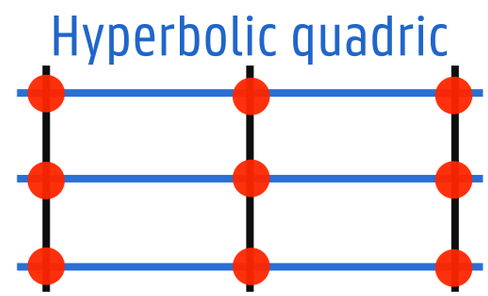


(Points, symps) is a projective plane.

Through each two points there is a unique symp.

Projective planes with polar spaces



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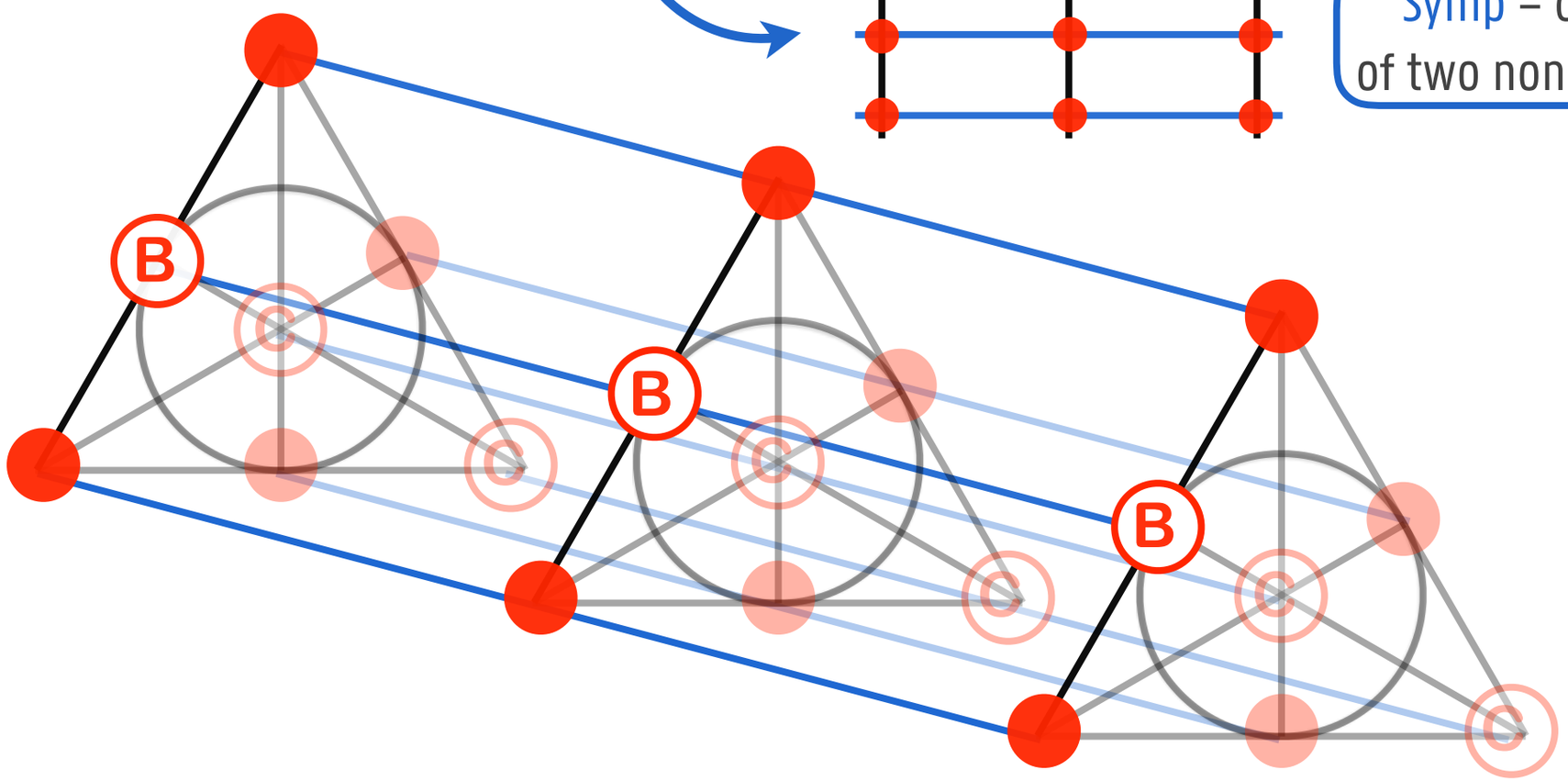

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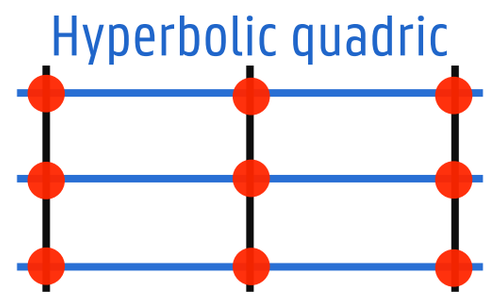

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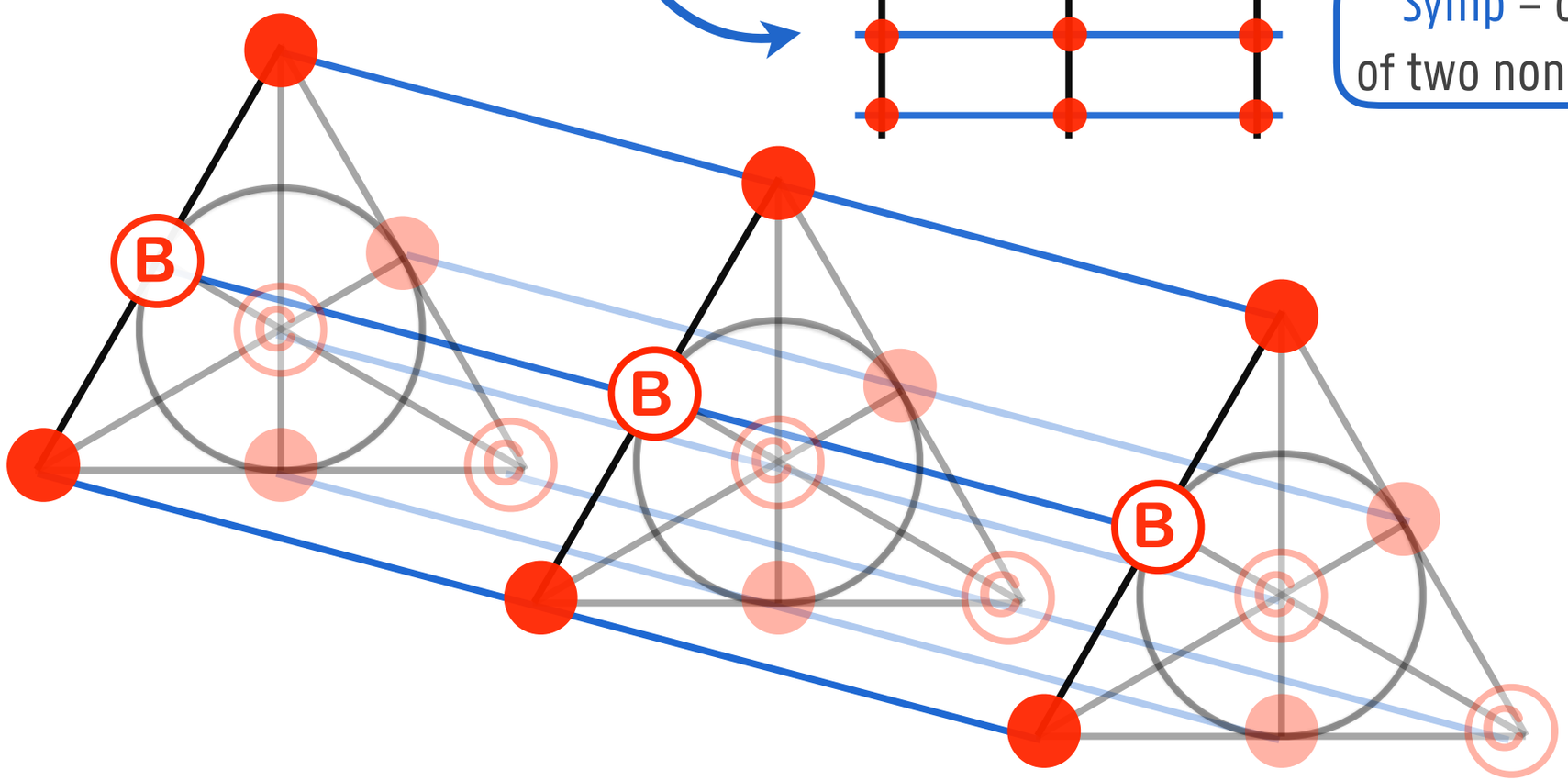

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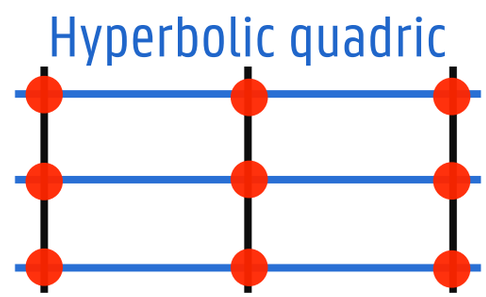
There is a frame (four points, no three on a symp).

Projective planes with polar spaces



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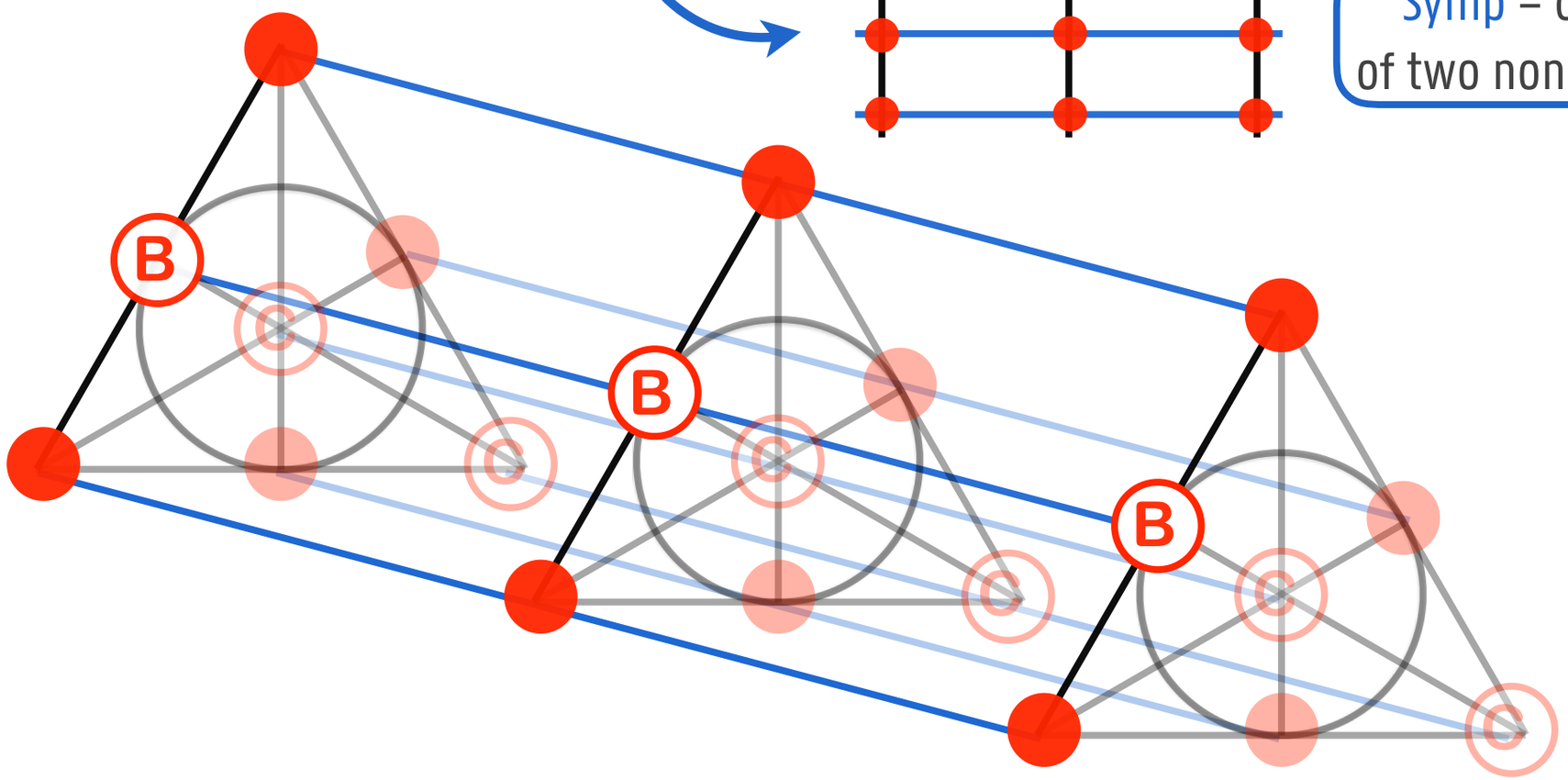

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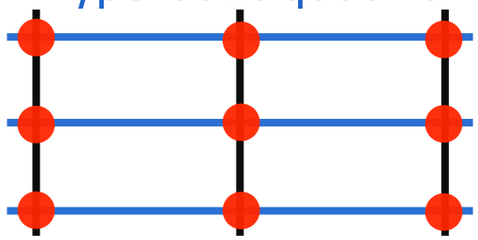
Not all points are on one symp.

weaker version

Projective planes with polar spaces

Hyperbolic quadric



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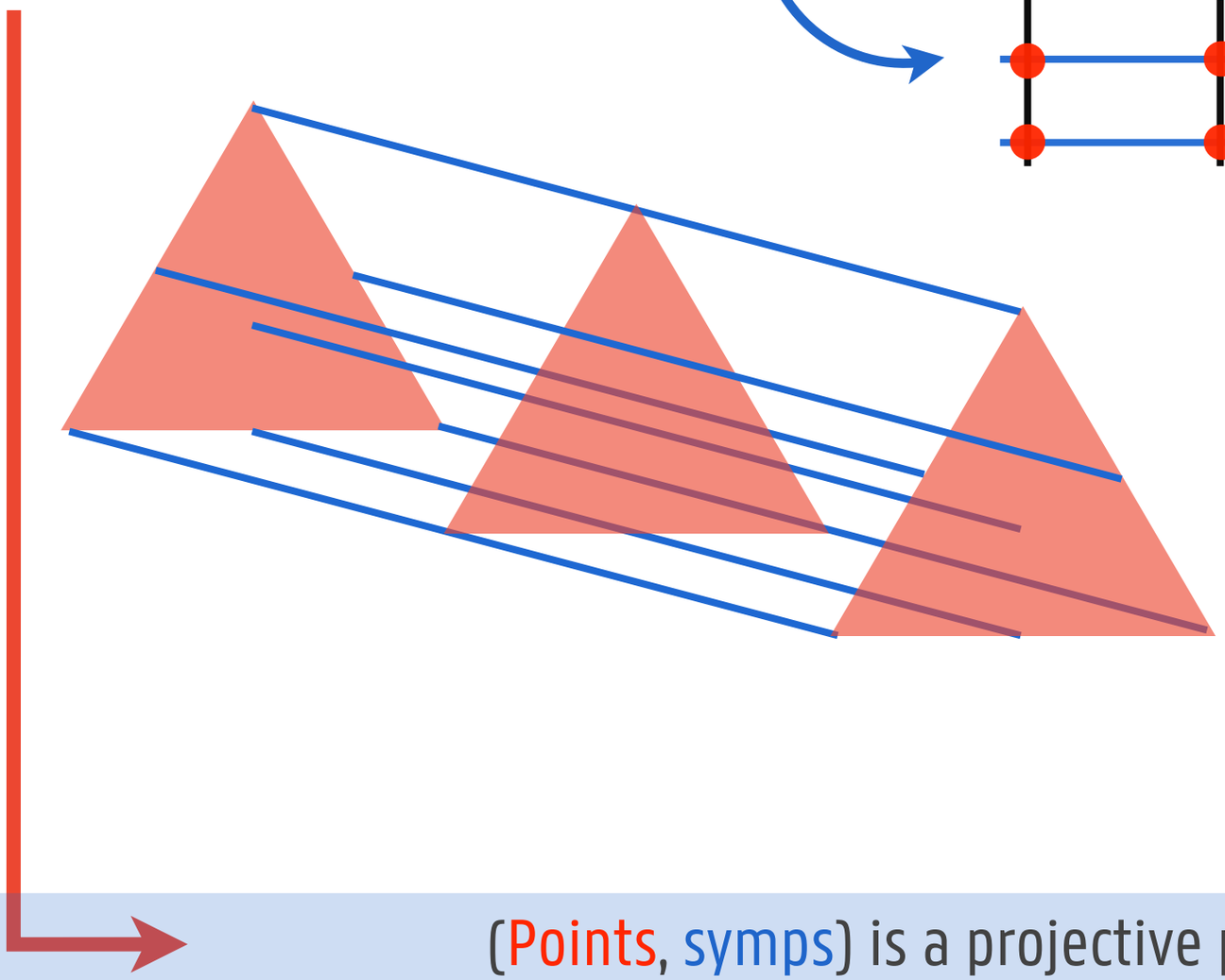

Direct product of

any projective plane π

and any projective line L

(Segre variety $S_{1,2}(K)$)

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Projective planes with polar spaces

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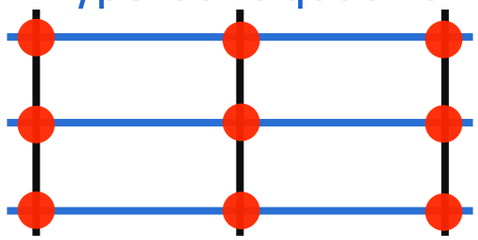

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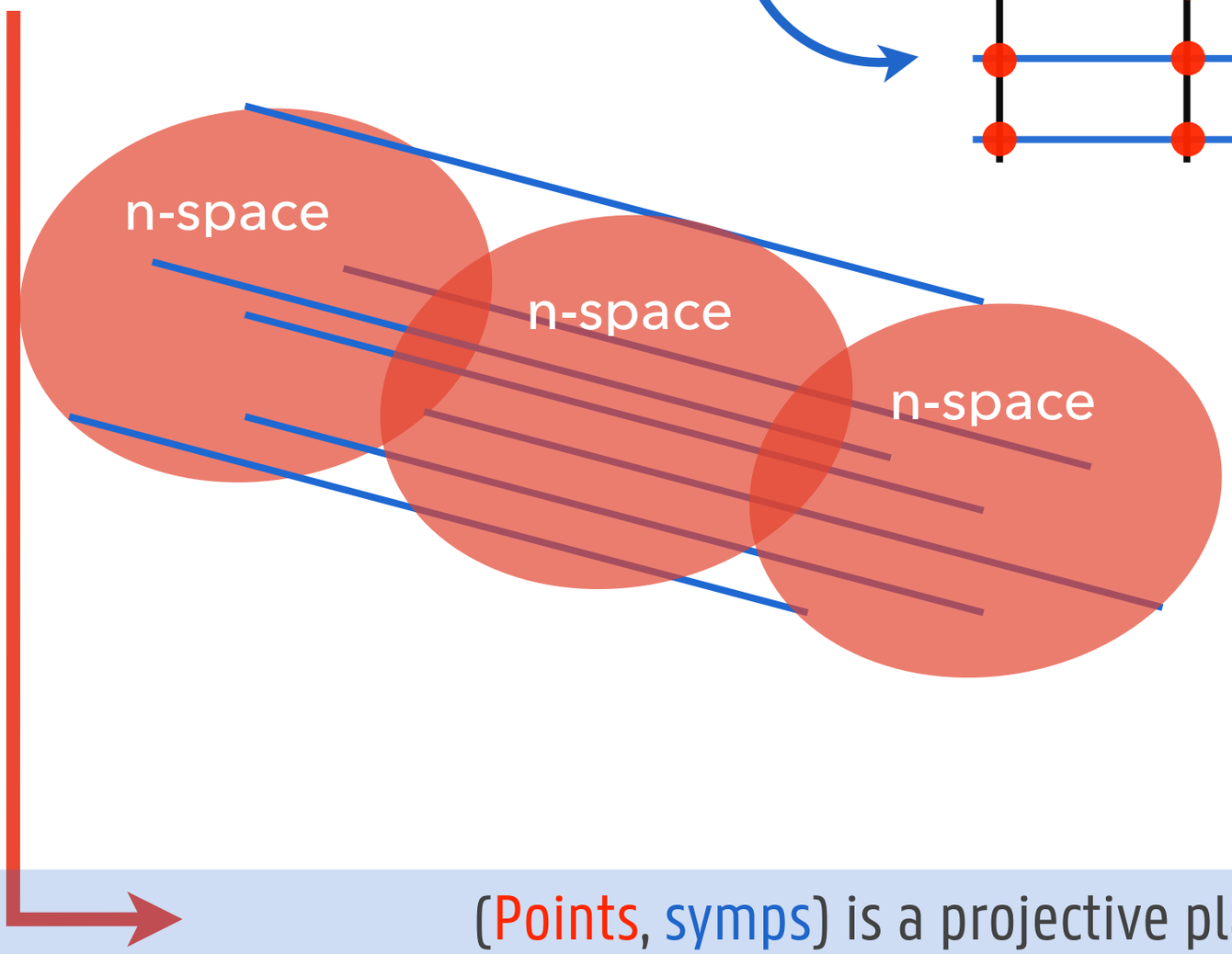

Direct product of

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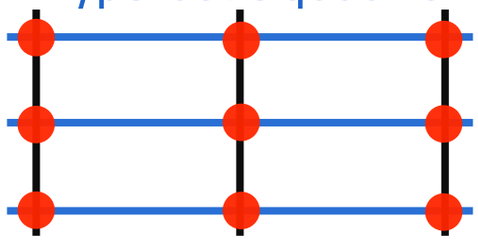

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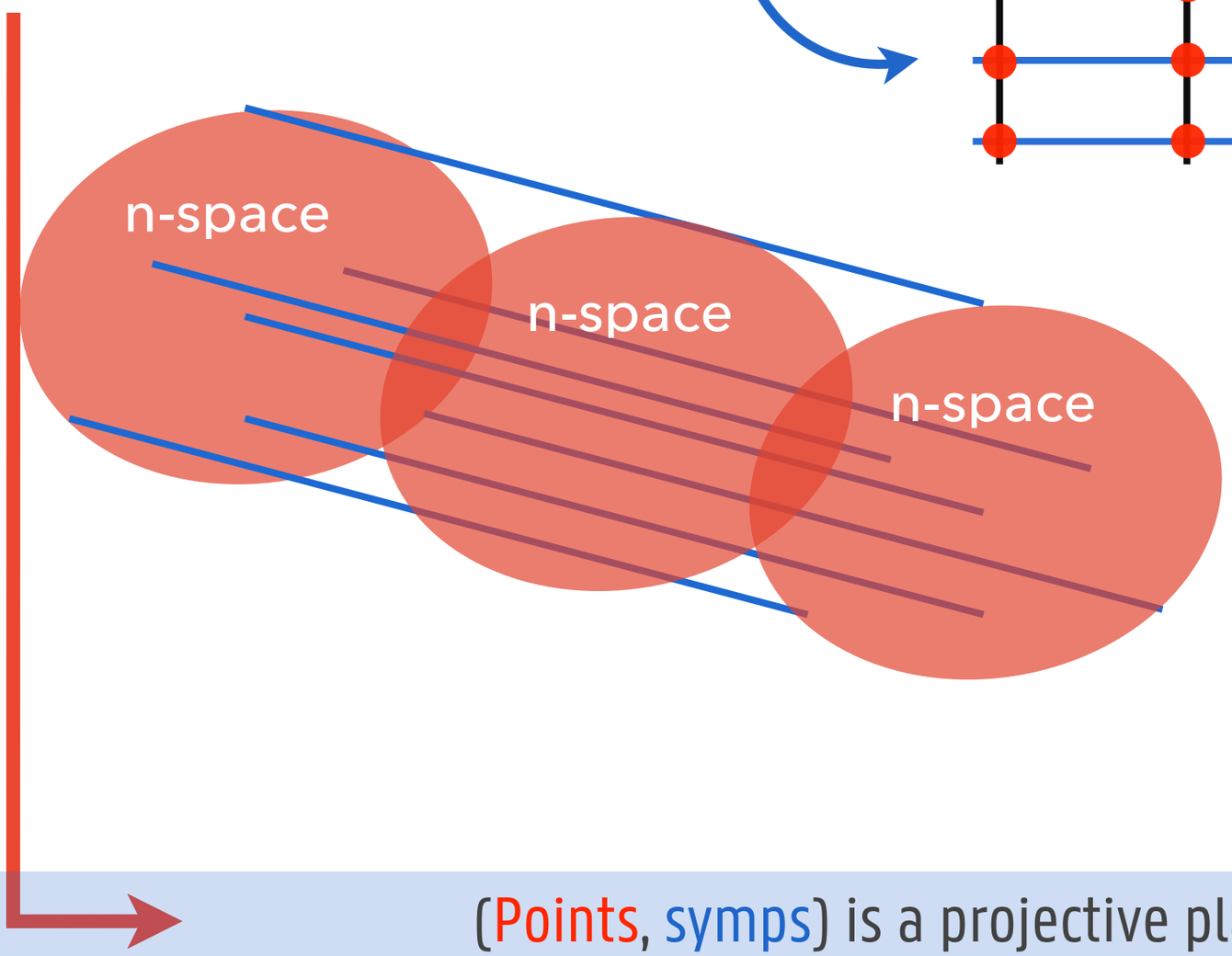

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This is not true if n > 2!

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Projective planes with polar spaces

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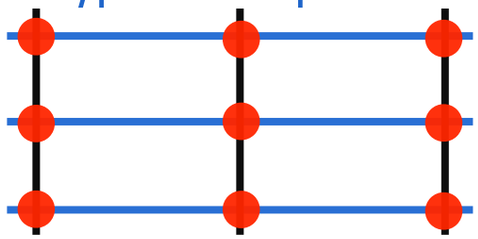

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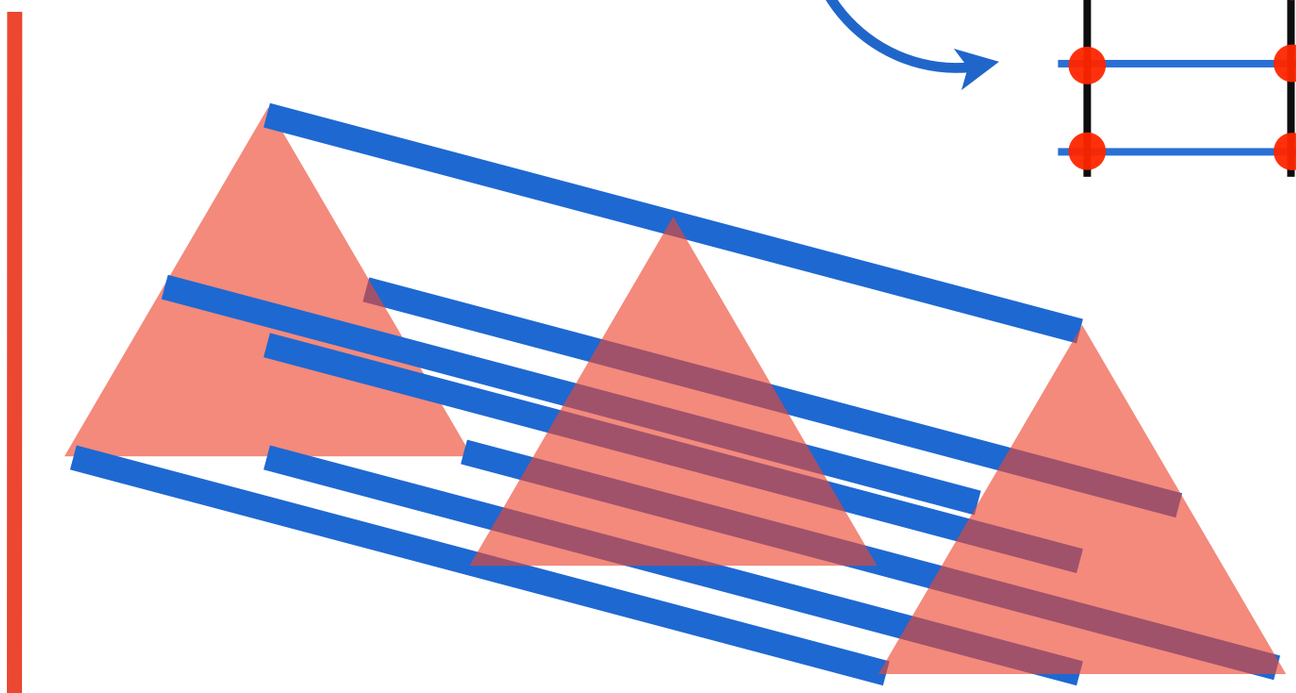

Direct product of

any projective plane π

and any projective plane π'

(Segre variety $S_{2,2}(K)$)

(Points, symps) is a projective plane.

Through each two points there is a unique symp.

Each two symps meet in at least one point.

Not all points are on one symp.

Which point-symp-geometries are such that:

Symps are convex in point-line geometry

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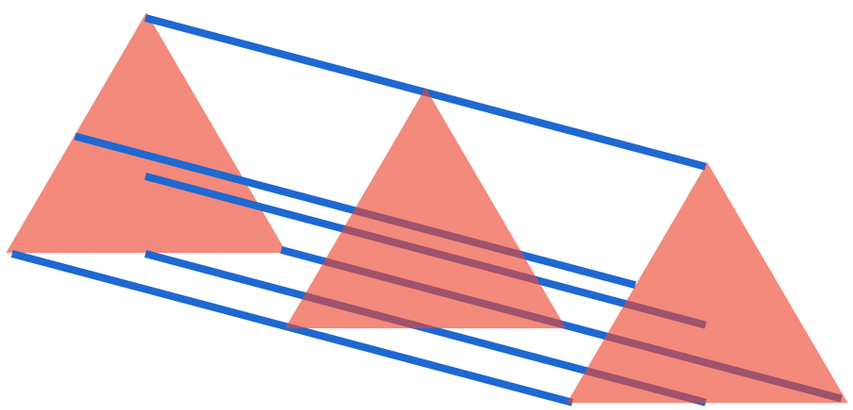

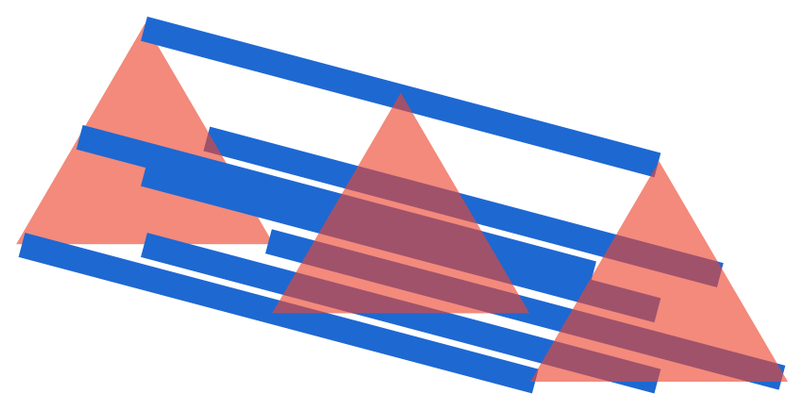

Which point-symp-geometries are such that:

symps can be of any kind, and need not be isomorphic or of same rank

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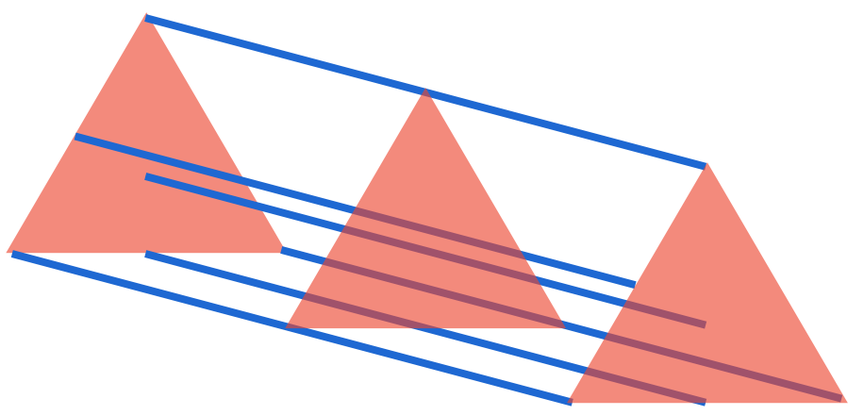

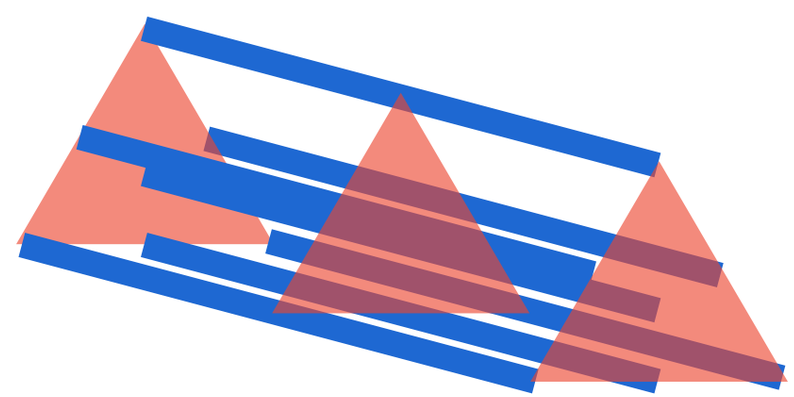

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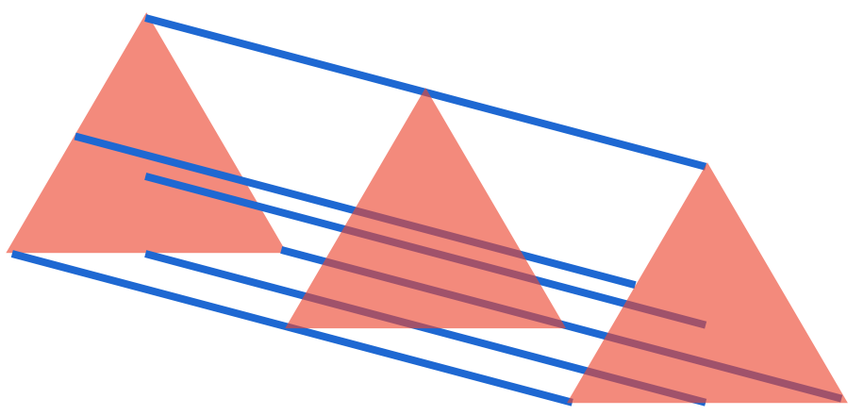

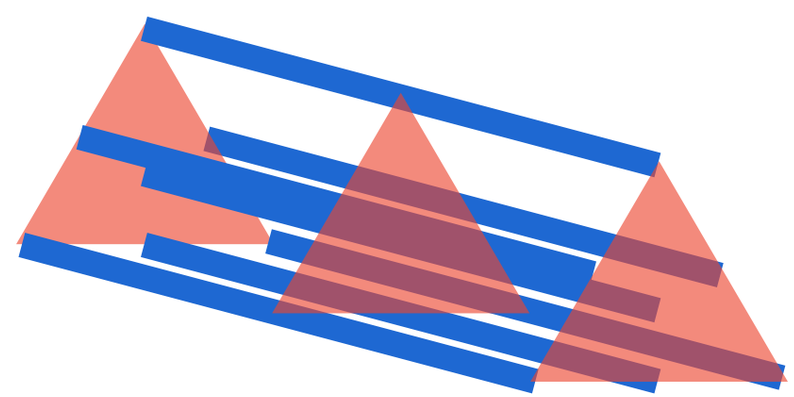

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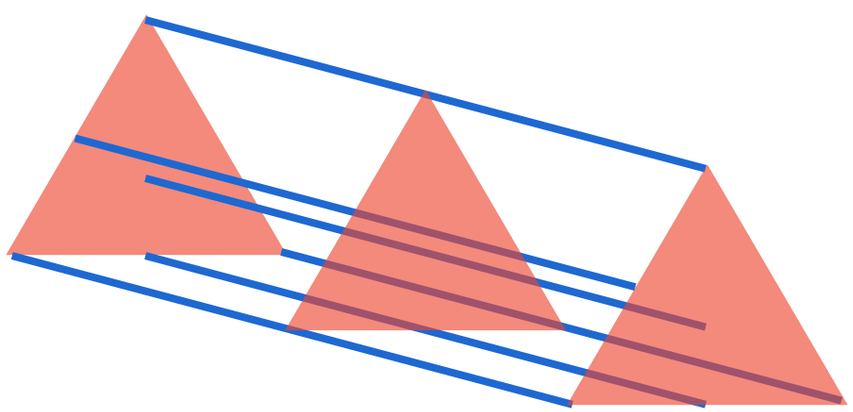
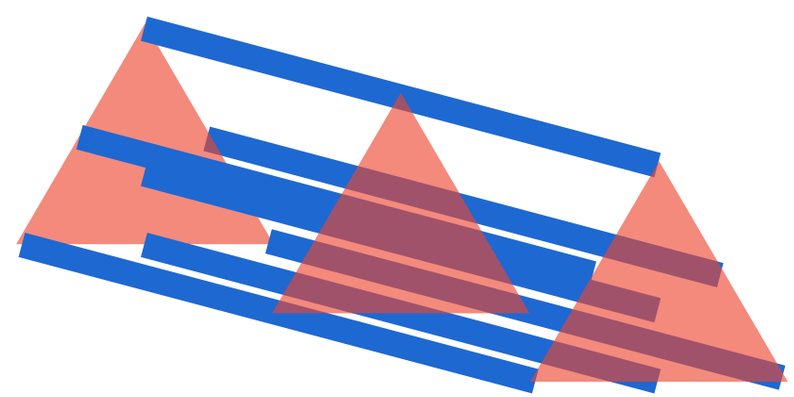
If there is a symp of rank 2:

only the **direct products** L × π and π × π'





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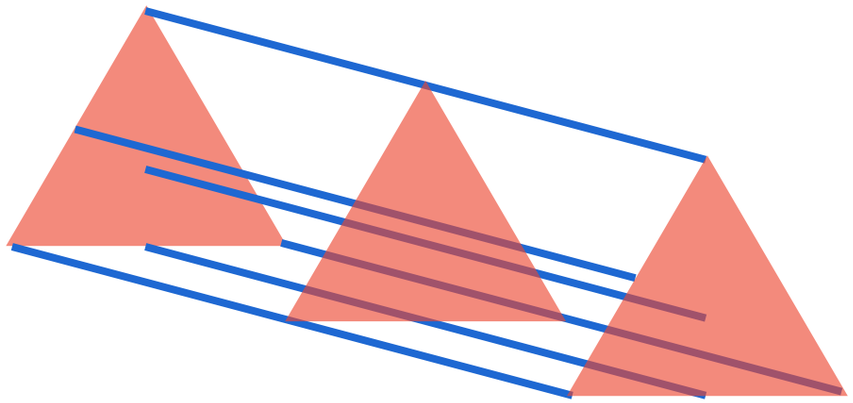
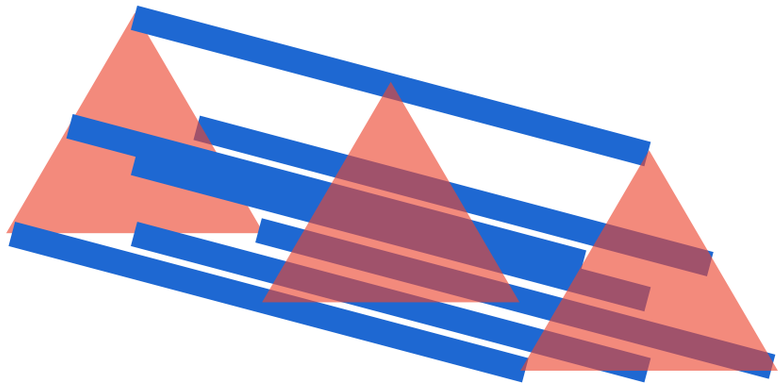
Both examples share that some lines are contained in a **unique** symp. This we want to **avoid** if there are no symps of rank 2.





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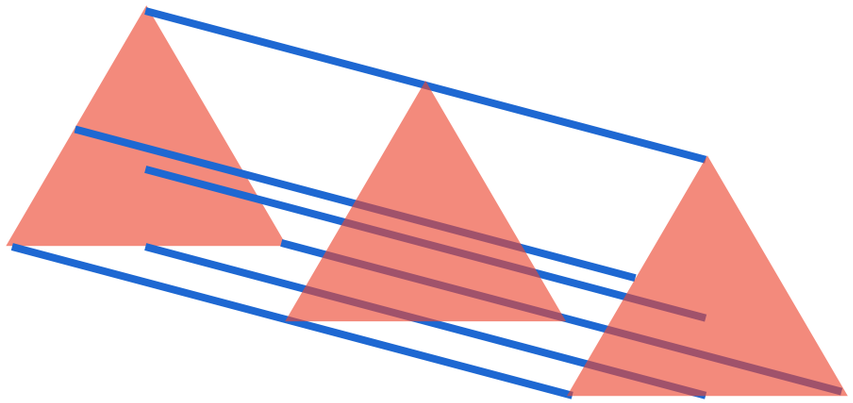

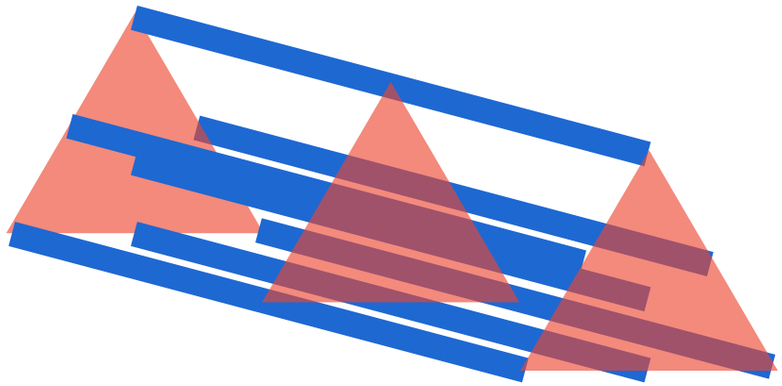

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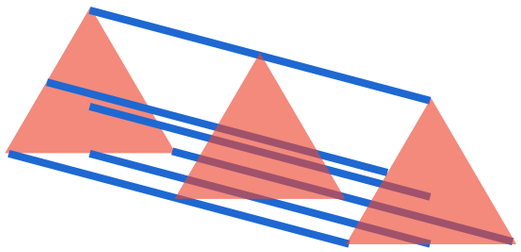

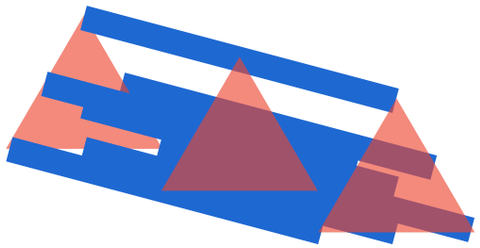

IF all symps have rank >2, there is a line contained in at least two symps.

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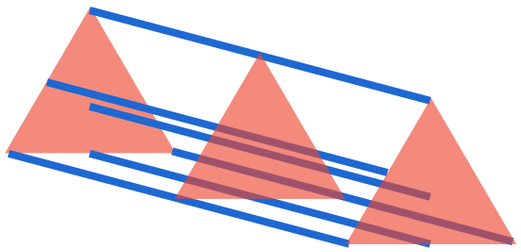

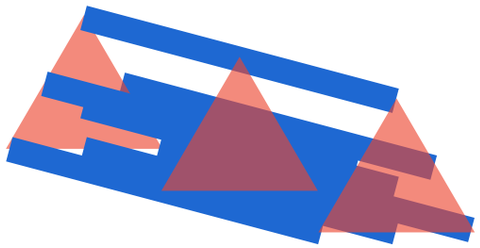

IF all symps have rank >2, there is a line contained in at least two symps.

Only the **line Grassmannian $A_{n,2}(L)$** of a projective n-space with $n \in \{4,5\}$ and the exceptional **$E_{6,1}(K)$-variety**.

Consider the **line Grassmannian** of a projective **n**-space. We verify the **axioms**:

Symps are convex in point-line geometry

Through each two points there is a ~~unique~~ symp.

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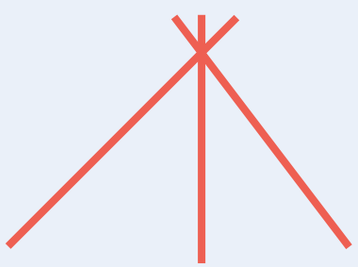

distance 1

(new) lines:
planar point pencils

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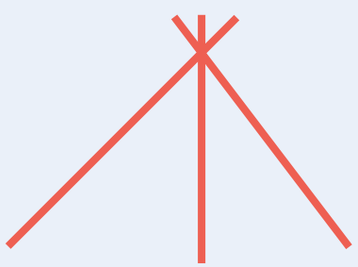

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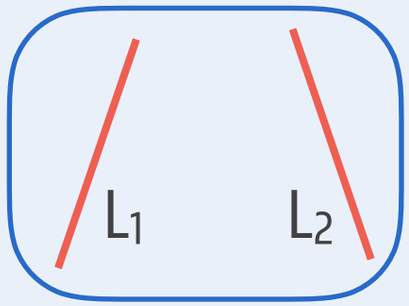

distance 2

Symp: convex closure of L_1, L_2

➜ all lines in $\langle L_1, L_2 \rangle$

➜ Klein quadric $Q^+(5,K)$

Consider the **line Grassmannian** of a projective **n**-space. We verify the **axioms**:

Symps are convex in point-line geometry

Through each two points there is a ~~unique~~ symp. ✓ n>2

Each two symps meet in at least one point.

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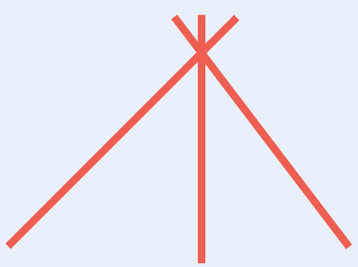

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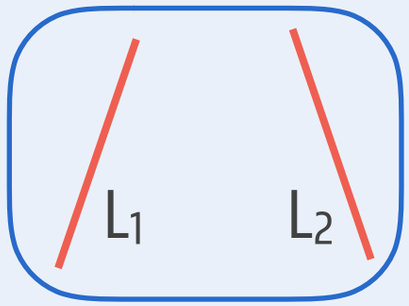

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Symp: convex closure of L_1, L_2
➜ all lines in $\langle L_1, L_2 \rangle$
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n=2: projective plane (no symps)

Consider the **line Grassmannian** of a projective **n**-space. We verify the **axioms**:

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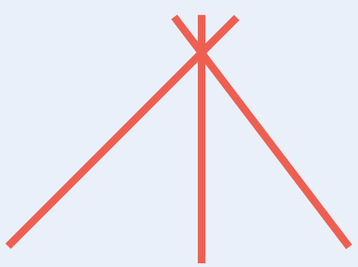
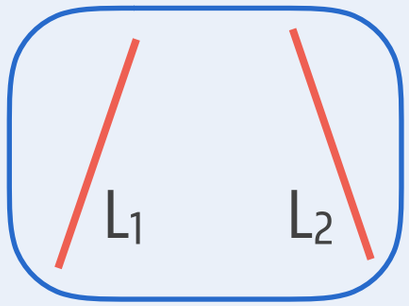


distance 1

(new) lines:
planar point pencils



distance 2

Symp: convex closure of L_1, L_2
➜ all lines in $\langle L_1, L_2 \rangle$
➜ Klein quadric $Q^+(5,K)$

n=2: projective plane (no symps)
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Consider the **line Grassmannian** of a projective **n**-space. We verify the **axioms**:

Symps are convex in point-line geometry

Through each two points there is a ~~unique~~ symp. ✓ n>2

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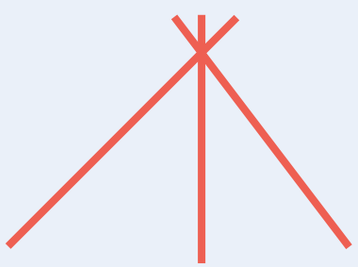

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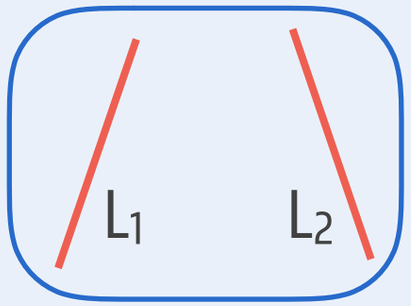

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3 < n < 6: each two 3-spaces share a line
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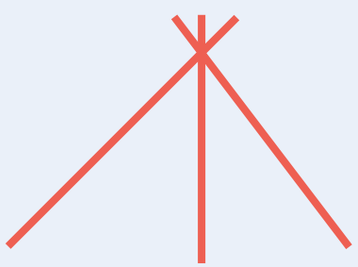
IF all symps have rank >2, there is a line contained in at least two symps. ✓ **n > 3**



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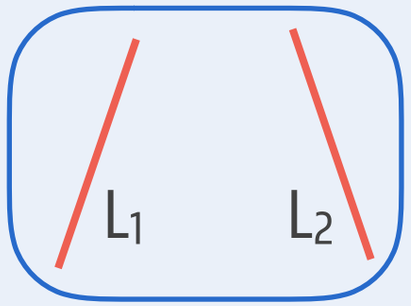

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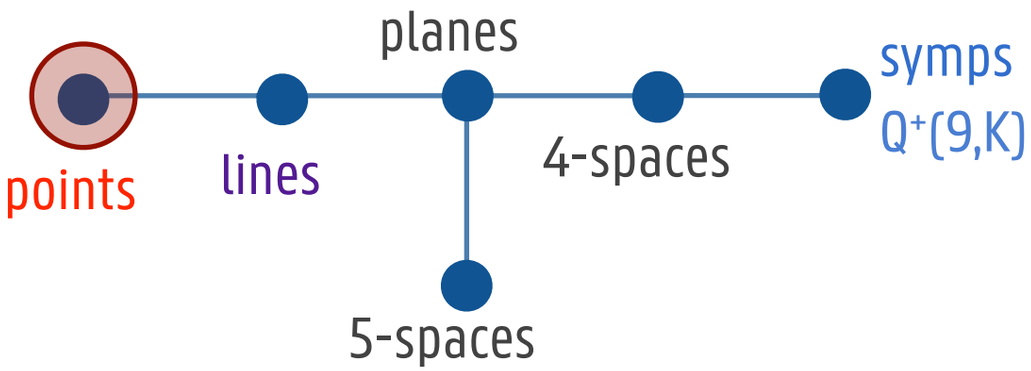

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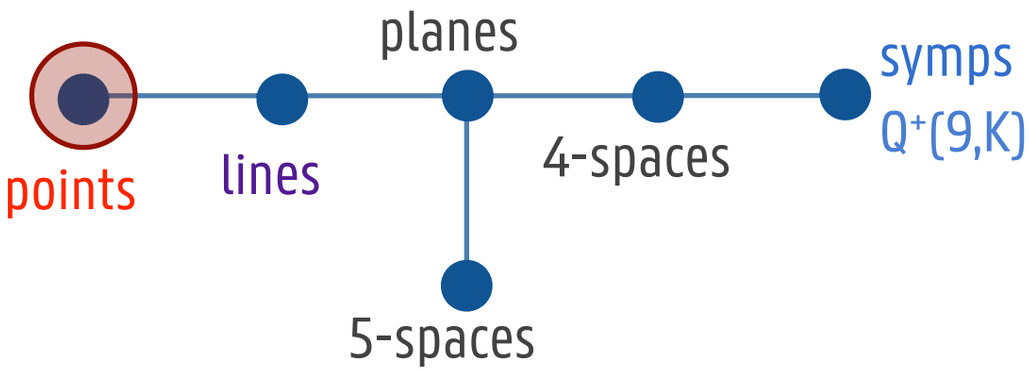

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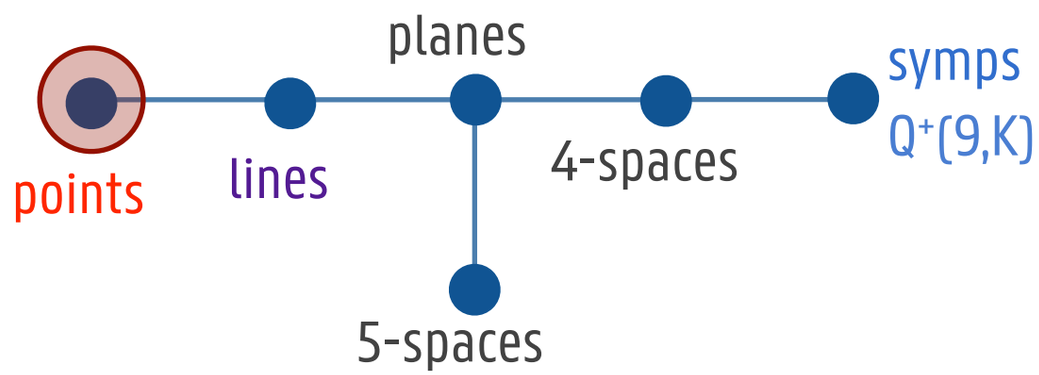

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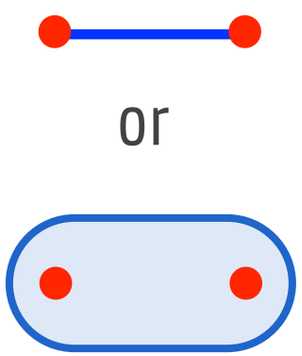

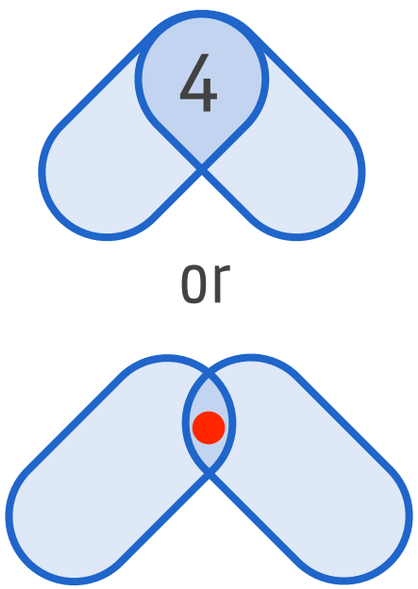

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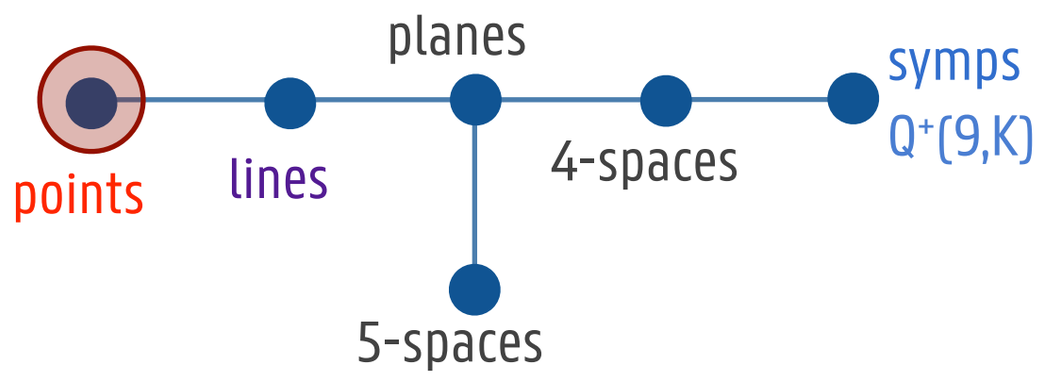

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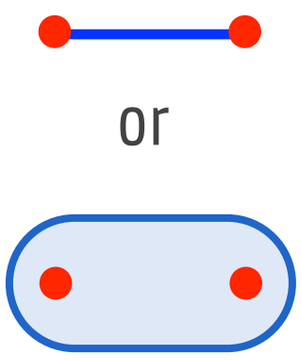

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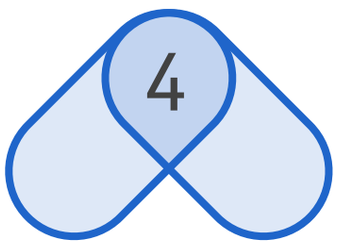

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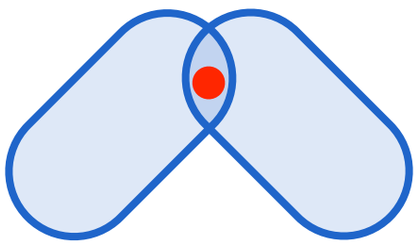

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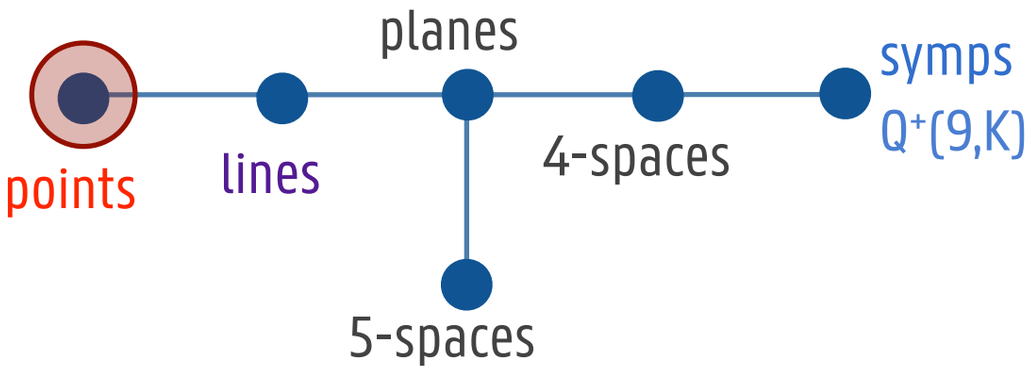

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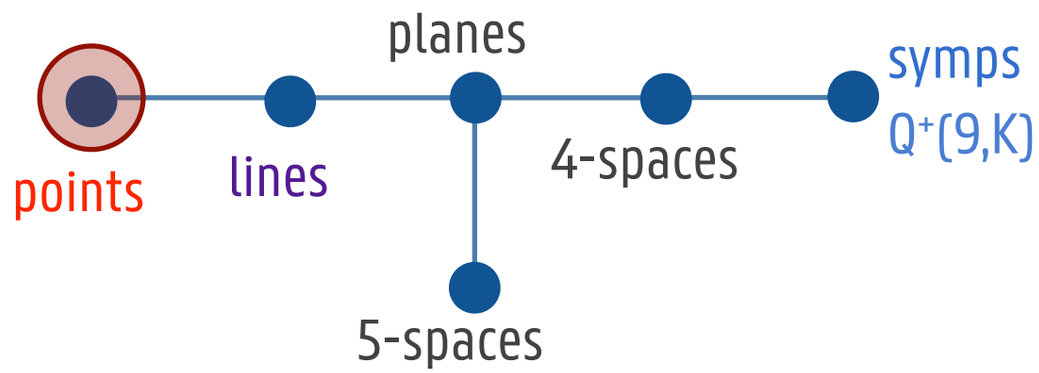

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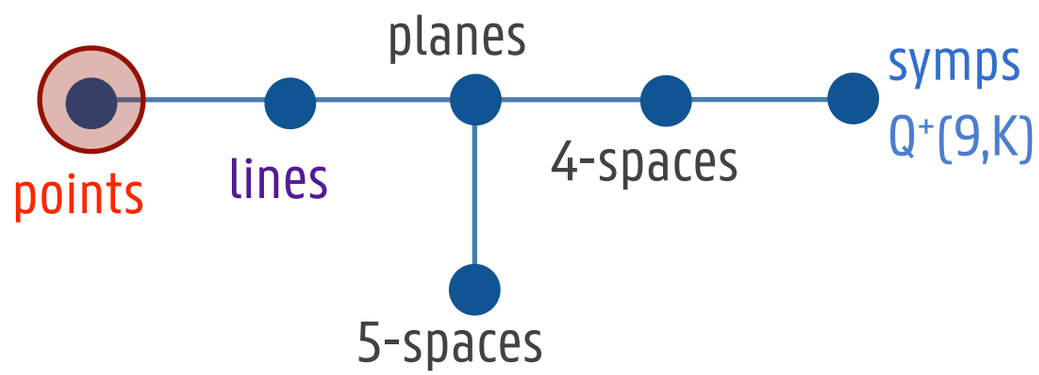

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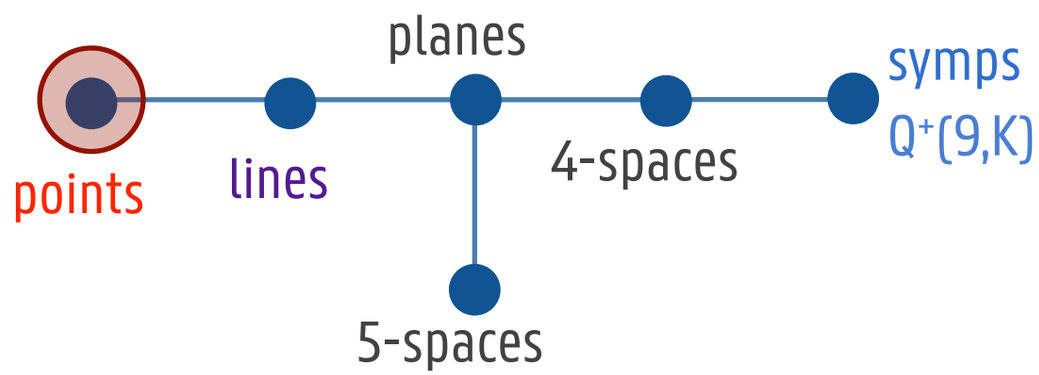

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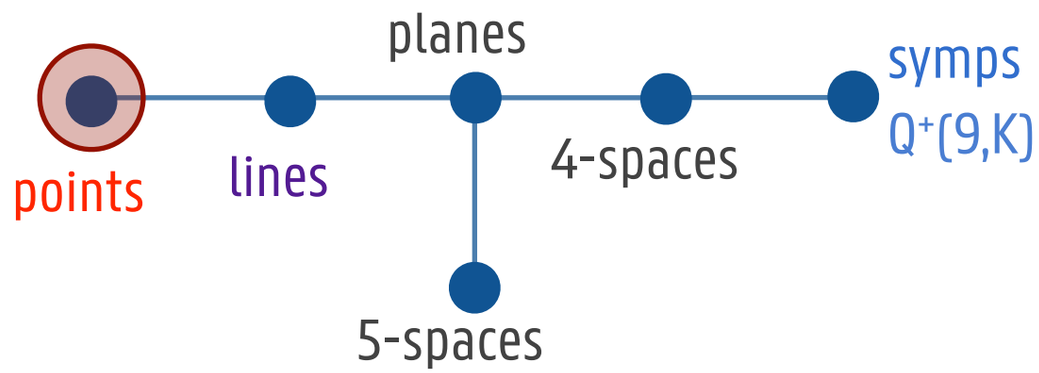

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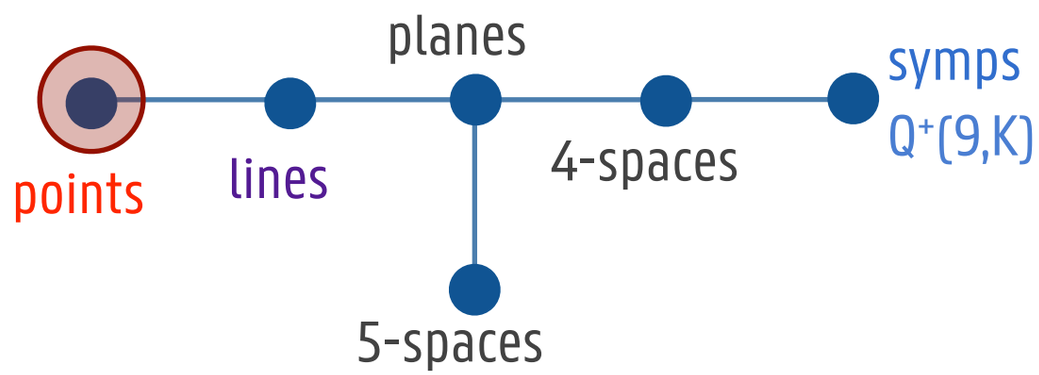

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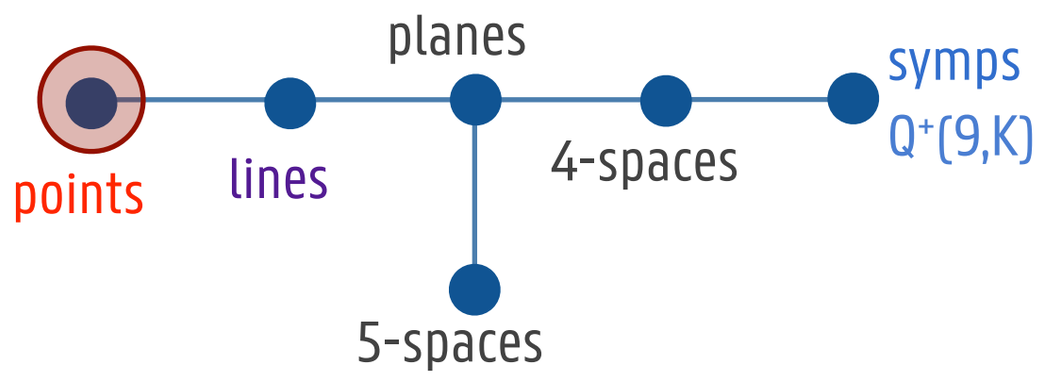

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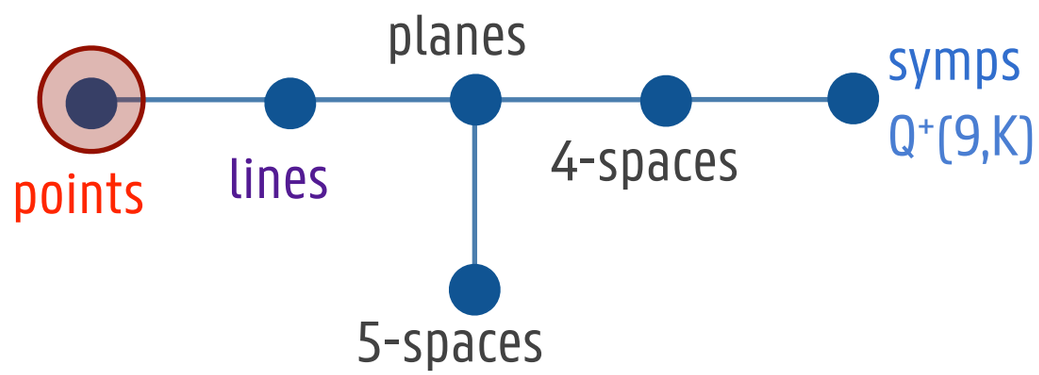

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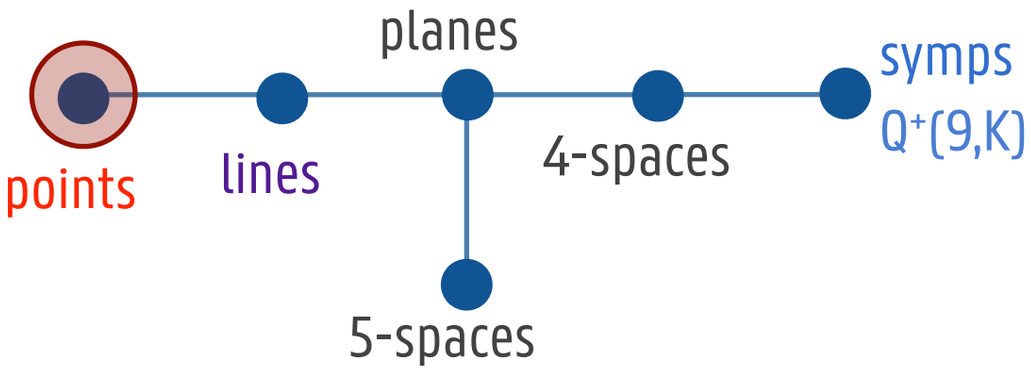

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Which point-symp-geometries are such that:

symps can be of any kind, and need not be isomorphic or of same rank

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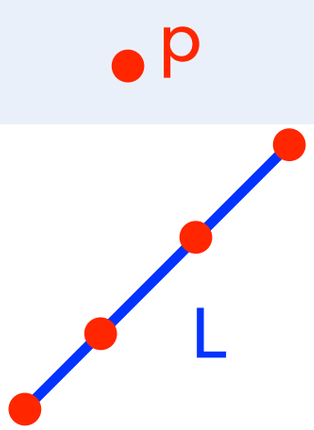

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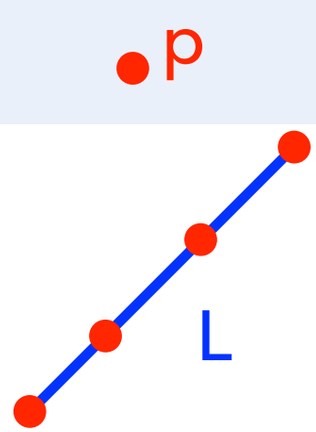

1

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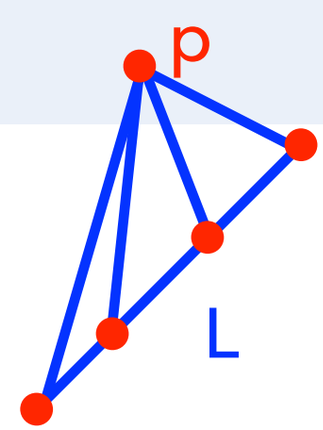

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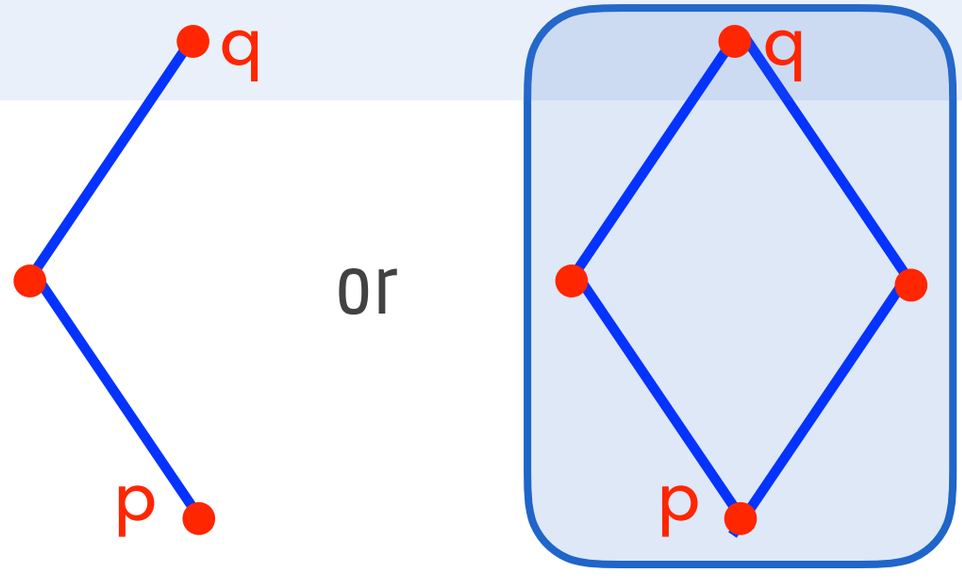

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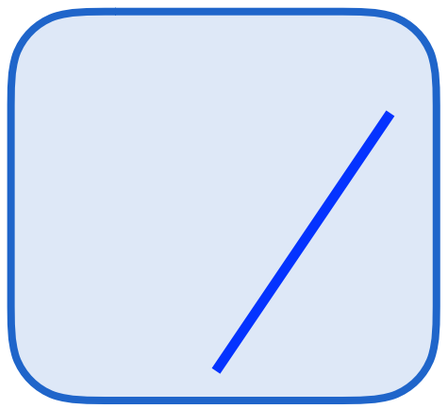

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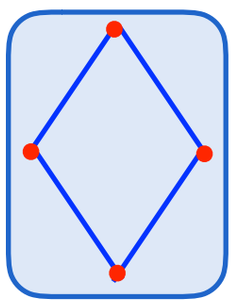

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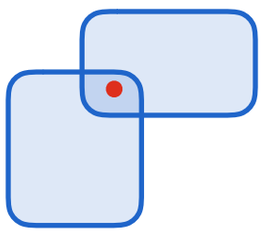

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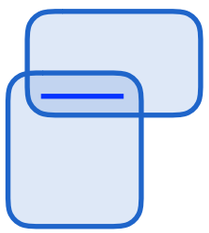

The only **parapolar spaces** which satisfy the **additional properties**

strong: always option (ii) and **diameter 2**

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- the **direct products** of a projective **line/plane** with a projective **plane**
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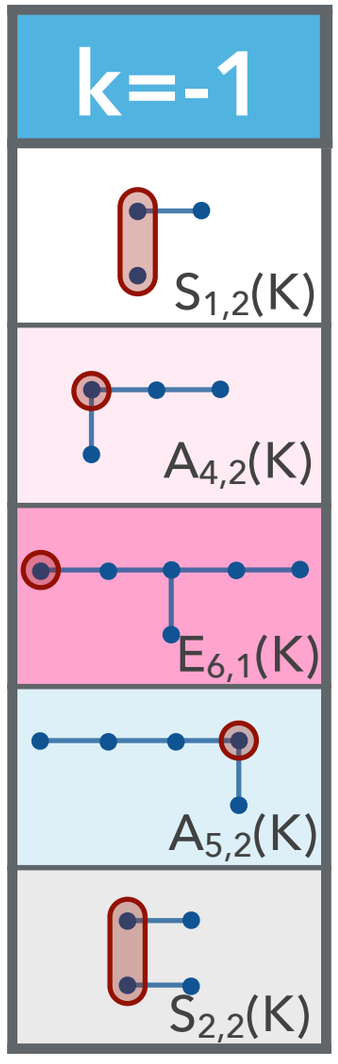

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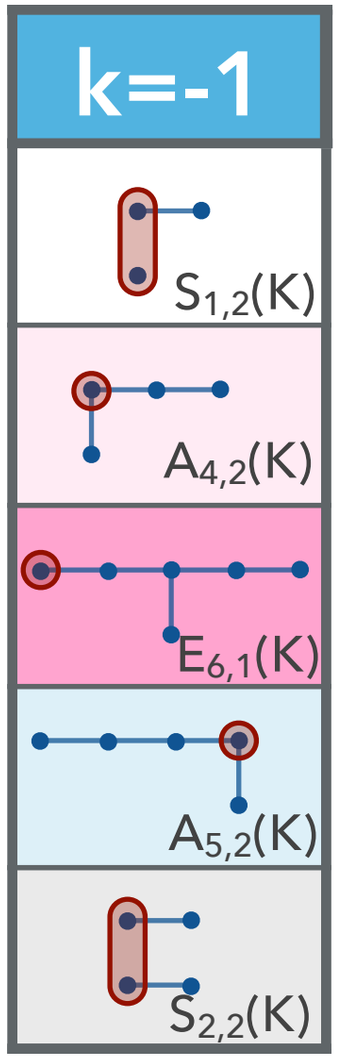

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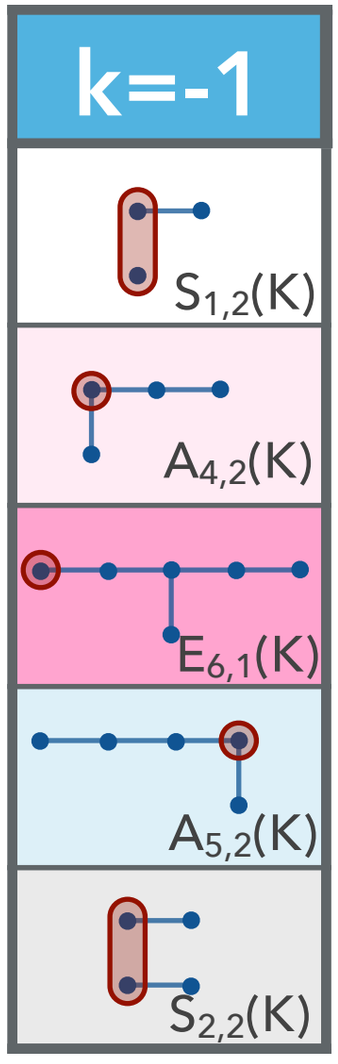

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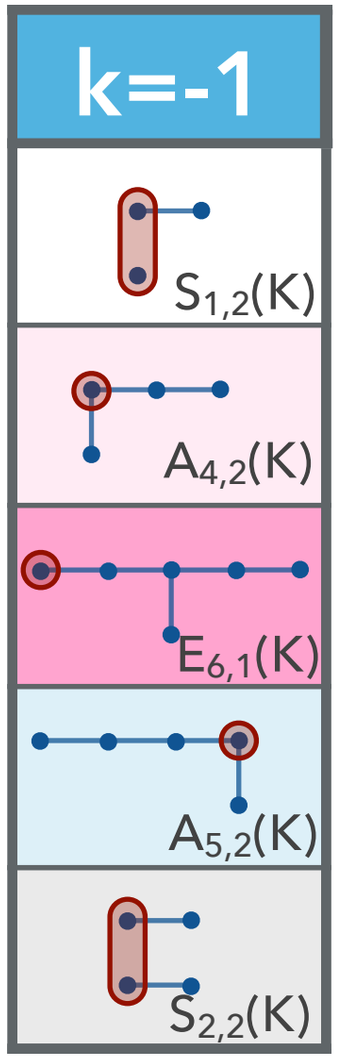

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Take **any** integer $k \geq 0$

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Each symp has rank at least k+3

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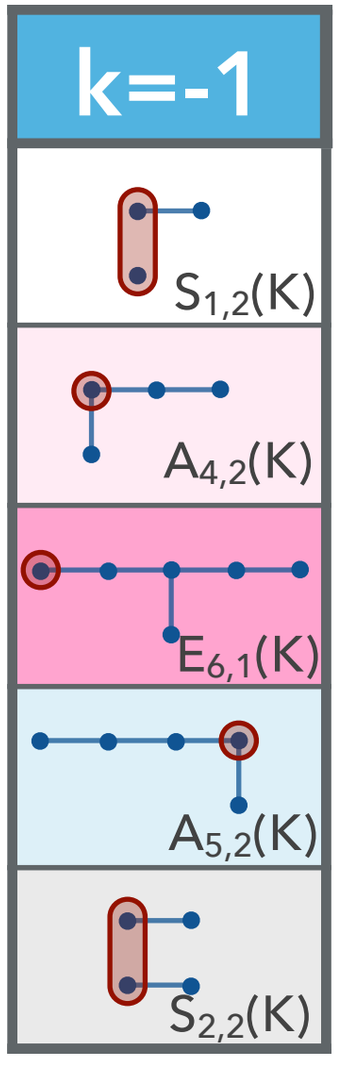

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locally connected

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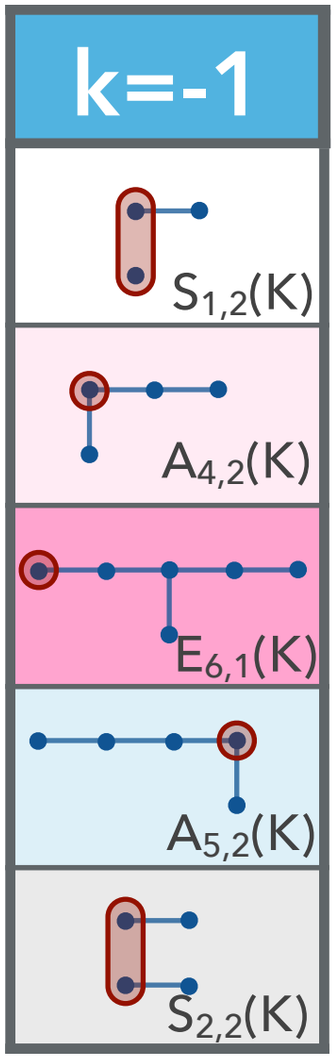

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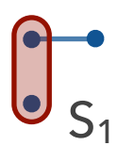
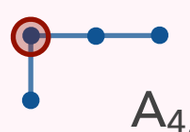
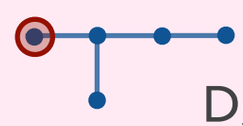
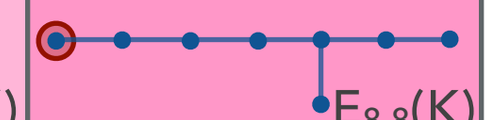
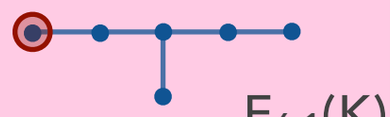
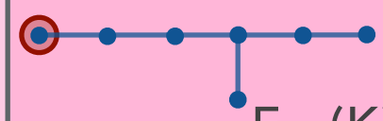
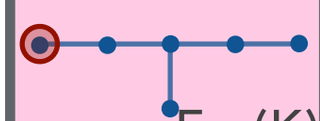
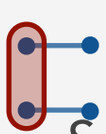
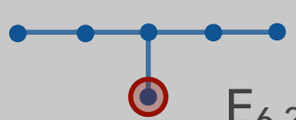
k=-1	0	1	2	3	4
 $S_{1,2}(K)$	 $A_{4,2}(K)$	 $D_{5,5}(K)$	 $E_{6,1}(K)$	 $E_{7,7}(K)$	 $E_{8,8}(K)$
 $A_{4,2}(K)$	 $D_{5,5}(K)$	 $E_{6,1}(K)$	 $E_{7,7}(K)$	 $E_{8,8}(K)$	
 $E_{6,1}(K)$	 $E_{7,7}(K)$	 $E_{8,8}(K)$			
 $A_{5,2}(K)$	 $D_{6,6}(K)$	 $E_{7,1}(K)$			
 $S_{2,2}(K)$	 $A_{5,3}(K)$	 $E_{6,2}(K)$			

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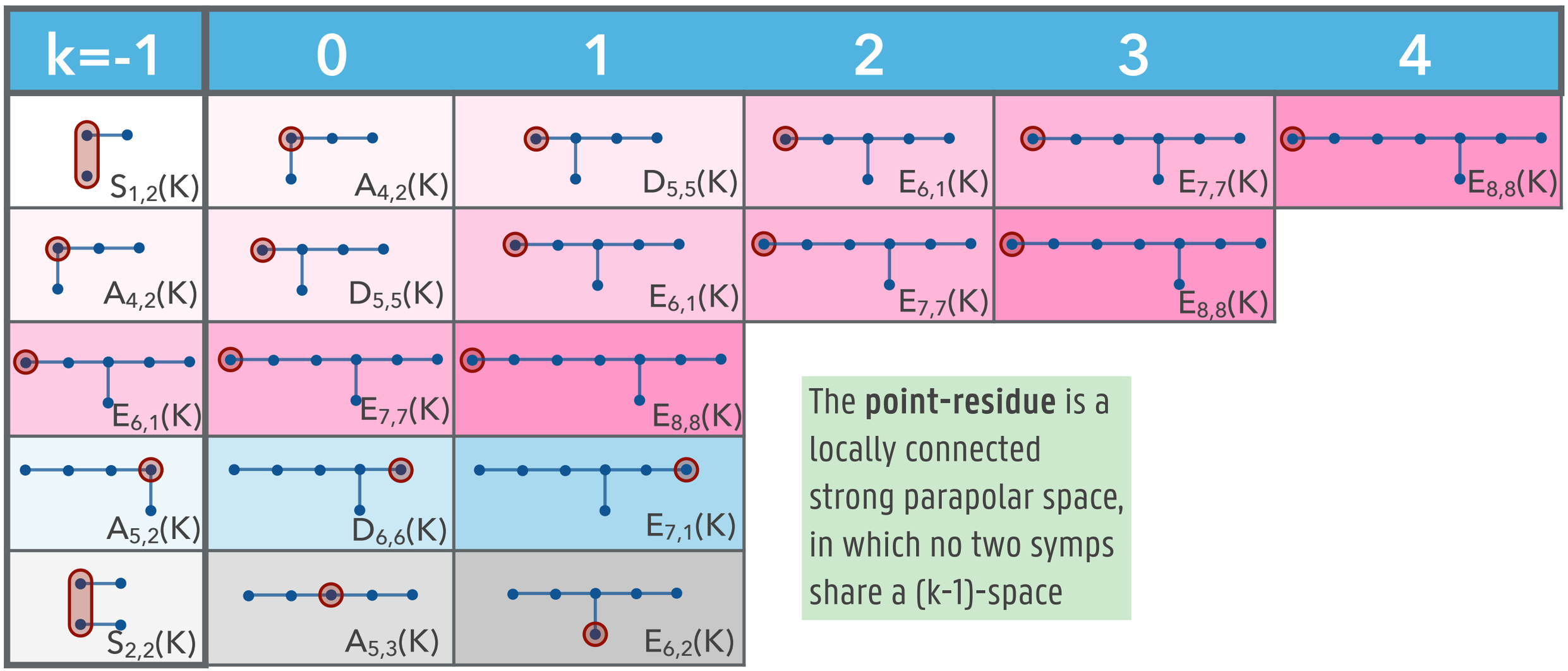

The **point-residue** is a locally connected strong parapolar space, in which no two symps share a (k-1)-space

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(not a real condition)

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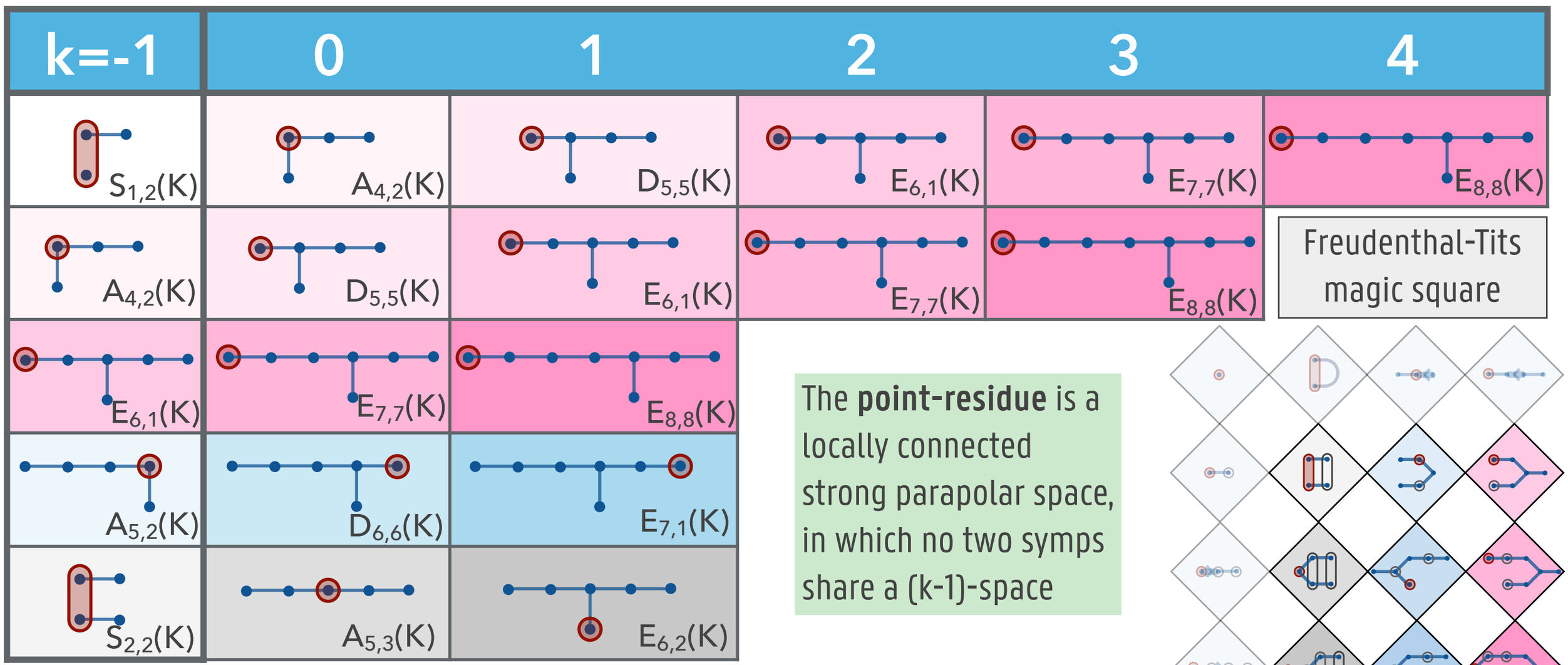

Freudenthal-Tits magic square

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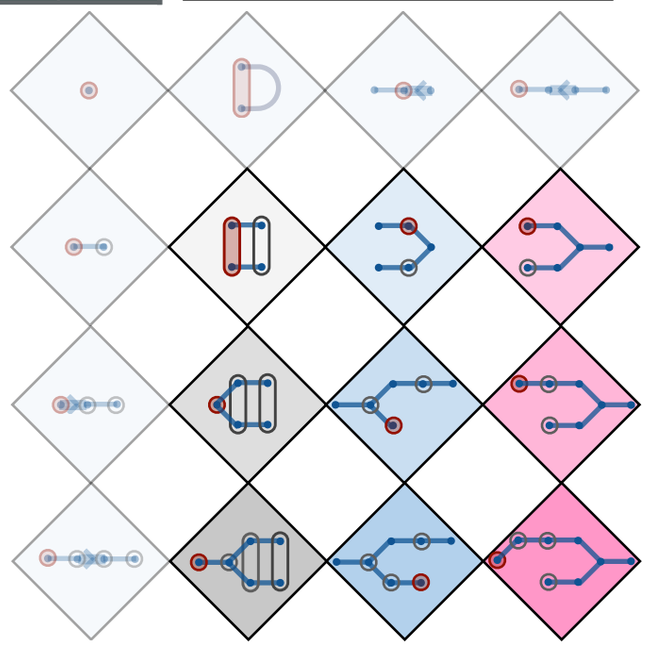

Thank you for your attention!