1. Can one find 4004 positive integers such that the sum of any 2003 of them is not divisible by 2003?

2. Let $\triangle ABC$ be a triangle with $AB \neq AC$, and let $D$ be the point where the tangent from $A$ to the circumcircle of $\triangle ABC$ meets $BC$. Consider the points $E, F$ which lie on the perpendiculars raised from $B$ and $C$ on $BC$ respectively, and on the perpendicular bisectors of $[AB]$, respectively $[AC]$. Prove that $D, E$ and $F$ are collinear.

3. Find all functions $f : \mathbb{Q} \to \mathbb{R}$ which fulfill the following conditions:
   a) $f(1) + 1 > 0$;
   b) $f(x + y) - x f(y) - y f(x) = f(x) f(y) - x - y + xy$, for all $x, y \in \mathbb{Q}$;
   c) $f(x) = 2 f(x + 1) + x + 2$, for every $x \in \mathbb{Q}$.

4. Let $ABCD$ be a rectangle of lengths $m, n$, made up of $m \times n$ unit squares, where $m, n$ are two odd and coprime positive integers. The main diagonal $AC$ intersects the unit squares in the points $A_1, A_2, \ldots, A_k$, where $k$ is a positive integer, $k \geq 2$, and $A_1 = A$, and $A_k = C$. Prove that

$$A_1A_2 - A_2A_3 + A_3A_4 - \cdots + (-1)^k A_{k-1}A_k = \frac{\sqrt{m^2 + n^2}}{mn}.$$