

Saturating linear sets and the rank covering problem

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Saturating sets, interesting combinatorial structures in projective spaces over finite fields, have been a subject of extensive research over the past three decades. They are closely intertwined with the covering problem of codes in the Hamming metric. In this talk, we delve into the relatively recent concept of linear saturating sets, which are linked to the covering problem in the rank metric.

Our primary focus in this context revolves around two fundamental questions: how small can the rank of a saturating linear set be? And how can we construct saturating linear sets with minimal rank? In [3], we established a lower bound on the rank of saturating linear sets within a specific projective space, demonstrating its tightness in certain scenarios. Moreover, we showed an interesting relation with minimal codes in the rank metric, introduced in [1]. In [2], we showcased the construction of other saturating linear sets that meet this lower bound, shedding light on the connection between the saturating property and the scatteredness of linear sets.

In this talk, we aim to provide an introduction to this topic while also presenting potential developments and open questions.

References

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- [2] D. Bartoli, M. Borello, G. Marino. *Saturating linear sets of minimal rank*. Arxiv:2306.17081 (2023).
- [3] M. Bonini, M. Borello, E. Byrne. *Saturating systems and the rank-metric covering radius*. Journal of Algebraic Combinatorics (2023).

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