Additive MDS codes with linear projections

Sam Adriaensen

Vrije Universiteit Brussel, Belgium

(Joint work with Simeon Ball)

Let A be a set of size q. An $(n, M, d)_q$ *code* with *alphabet* A is a subset $\mathcal{C} \subseteq A^n$, such that $|\mathcal{C}| = M$, and for any two distinct vectors $\mathbf{x}, \mathbf{y} \in \mathcal{C}$, there are at least d coordinate positions in which $\mathbf{x}$ and $\mathbf{y}$ have different entries. The Singleton bound tells us that $M \leq q^{n-d+1}$, and codes attaining this bound are called *maximum distance separable*, or *MDS* for short. A central question in the study of MDS codes is what the largest value of n is, such that an $(n, q^{n-d+1}, d)_q$ MDS code exists, if d and q are fixed. The *MDS conjecture* poses that except for some known exceptions, nothing outperforms the Reed-Solomon codes.

In this talk, we are interested in MDS codes $\mathcal{C}$ whose alphabet is a finite field $\mathbb{F}_q$, and which are *additive*, i.e. $\mathbf{x}, \mathbf{y} \in \mathcal{C} \implies \mathbf{x}+\mathbf{y} \in \mathcal{C}$. If $p = \mathrm{char}(\mathbb{F}_q)$, then $\mathcal{C}$ is additive if and only if it is linear over the subfield $\mathbb{F}_p$ of $\mathbb{F}_q$. For this reason, we study the slightly larger framework of MDS codes over $\mathbb{F}_{q^h}$ which are linear over the subfield $\mathbb{F}_q$.

The main result of our work states essentially that if an $\mathbb{F}_q$-linear MDS code $\mathcal{C}$ over $\mathbb{F}_{q^h}$ is long enough and has some *projections* (also called *shortenings*) which are equivalent to linear codes, then $\mathcal{C}$ itself must be equivalent to a linear code [1]. This is a step towards reducing the additive MDS conjecture to the linear MDS conjecture.

It is worth noting that similar results have been obtained in a different context. There are certain point-line geometries called *generalised quadrangles*. A subclass of these (the translation generalised quadrangles with parameters (q^h, q^h)) are equivalent to additive $(q^h + 1, q^{3h}, q^h - 1)_{q^h}$ MDS codes. These generalised quadrangles have also been studied through their projections.

References

[1] S. Adriaensen, S. Ball. *On additive MDS codes with linear projections.* Finite Fields Appl. **91** (2023), Paper No. 102255, 25 pp.

Vrije Universiteit Brussel, Department of Mathematics and Data Science
Pleinlaan 2, 1050 Elsene, Belgium
sam.adriaensen@vub.be